A brief introduction to the logic of relational propositions

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Abstract

The logic of relational propositions is a simple method for exploring the logical coherence of text. Any text analyzable using RST can be restated as a relational proposition, and any relational proposition can be restated as a logical expression. Exploring these expressions in accordance with accepted rules of inference shows that the underlying coherence of discourse has a basis in logic. This demonstrates that the approach can be used for the logical analysis of discourse. This paper provides a brief introduction to the topic.

1 Introduction

The logic of relational propositions is a simple method for exploring the logical coherence of text. This paper provides a brief overview of the topic. More complete treatments of this topic are available elsewhere (Potter, 2019, 2020, 2021, 2022). The method introduced here has its basis in *Rhetorical Structure Theory*, as defined by (Mann & Thompson, 1987, 1988). RST is a tool for describing and characterizing texts in terms of the relations that hold among the clauses comprising the text. Figure 1 shows an example of an RST analysis. This simple example is used throughout this short paper.

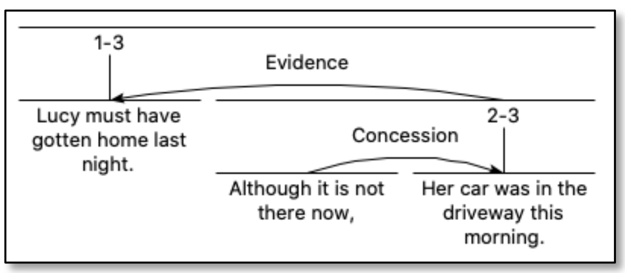

Figure 1: Example of an RST analysis

The example shows analysis of a text consisting of three sentences (sometimes called *units*). The CONCESSION and EVIDENCE relations show how these units relate to one another. Unit 2 is in a concessive relation with unit 3, and the span of units 2-3 provide evidence in support of unit 1. RST defines a rich set of relations for analyzing texts (e.g., ANTITHESIS, ELABORATION, ENABLEMENT, MOTIVATION, PURPOSE…).

Another key part of the method presented here is the theory of *relational propositions*. Relational propositions provide a propositional analog to RST structures, with relations being expressed as propositions. These propositions are implicit coherence-producing assertions occurring between clauses in a text and are essential to the effective functioning of the text (Mann & Thompson, 1986). A relational proposition consists of a relation (or predicate) and two variables, one of which corresponds to the RST satellite and the other to the nucleus. Complex relational propositions can be expressed using a predicate notation described in an earlier paper (Potter, 2019). Such complex expressions are referred to as nested relational propositions, since one or more propositions is nested, or enclosed, within another. This supports the representation of complex RST structures in compact functional form.

The final essential ingredient is *propositional logic*. Propositional logic is used to define these assertions as logical expressions constructed of propositions and logical operators. Units correspond to propositions, and the commonly used operators are

Negation	$\neg$
Conjunction	$\wedge$
Disjunction	$\vee$
Implication	$\rightarrow$
Equivalence	$\leftrightarrow$

In presenting this brief introduction, I have sought to avoid introducing anything new, but simply to explain established concepts as briefly and simply as possible.

2 A Simple Example

The logic of relational propositions is a method for analyzing the logical coherence of text. We start with a short text consisting of two sentences:

Lucy must have gotten home last night. Her car was in the driveway this morning.

For the purpose of analysis, we can assign numbers these individual sentences, or units, as follows:

1. *Lucy must have gotten home last night.*
2. *Her car was in the driveway this morning.*

The presence of Lucy's car in the driveway is offered as *evidence* that Lucy has gotten home from somewhere. So we say that the two sentences are related to one another using an *evidence* predicate. This corresponds to the RST EVIDENCE relation shown in Figure 1 and can be stated as a relational proposition:

evidence(2,1)

which means that unit 2 is *evidence* in support of 1. That is, if sentence 2 is accepted, then sentence 1 is also likely to be accepted. Another way to express that is

2 implies 1

But as stated, this is merely hypothetical: *if 2, then 1*. But our text actually provides more information than just the if-then statement. It does not say *if Lucy's car was in the driveway this morning, she Lucy must have gotten home last night.* It asserts that *her car was in the driveway this morning and therefore she must have gotten home last night*. That is,

2 implies 1
And 2
Therefore 1

In logical notation that would be

$(2 \rightarrow 1)$
1
$\therefore 2$

Or as a single linear expression:

$$(((2 \rightarrow 1) \wedge 2) \rightarrow 1)$$

So long as it is true that $(2 \rightarrow 1)$ and 2, it follows that 1. So the evidence predicate implements the *modus ponens* rule of inference.

For those familiar with logic, it might seem strange to be identifying the units numerically rather than with alphabetic symbols, as in common in propositional logic (p, q, r…). This practice is adopted in order to facilitate mapping from a segmented text to the RST analysis, from the RST analysis to the relational proposition, and from the relational proposition to the logical expression.

3 Extending the Example

Let's make the example a little more complicated:

1. *Lucy must have gotten home last night.*
2. *Although it is not there now,*
3. *Her car was in the driveway this morning.*

Intuitively, this reasoning makes sense. But the name of the game here is to boil it down to raw logic. We start we clauses 2 and 3

2. *Although it is not there now,*
3. *Her car was in the driveway this morning.*

These two clauses are related using the *concession* predicate. The relational proposition is:

concession(2,3)

To understand the logic of concession, it is helpful to consider the situation in which the *concession* is presented. In their paper on concessive relations, Thompson and Mann (1986, p. 441) wrote:

> Only in terms of its discourse context can we understand how concession is a 'conceding' of something: it concedes the potential incompatibility of two situations in order to forestall an objection that could interfere with the reader's belief of the point the writer wants to make.

The two potentially incompatible situations in our example are the presence of the car in the driveway this morning and its absence now. But the situations are not really incompatible. Just because the car is not there right now does not mean it was not there earlier this morning. That is to say, it is not the case that unit 2 implies the negation of unit 3:

$$\neg(2 \rightarrow \neg 3)$$

Having forestalled the potential incompatibility, we can infer the claim that the car was in fact in the driveway earlier:

$\neg(2 \rightarrow \neg 3) \rightarrow 3$
$\neg(2 \rightarrow \neg 3)$
$\therefore 3$

Or linearly:

$$(((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3)$$

and if the car was there earlier this morning, then Lucy must have gotten home last night. So the complex expression $(((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3)$ becomes evidence for the claim made in unit 1. The relational proposition is

evidence(concession(2,3),1)

and the corresponding logical expression is:

$((((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3) \rightarrow 1$
$(\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3$
$\therefore 1$

Or linearly,

$$((((((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3) \rightarrow 1) \land (((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3)) \rightarrow 1)$$

Of course it is always possible that something extratextual is going on. Maybe someone stole Lucy's car while she was on vacation, drove to her house, broke in, raided her refrigerator, slept on the living room couch, rose mid-morning, and drove away. Or maybe that wasn't Lucy's car after all. The possibilities are myriad. But what we are interested here is a simple method for logical analysis of the text. The text delimits the universe of discourse.

4 The Multiplicity of Boolean domains

In the foregoing example, the reasoning maps fairly neatly to our usual understanding of propositional logic as dealing exclusively with expressions that may be true or false. But a peculiarity of the logic of relational propositions is that it is not limited to this single Boolean domain. There are a multiplicity of Boolean domains reflecting the full range of rhetorical intentionalities specified by the relational predicates. The logic operates seamlessly across a range of intentionalities, such as belief and disbelief, action and inaction, acceptance and rejection, and comprehension and incomprehension, traversing from one inference to the next, as the writer develops an intended effect. As such, what might seem a peculiarity for logic is an ordinary feature in discursive reasoning. The logic of relational propositions provides a framework for subsuming these Boolean domains as a generalized Boolean data type. Suppose we extend the example once more:

1. *Lucy must have gotten home last night.*
2. *Although it is not there now,*
3. *Her car was in the driveway this morning.*
4. *So let's check in with her tonight.*

With this example, Lucy's return home presents the opportunity to pay her a visit. This brings another predicate into play.

enablement(evidence(concession(2,3),1),4)

The *enablement* predicate applies when the satellite of the predicate increases the reader's ability to perform the action specified in the nucleus. In this example, the satellite is *evidence(concession(2,3),1)* and the satellite is unit *4*. The logic follows the same model as *evidence*, but since the relational proposition is nested three predicates deep, the logical expression is complex.

$$((((((((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3) \rightarrow 1) \land (((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3)) \rightarrow 1) \rightarrow 4) \land ((((((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3) \rightarrow 1) \land (((\neg(2 \rightarrow \neg 3) \rightarrow 3) \land \neg(2 \rightarrow \neg 3)) \rightarrow 3)) \rightarrow 1)) \rightarrow 4)$$

The longer and more complex the text, the deeper the nesting, and the more complex the logical expression. That multiple Boolean domains should be subsumed under the generalized data type of intentional coherence may be unique, but when taken in perspective, this should not seem inordinate. Essentially, propositional logic is an application of the Boolean algebra. The theorems of propositional logic correspond to logical operations of the Boolean algebra, and elementary propositions are reducible to Boolean variables. The units of discourse are Boolean variables, not propositions. Boolean logic is more general and lends itself to a variety of applications, including the design of switching circuits, mathematics, set theory, digital logic, and database query languages. And to discursive reasoning.

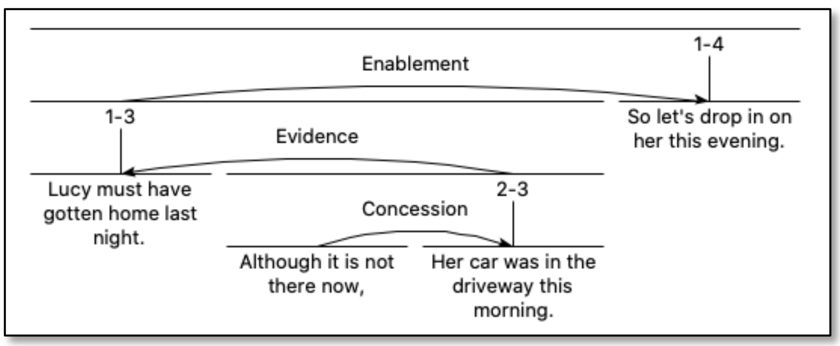

Figure 2: Completed example of an RST analysis

5 Conclusion

Any text analyzable using RST can be restated as a relational proposition, and any relational proposition can be restated as a logical expression. Exploring these expressions in accordance with accepted rules of inference shows that the underlying coherence of discourse has a basis in logic. This demonstrates that the approach can be used for the logical analysis of discourse. This in and of itself seems interesting.

But number of potential applications have been also mentioned. If integrated with computational methods for generating RST analyses (e.g., Corston-Oliver, 1998; Hernault, Prendinger, duVerle, & Ishizuka, 2010; Pardo, Nunes, & Rino, 2004; Soricut & Marcu, 2003), the method presented here could lead to useful tools for scalable analysis of large text collections. Some other potential uses include contributions to knowledge representation, automated reasoning, controlled natural languages, cross-document analysis (Cardoso, Jorge, & Pardo, 2015; Radev, 2000), and integration with research in semantic equivalence, entailment, and knowledge extraction (Androutsopoulos & Malakasiotis, 2010; Gangemi, 2013; Zhang & Patrick, 2005). Some areas for future study include development of a computational framework for the study of the logic of RST analyses, studies in the interrelationship between logic and rhetoric, a more in-depth look at multinuclear relations, and investigation of various genres of discourse using the methods described in this paper.

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