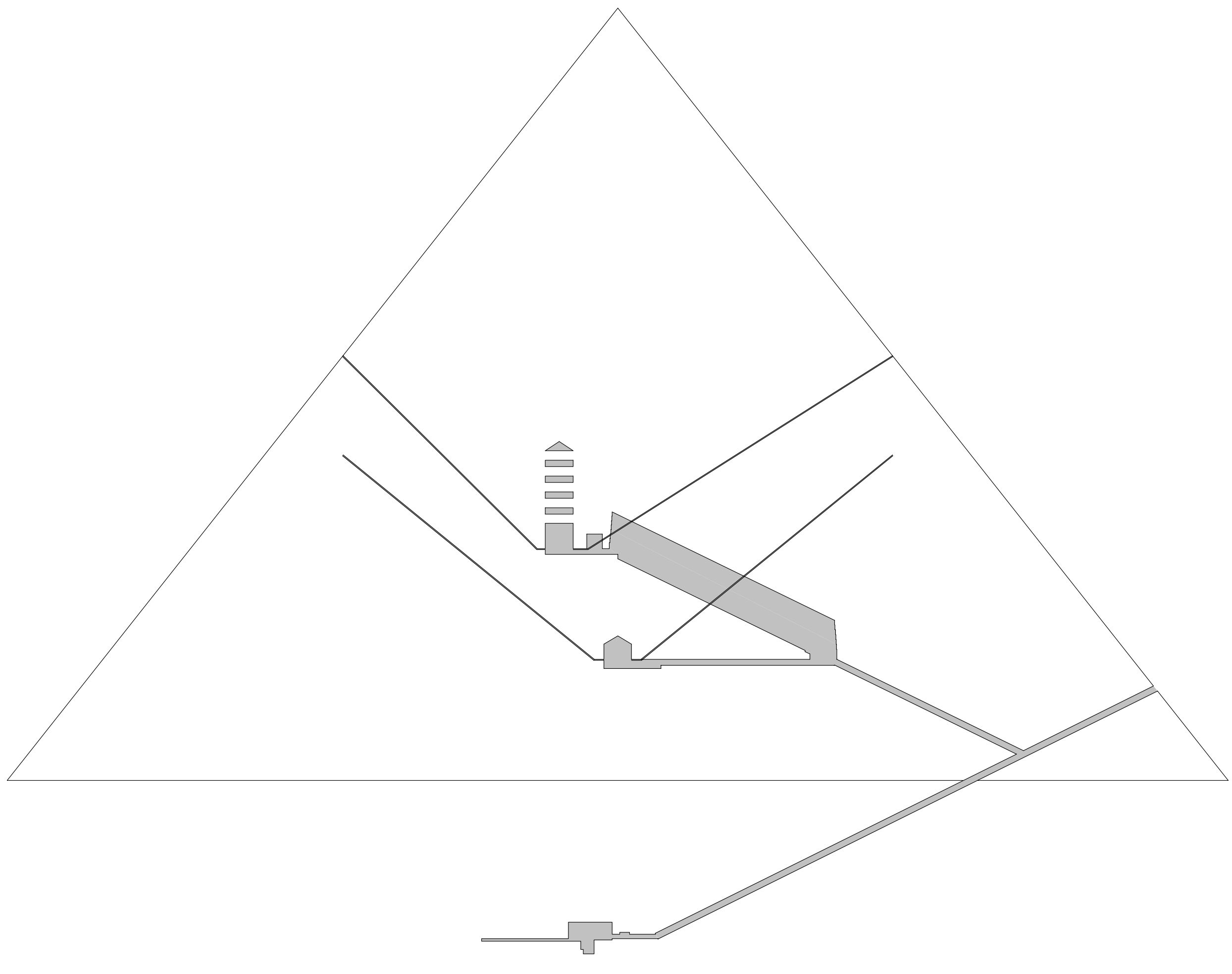
MEASURING THE GREAT PYRAMID

BY JIM ALISON

2019



Part One - The Entrance Passage

In *The Pyramids and Temples of Gizeh*, Flinders Petrie published his survey and commented on theories explaining the dimensions of the Great Pyramid. The total original length of the floor of the entrance passage is 4143 inches and the length of the roof of the passage is 4133 inches.[1] The height of the floor at the outer casing of the pyramid is 668.2 inches above ground level[2] and the height of the roof at the outer casing is 705.97 inches above ground level.[3] Petrie commented that "the angle of slope of the entrance passage is 1 rise on 2 base."[4]

"The old theory of 1 rise on 2 base, or an angle of 26º 33' 54"; is far within the variations of the entrance passage angle, and is very close to the observed angle of the whole passage, which is 26º 31' 23"; so close to it, that two or three inches on the length of 350 feet is the whole difference; so this theory may at least claim to be far more accurate than any other theory."[5]

Given the height of 705.97 inches above ground level for the roof at the outer casing of the pyramid, and given the slope of 1 rise on 2 base, the length of the above ground section of the roof is 1578.6 inches ($705.97 \times \sqrt{5}$). The phi (φ) proportion, $(\sqrt{5} + 1)/2$, also known as the golden section or the golden mean, is 1.618...

$1578.6 \times \varphi = 2554.2$ inches for the length of the below ground section of the roof of the passage. $1578.6 + 2554.2 = 4132.8$ inches for the total length of the roof. The ratio between the below ground section and the above ground section of the roof equals the ratio between the total length of the roof and the below ground section: $2554.2/1578.6 = 4132.8/2554.2 = 1.618$. In 2007 I wrote an article pointing out the φ proportion in the roof of the passage.[6]

Michael Saunders responded to my article by pointing out the π proportion in the floor.[7] Given the height of 668.2 inches above ground level for the floor at the outer casing of the pyramid, the length of the above ground section of the floor is 1494.14 inches ($668.2 \times \sqrt{5}$). The square root of π is 1.7724.

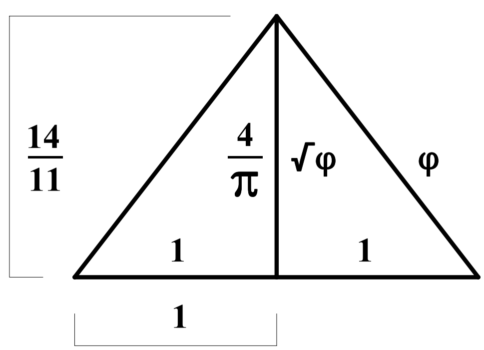
$1494.14 \times \sqrt{\pi} = 2648.3$ inches for the length of the below ground section of the floor of the passage. $1494.14 + 2648.3 = 4142.4$ inches for the total length of the floor. A circle with a radius equal to the above ground section of the floor has the same area as the square of the below ground section of the floor.

φ and π proportions are also associated with theories of the exterior dimensions of the pyramid. The mean base length of the sides of the pyramid at ground level is 9068.8 inches.[8] Petrie estimated a height of 5776.0 ±7.0 inches, based on measurements of the angle of the faces of the pyramid.[9] The ratio between a height of 5776 inches and a half base of 4534.4 inches is 1.2738.

One theory is that the ratio between the height and the half base is 14/11, or 1.2727. A half base of 4534.4 inches times 14/11 gives 5771 inches for the height of the pyramid. This theory generally claims a height of 280 ancient Egyptian cubits and a base length of 440 cubits. 9068.8 inches divided by 440 gives a cubit of 20.61 inches, in close accord with the length of the cubit obtained from the King's Chamber, the Queen's Chamber and the entrance passage.

Another theory of the exterior dimensions of the pyramid is that the area of each face of the pyramid is equal to the square area of the height of the pyramid. This requires a ratio between the height and the half base of $\sqrt{\varphi}/1$, or 1.272. A half base of 4534.4 inches times 1.272 gives 5768 inches for the height of the pyramid.

The theory of the π proportion is that the height of the pyramid as the radius of a circle produces a circumference equal to the perimeter of the pyramid. This requires a ratio between the height and the half base of $4/\pi$, or 1.2732. A half base of 4534.4 inches times 1.2732 gives 5773.3 inches for the height of the pyramid.



$14/11 \cong 4/\pi \cong \sqrt{\varphi}$

Diagram 1

Of these three proportions, $4/\pi$ is closest to the observed pyramid slope. Petrie commented that "For the whole form the π proportion (height is the radius of a circle = circumference of Pyramid) has been very generally accepted of late years, and is a relation strongly confirmed by the presence of the numbers 7 and 22 in the number of cubits in height and base respectively; 7 : 22 being one of the best known approximations to π. With these numbers (or some slight fractional correction on the 22) the designer adopted 7 of a length of 20 double cubits for the height; and 22 of this length for the half-circuit. The profile used for the work being thus 14 rise on 11 base." [10]

φ and π proportions are also associated with theories of the dimensions of the King's Chamber. The chamber is 20 cubits long and 10 cubits wide. The horizontal diagonal of the chamber is 22.36 cubits ($\sqrt{5} \times 10$). The walls are comprised of five courses of equal height, but the floor of the chamber is inserted between the walls, and the top of the floor is approximately one-quarter of a cubit above the base of the first course of the walls. As a result, there are two wall heights for the chamber. The height from the top of the floor and the height from the base of the first course of the walls. The height from the top of the floor to the ceiling is 11.18 cubits, equal to one-half of the diagonal of the chamber, or $\sqrt{5} \times 5$, or $10\varphi - 5$. The diagonal of the short side of the chamber from the floor to the ceiling is 15 cubits and the diagonal across the chamber from the floor to the ceiling is 25 cubits (the hypotenuse of a 3-4-5 right triangle with sides of 15 and 20 cubits). The volume of the chamber from the floor to the ceiling is 2236 cubits or $\sqrt{5} \times 1000$.

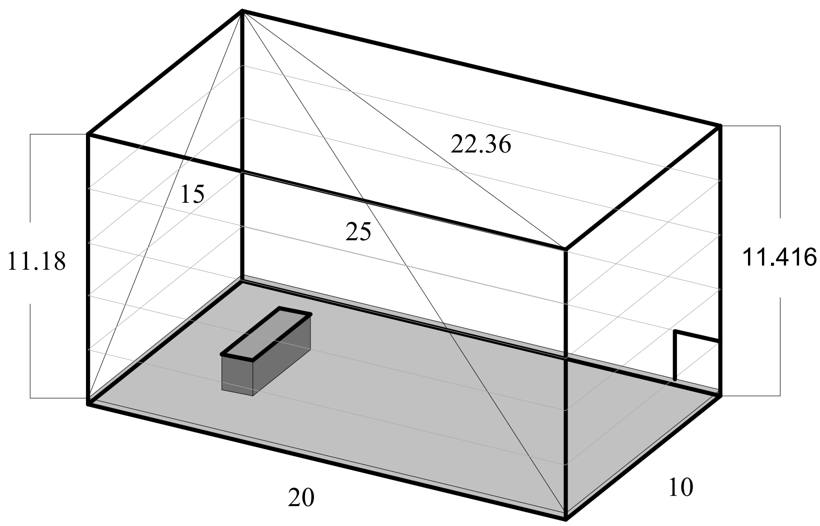


Diagram 2

According to Petrie's survey "the average variation of the thickness of the [King's Chamber] courses from their mean is .051 inches, the mean being 47.045 inches between similar joints."[11] The total height of the five courses is 235.2 ± .06.[12] "The theory of the height of the walls [from the base of the first course to the ceiling] is similar to one of the best theories of the outside of the Pyramid; it asserts that taking the circuit of the N. or S. walls, that will be equal to the circumference of a circle whose radius is the breadth of the chamber at right angles to those walls, or whose diameter is the length of those walls. Now by the mean original dimensions of the chamber the side walls are 412.25 inches long, and the ends 206.13, exactly half the amount. Taking, then, either of these as the basis of a diameter or radius of a circle, the wall height, if the sides are the circumference of such circle, will be 235.32 ±.10 [235.32/5 = 47.06 inch course height], and this only varies from the measured amount within the small range of the probable errors. This theory leaves nothing to be desired, therefore, on the score of accuracy, and its consonance with the theory of the Pyramid form, strongly bears it out."[13]

Perpendicular measurements from the floor to the roof of the inclined passages in the pyramid vary from 46.2 inches to 48.6 inches.[14] Petrie measured a perpendicular height of 47.26 inches for the entrance passage near the damaged present exterior of the pyramid, approximately 100 inches inside of the original outer casing.[15] Petrie comments that the perpendicular height of the entrance passage is “the same as the very carefully wrought courses of the King's Chamber, with which the passage is clearly intended to be identical.”[16] “In considering any theory of the height of the entrance passage, it can not be separated from the similar passages, or from the most accurately wrought of all such heights, the course height of the King's Chamber.”[17]

“From the entrances of the Third pyramid, the south pyramid of Dahshur, and the pyramid of Medum, all of which retain their casing, there seemed scarcely a question but that the rule was for the doorway of a pyramid to occupy the height of exactly one or two courses on the outside. That the casing courses were on the same levels as the present core courses is not to be doubted, as they are so in the other pyramids which retain their casing, and at the foot of the Great pyramid.”[18]

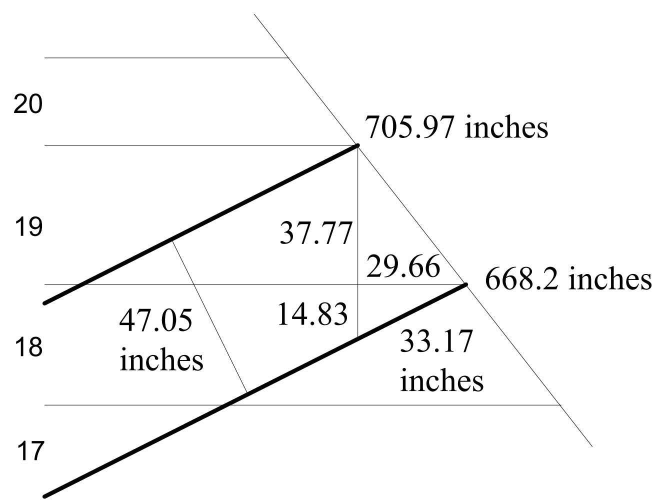


Diagram 3

The entrance passage exited the finished outer casing of the pyramid at the 19th course of masonry.[19] The bottom of the 19th course is 668.2 inches above ground level[20] and the top of the 19th course is 705.97 inches above ground level,[21] giving 37.77 inches for the height of the 19th course. Given a slope of 1.2732 ($4/\pi$) for the original outer casing of the pyramid, the horizontal distance from the floor of the passage at the outer casing to a point vertically below the roof of the passage at the outer casing is 29.66 inches (37.77/1.2732 = 29.66). Given the passage slope of one over two, the vertical distance from this point to the floor of the passage is 14.83 inches (29.66/2 = 14.83). This gives a vertical height of 52.60 inches (37.77 + 14.83) from the floor to the roof of the passage. Given the one over two slope of the passage, the perpendicular height of the passage is 47.05 inches ($52.60 \times 2/\sqrt{5} = 47.05$).

Petrie states that "The entrance passage has a flat end, square with its axis (within at least 1°), and out of this end a smaller horizontal passage proceeds, leaving a margin of the flat end along the top and two sides. This margin is 4.5 wide at E., 3.2 at W., and 5.4 to 6.0 from E. to W. along the top."[22] The length of the end of the passage is the same as the perpendicular height of the passage (47.05 inches). Given the one over two slope of the passage, the vertical height of the end of the passage is 42.08 inches (47.05 × 2/√5).

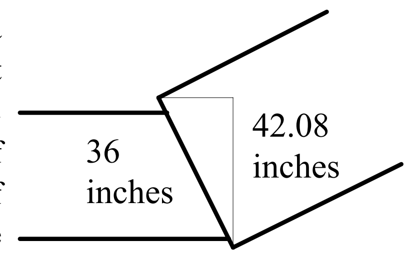


Diagram 4

Adam Rutherford reported that there is also a margin between the bottom of the passage and the floor of the smaller horizontal passage and reported a height of 35.7 inches and a width of 33.5 inches for the horizontal passage.[23] At the northern entrance to the subterranean chamber, Petrie reported a height of 35.5 - 36 inches and a width of 32.0 - 33.3 inches for the horizontal passage.[24] 33.3/φ = 20.61 inches and 35.7/√3 = 20.61 inches.

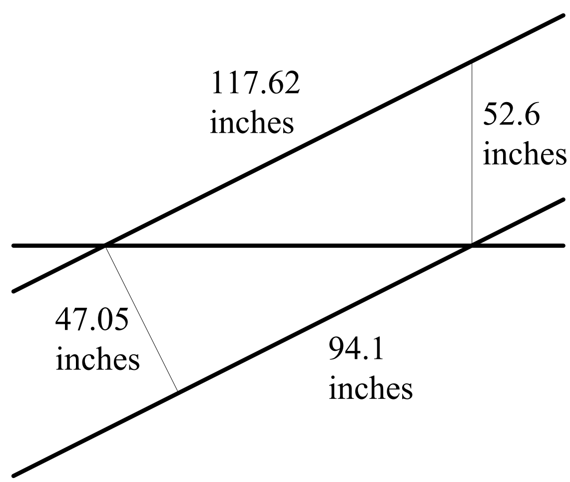


Diagram 5

Diagram 5 shows the entrance passage at ground level. Because the lower end is perpendicular to the passage (diagram 4), the section of the floor that is below ground level is 94.1 inches longer than the section of the roof that is below ground level (47.05 × 2 = 94.1). The floor continues for 33.17 inches to the outer casing from the point on the floor vertically beneath the roof at the outer casing (diagram 3). The roof continues for 117.62 inches to ground level from the point on the roof vertically above the floor at ground level (52.6 × √5 = 117.62). The section of the roof that is above ground level is 84.45 inches longer than the section of the floor above ground level (117.62 − 33.17 = 84.45). Given that the floor below ground level is 94.1 inches longer than the roof below ground level, and given that the roof above ground level is 84.45 inches longer than the floor above ground level, the total length of the floor is 9.65 inches longer than the total length of the roof. This is in accordance with Petrie's measurements of 4143 inches for the floor and 4133 inches for the roof.

Given the perpendicular height of the passage of 47.05 inches, the perpendicular lower end, the one over two slope of the passage and the exterior angle of the pyramid, an equation will provide the beginning heights above ground level and the total lengths of the floor and the roof that are required for both the φ and π proportions to be present. If the above ground length of the floor is $\boldsymbol{x}$, then the below ground length of the floor is $\sqrt{\pi}\,\boldsymbol{x}$.

The above ground length of the roof is 84.45 inches longer than the above ground length of the floor (diagrams 3 and 5). If the above ground length of the floor is $\boldsymbol{x}$, the above ground length of the roof is $\boldsymbol{x}$ plus 84.45 inches. The below ground length of the roof is $\varphi\,\boldsymbol{x}$ plus $\varphi \times 84.45$ inches. 84.45 inches times φ equals 136.64 inches, so the below ground length of the roof is $\varphi\,\boldsymbol{x}$ plus 136.64 inches. The below ground length of the roof is 94.1 inches shorter than the below ground length of the floor (diagram 5). The below ground length of the floor ($\sqrt{\pi}\,\boldsymbol{x}$) equals the below ground length of the roof ($\varphi\,\boldsymbol{x}$ plus 136.64 inches) plus 94.1 inches:

$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 136.64$ inches $+ 94.1$ inches

$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 230.74$ inches

$\sqrt{\pi}\,\boldsymbol{x} = 1.7724\,\boldsymbol{x}$

$\varphi\,\boldsymbol{x} = 1.6180\,\boldsymbol{x}$

$1.7724\,\boldsymbol{x} = 1.6180\,\boldsymbol{x} + 230.74$ inches

$1.7724\,\boldsymbol{x} - 1.6180\,\boldsymbol{x} = 230.74$ inches

$1.7724 - 1.6180 = .1544$

$.1544\,\boldsymbol{x} = 230.74$ inches

$\boldsymbol{x} = 230.74$ inches/.1544

$\boldsymbol{x} = 1494.24$ inches (the above ground length of the floor of the passage).

$1494.24 \times \sqrt{\pi} = 2648.47$ inches (below ground length of the floor).

$1494.24 + 2648.47 = 4142.7$ inches (total length of the floor).

$1494.24/\sqrt{5} = 668.24$ inches (height of the floor at the outer casing of the pyramid).

$1494.24 + 84.45 = 1578.69$ (above ground length of the roof of the passage).

$1578.69 \times \varphi = 2554.37$ (below ground length of the roof).

$1578.69 + 2553.92 = 4133.06$ inches (total length of the roof).

$1578.41/\sqrt{5} = 706.01$ inches (height of the roof at the outer casing of the pyramid).

The King's Chamber is 10 cubits wide and 20 cubits long. Petrie gives the following measures for the lengths of the sides of the King's Chamber at the top, the mean and the base: [25]

	North wall	East wall	South wall	West wall		
Top	412.14	206.30	411.88	206.04	=	1235.36 / 6 = 205.89
Mean	412.40	206.29	412.11	205.97	=	1236.77 / 6 = 206.13
Base	412.78	206.43	412.53	206.16	=	1237.90 / 6 = 206.32

Piazzi Smyth measured the length of each of the first three courses and also published the Aiton-Inglis measures of the upper two courses of the chamber: [26]

	North wall	East wall	South wall	West wall		
5th course	412.0	205.9	412.0	205.9	=	1235.8 / 6 = 205.96
4th course	411.9	205.9	411.9	205.8	=	1235.5 / 6 = 205.91
3rd course	411.9	206.2	411.6	206.0	=	1235.7 / 6 = 205.95
2nd course	412.6	206.1	412.2	206.4	=	1237.3 / 6 = 206.22
1st course	413.3	206.2	412.8	206.4	=	1238.7 / 6 = 206.45

The mean of all three surveys for all five courses is 20.61 inches per cubit. Both surveys show that the length of the base of the chamber is greater than the other four courses. Petrie gave more weight to the length of the chamber at the base, commenting that "probably the base of the chamber was the part most carefully adjusted and set out" [27] The walls, the floor and the ceiling of the King's chamber are made of large blocks of granite. The blocks in the floor of the chamber were inserted into and rose above the bottom of the first course. The length of the chamber at the base may be slightly greater to insure that the flooring blocks fit inside the base of the walls.

Given the 20 cubit wall length of the King's Chamber and the wall length as a diameter giving a circumference equal to the perimeter of the wall, the length of the perimeter of the wall is 20π cubits. Subtracting the 40 cubit length of the base and the ceiling leaves $20\pi - 40$ for both wall heights, or $10\pi - 20$ for the wall height on each side, or $2\pi - 4$ for the height of each course of the walls. $2\pi - 4 = 2.283$ cubits. Given a height of 47.05 inches for each course, 47.05/2.283 = 20.607 inches/cubit.

The measured width of the all of the passages in the pyramid, including the entrance passage, varies from 40.6 to 42.6 inches.[28] The intended width of the passages is 2 cubits, or 41.21 inches. The perimeter of the entrance passage is twice the $2\pi - 4$ height of the walls plus the 2 cubit width of the floor and the roof, or $4\pi - 4$. The perimeter of the short walls of the King's Chamber is twice the $10\pi - 20$ height of the walls plus the 10 cubit width of the base and the roof, or $20\pi - 20$. The short walls of the King's Chamber have the same proportion, five times larger, than the entrance passage. The area of the short walls of the chamber is 25 times the area of the perpendicular section of the entrance passage.

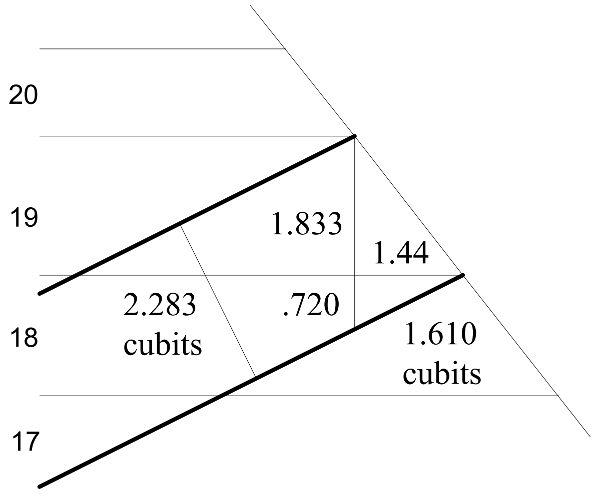


Diagram 6

The perpendicular height of the entrance passage equals the course height of the walls of the King's Chamber, or $2\pi - 4 = 2.283$ cubits. The vertical height of the entrance passage from the floor to the roof of the passage at the outer casing is $2.283 \times \sqrt{5}/2 = 2.553$ cubits. The slope of the pyramid is $4/\pi$ or 1.273. The ratio between the vertical height from the roof and the horizontal distance to the floor at the outer casing is 1.273/1 or 1.833 cubits/1.44 cubits. The ratio between the vertical height above the floor at the outer casing and the vertical height below the floor at the outer casing is 1.273/.5 or 1.833 cubits/.720 cubits. The vertical height of the passage is $1.833 + .720 = 2.553$ cubits. The length of the floor of the entrance passage from the outer casing to the vertical line from the roof is 1.610 cubits ($.720 \times \sqrt{5}$).

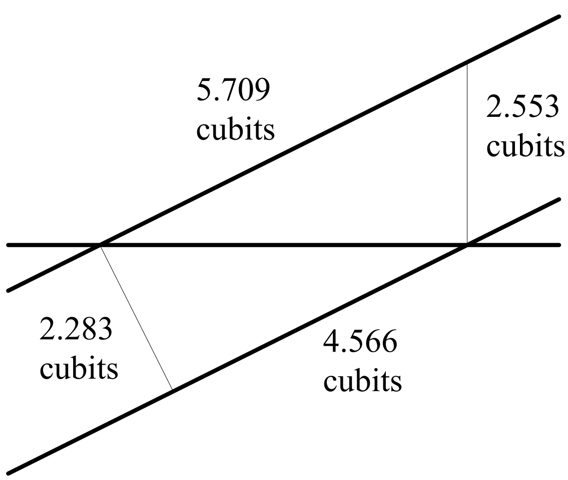


Diagram 7

Diagram 7 shows the entrance passage at ground level. Because the lower end of the passage is perpendicular to the passage (diagram 4), the section of the floor that is below ground level is 4.566 cubits longer than the section of the roof that is below ground level ($2.283 \times 2 = 4.566$).

The floor continues for 1.610 cubits to the outer casing from the point on the floor vertically beneath the roof at the outer casing (diagram 6). The roof continues for 5.709 cubits to ground level from the point on the roof vertically above the floor at ground level ($2.553 \times \sqrt{5} = 5.709$). The section of the roof that is above ground level is 4.099 cubits longer than the section of the floor above ground level ($5.709 - 1.610 = 4.099$).

Given the perpendicular height of the passage of 2.283 cubits, the perpendicular lower end, the one over two slope of the passage and the exterior angle of the pyramid, the same equation will provide the beginning heights above ground level in cubits and the total lengths of the floor and the roof in cubits that are required for both the φ and π proportions to be present. If the above ground length of the floor is $\boldsymbol{x}$, then the below ground length of the floor is $\sqrt{\pi}\,\boldsymbol{x}$.

The above ground length of the roof is 4.099 cubits longer than the above ground length of the floor (diagrams 6 and 7). If the above ground length of the floor is $\boldsymbol{x}$, the above ground length of the roof is $\boldsymbol{x}$ plus 4.099 cubits. The below ground length of the roof is $\varphi\,\boldsymbol{x}$ plus $\varphi \times 4.099$ cubits. 4.099 cubits times φ equals 6.632 cubits, so the below ground length of the roof is $\varphi\,\boldsymbol{x}$ plus 6.632 cubits. The below ground length of the roof is 4.566 cubits shorter than the below ground length of the floor (diagram 7). The below ground length of the floor ($\sqrt{\pi}\,\boldsymbol{x}$) equals the below ground length of the roof ($\varphi\,\boldsymbol{x}$ plus 6.632 cubits) plus 4.566 cubits:

$$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 6.632 \text{ cubits} + 4.566 \text{ cubits}$$

$$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 11.198 \text{ cubits}$$

$$\sqrt{\pi}\,\boldsymbol{x} = 1.7724\,\boldsymbol{x}$$

$$\varphi\,\boldsymbol{x} = 1.6180\,\boldsymbol{x}$$

$$1.7724\,\boldsymbol{x} = 1.6180\,\boldsymbol{x} + 11.198 \text{ cubits}$$

$$1.7724\,\boldsymbol{x} - 1.6180\,\boldsymbol{x} = 11.198 \text{ cubits}$$

$$1.7724 - 1.6180 = .1544$$

$$.1544\,\boldsymbol{x} = 11.198 \text{ cubits}$$

$$\boldsymbol{x} = 11.198 \text{ cubits}/.1544$$

$\boldsymbol{x} = 72.516$ (the above ground length of the floor of the passage).

$72.516 \times \sqrt{\pi} = 128.531$ cubits (below ground length of the floor).

$72.516 + 128.531 = 201.041$ cubits (total length of the floor).

$72.516/\sqrt{5} =$ **32.43** cubits (height of the floor at the outer casing of the pyramid).

$72.516 + 4.099 = 76.615$ (above ground length of the roof of the passage).

$76.615 \times \varphi = 123.966$ (below ground length of the roof).

$76.615 + 123.966 = 200.482$ cubits (total length of the roof).

$76.615/\sqrt{5} =$ **34.26** cubits (height of the roof at the outer casing of the pyramid).

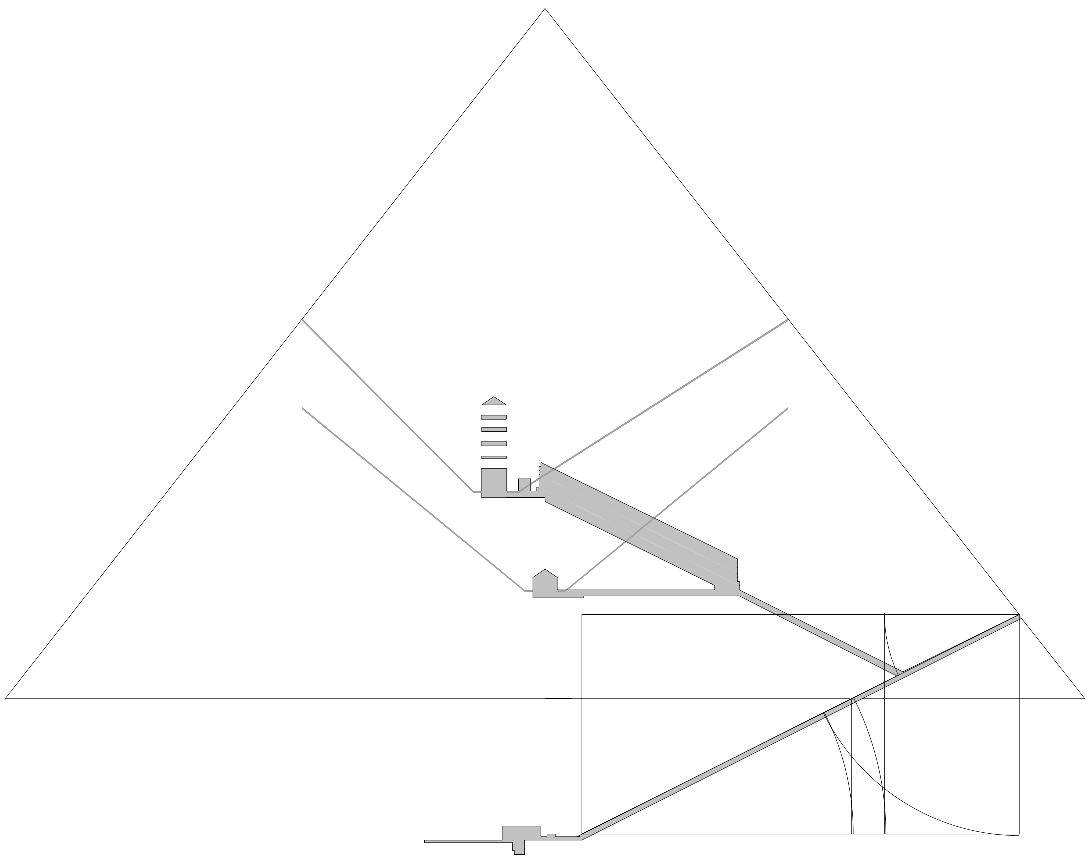


Diagram 8

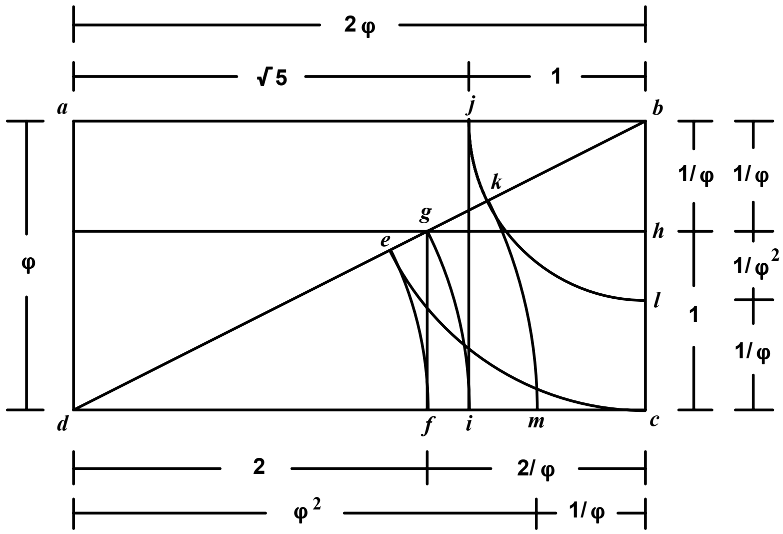


Diagram 9

abcd is a double square. By arcing *bc* to *e* and arcing *de* to *f*, *cd* is divided into a golden section by *f*; *bd* is divided into a golden section by the projection of *f* to *g*; and *bc* is divided into a golden section by the projection of *g* to *h*. $ch/bh = df/cf = dg/bg = \varphi$. *gh* marks ground level of the great pyramid. *bd*, the diagonal of the double square, is the axis of the entrance passage. *dg* is arced to *i*, *i* is projected to *j*, and *bj* is arced to *l*. *k* divides the axis of the entrance passage into the ratio of φ^2 to one ($dk/bk = \varphi^2$). *k* marks the intersection of the axis of the entrance passage with the projection of the floor of the ascending passage. *k* is the axis point in the following diagrams. The length of 1 in diagram 9 equals 55 cubits in the great pyramid. $\varphi = 88.99$ cubits, $\varphi^2 = 143.99$ cubits, $1/\varphi = 33.99$ cubits, $1/\varphi^2 = 21.01$ cubits, $\sqrt{\varphi} = 69.96$ cubits and $\sqrt{\varphi} \times \sqrt{2} = 98.94$ cubits.

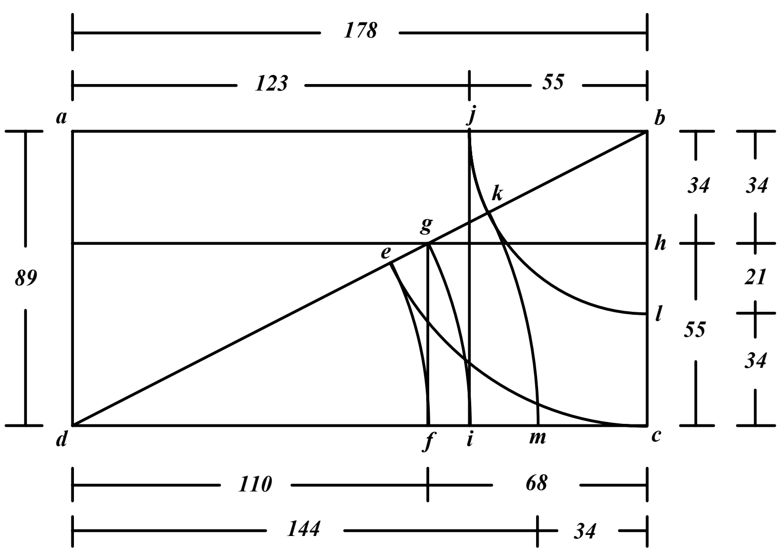


Diagram 10

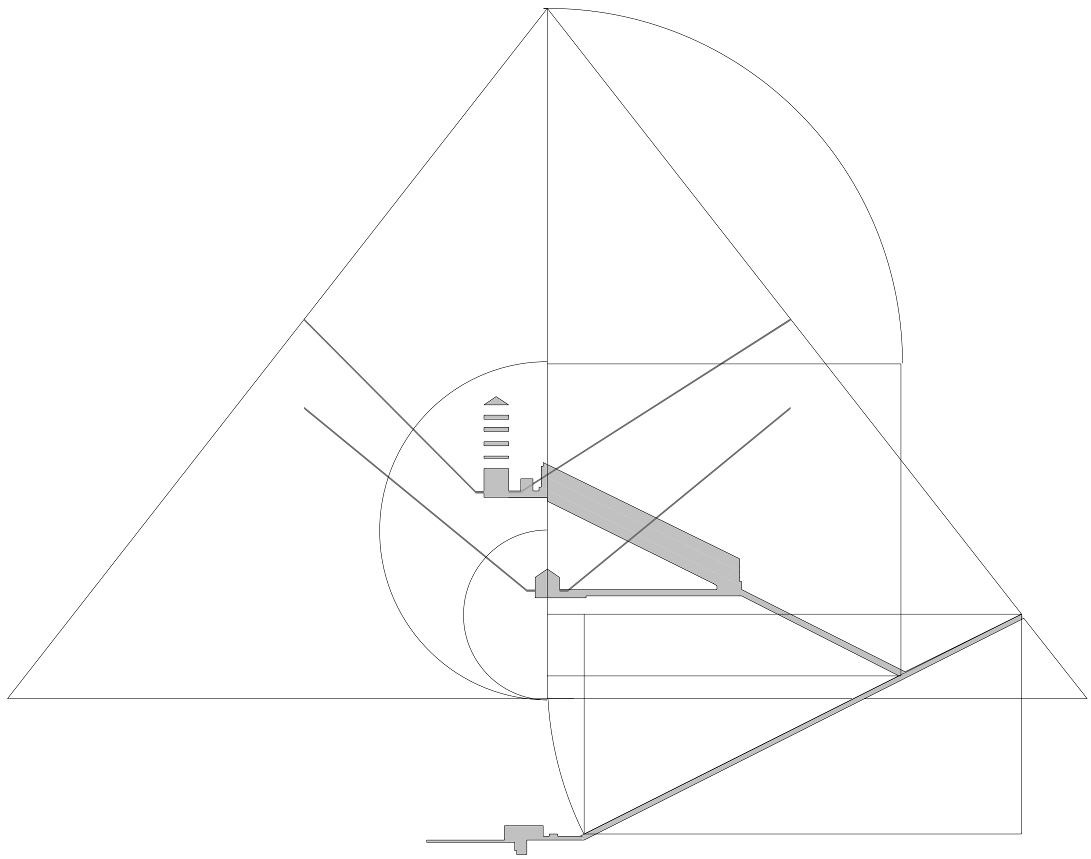


Diagram 11

The sloping length from the axis point to the end of the axis of the entrance passage is 143.99 cubits. The axis point is also 143.99 cubits north of the vertical midline of the pyramid. Arcing the length from the axis point to the end of the axis of the entrance passage, to the horizontal projection of the axis point, marks the vertical midline of the pyramid. The axis of the entrance passage exits the pyramid 33.99 cubits above ground level. This height times four (33.99×4 = 135.97), plus the 143.99 cubit length from the axis point to the vertical midline of the pyramid, marks the 279.96 height of the pyramid, as shown in diagram 11. The height of the pyramid, up from the height the axis of the entrance passage exits the pyramid, is 33.99×3 plus 143.99 = 245.96 cubits. The horizontal distance from the point the axis of the entrance passage exits the pyramid to the vertical midline of the pyramid is 193.18 cubits (given the 1 over 2 rise of the entrance passage, the sloping distance of 55 cubits from the axis point to the outer casing produces a length of 55 ×2/√5, or 49.19 cubits for the horizontal distance from the axis of the entrance passage at the outer casing, to a point vertically above the axis point. 49.19 plus 143.99 =193.18. 245.96/193.18 = 1.2732 for the pyramid slope. 4/1.2732 = 3.1416 and $6/5\varphi^2$ = 3.1416. A line from the top of the pyramid, to the point the axis of the entrance passage exits the pyramid, extended to ground level, produces a baselength for the pyramid at ground level of 219.89 cubits (279.96/1.2732 = 219.89).

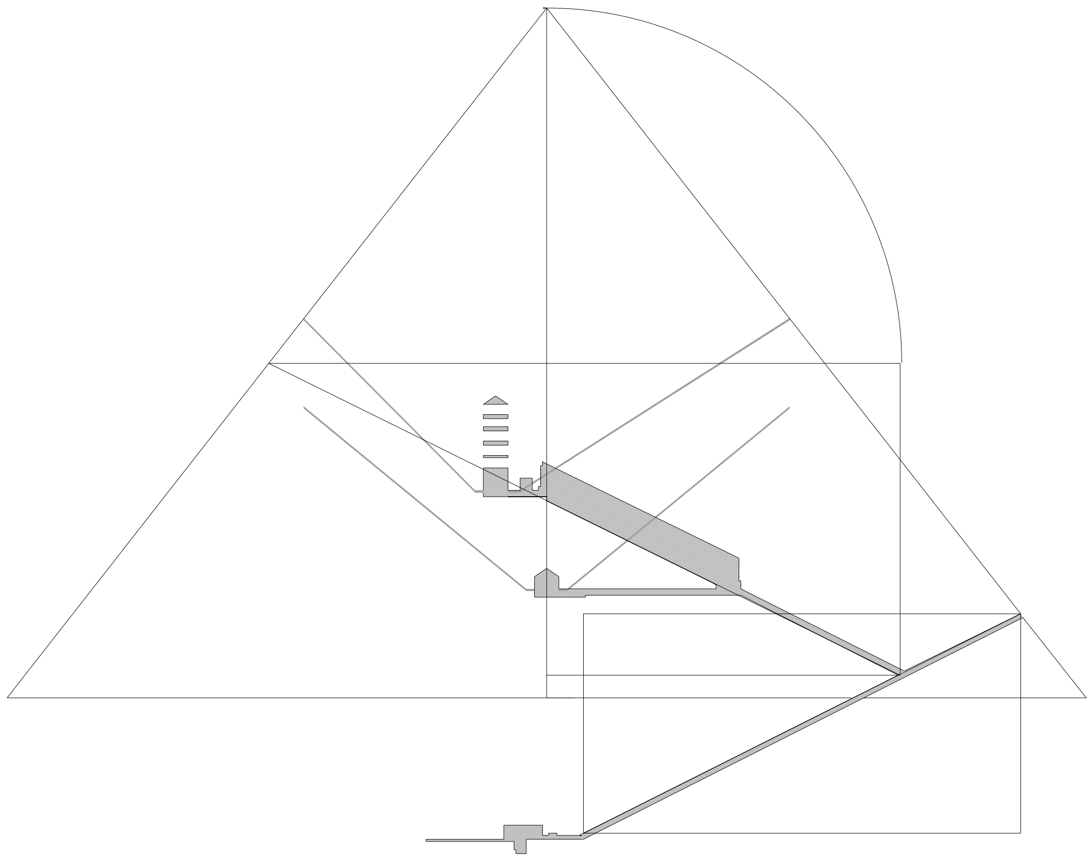


Diagram 12

The horizontal line 135.97 cubits above ground level is extended to the southern edge of the pyramid. A line from this point to the axis point marks the floor of the ascending passage and the grand gallery. According to Petrie's survey, 26º 12' 50" is the "mean angle of both passage and gallery together," [1] and "the angle 26º 12' 50" (by which the end of the gallery was calculated from the plug-blocks)." [2] The φ^2 division of the axis of the entrance passage in the constructions above produces an angle of 26° 12' 47" for the passage and the gallery. The axis point is 55 cubits from the edge of the pyramid. The 1/2 slope of the passage means the vertical distance from the axis point to the exit point of the axis is 24.59 cubits ($55/\sqrt{5}$). The axis point is 9.4 cubits above ground level (33.99-24.59). The projection of the ascending passage exits the pyramid 135.97 cubits above ground level and143.99 cubits below the top of the pyramid. The horizontal distance from the ascending passage exit point to the midline of the pyramid is 143.99 divided by the 1.2732 slope of the pyramid. Because the horizontal distance from the axis point to the midline of the pyramid is also 143.99 cubits, the projection of the ascending passage from the axis point to the edge of the pyramid is divided by the midline of the pyramid in the same proportion of 1/1.2732. The height of the passage from the axis point to the edge of the pyramid is 126.57 cubits (135.97 minus 9.4). The passage is 80.29 cubits above ground level at the midline of the pyramid ($126.57 \times 1.2732/2.2732 = 70.89$ and $70.89 + 9.4 = 80.29$ cubits). The slope of the passage and gallery is 70.89/143.99 or 1/2031.

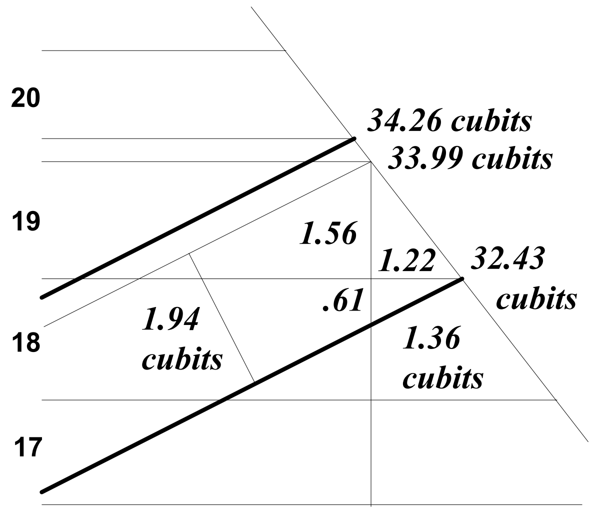


Diagram 13

The height above ground level of the beginning of the roof of the entrance passage is fixed by the perpendicular height of the passage and the φ and √π ratios in the roof and the floor. The beginning of the axis of the entrance passage is .27 cubits below the roof at the outer casing of the pyramid. Like the roof, the axis of the entrance passage is divided into a golden section by ground level.

The floor of the passage at the outer casing is 32.43 cubits above ground level and the axis of the entrance passage is 33.99 cubits above ground level at the casing. The vertical height from the floor at the outer casing to the axis at the casing is 1.56 cubits (33.99 − 32.43).

Given the pyramid slope of 1.2732 (4/π), the horizontal distance from the floor of the passage at the casing to a point vertically below the axis of the passage at the casing is 1.22 cubits (1.56/1.2732 = 1.22). Given the passage slope of one over two, the vertical distance from this point to the floor of the passage is .61 cubits (1.22/2 = .61). The length of the floor of the passage from the casing to the point vertically below the axis at the outer casing is 1.36 cubits (.61 × √5).

The vertical height from axis to floor is 2.17 cubits (1.57 + .61). Given the one over two slope of the passage, the perpendicular height from the axis to the floor is 1.94 cubits (2.17 × 2/√5).

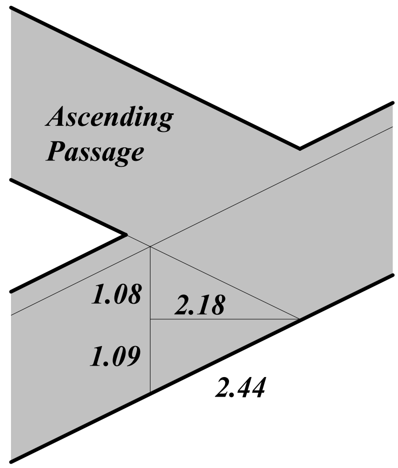


Diagram 14

In diagram 14 the floor of the ascending passage is projected to the floor of the entrance passage. The slope of the ascending passage is 1/2.03. The sides of the right triangle formed by the axis of the entrance passage and the projection of the floor of the ascending passage have the same 1/2.03 ratio. The slope of the entrance passage is one over two. The vertical side of the right triangle formed by the floor of the entrance passage is ½ the length of the horizontal side, or 1.015 times the vertical side of the right triangle formed by the projection of the ascending passage floor. The vertical height from the axis to the floor of the entrance passage is 2.17 cubits. 2.17/2.015 = 1.08. 2.17×1.015/2.015 = 1.09. The length of the floor of the entrance passage from the intersection point of the floors to the point vertically below the intersection of the ascending passage and the axis is 2.44 cubits (1.09 × $\sqrt{5}$).

The length of the axis of the entrance passage from the outer casing to the intersection with the projection of the ascending passage is 55 cubits. The length along the floor of the entrance passage from the outer casing to the intersection of the floor of the entrance passage with the projection of the floor of the ascending passage is 53.91 cubits (55+ 1.36 − 2.44 = 53.92). Petrie measured 1110.64 inches along the floor of the entrance passage from the outer casing to the intersection of the floor of the entrance passage with the projection of the floor of the ascending passage.[3] 1110.64 inches divided by 53.92 cubits equals 20.6 inches per cubit.

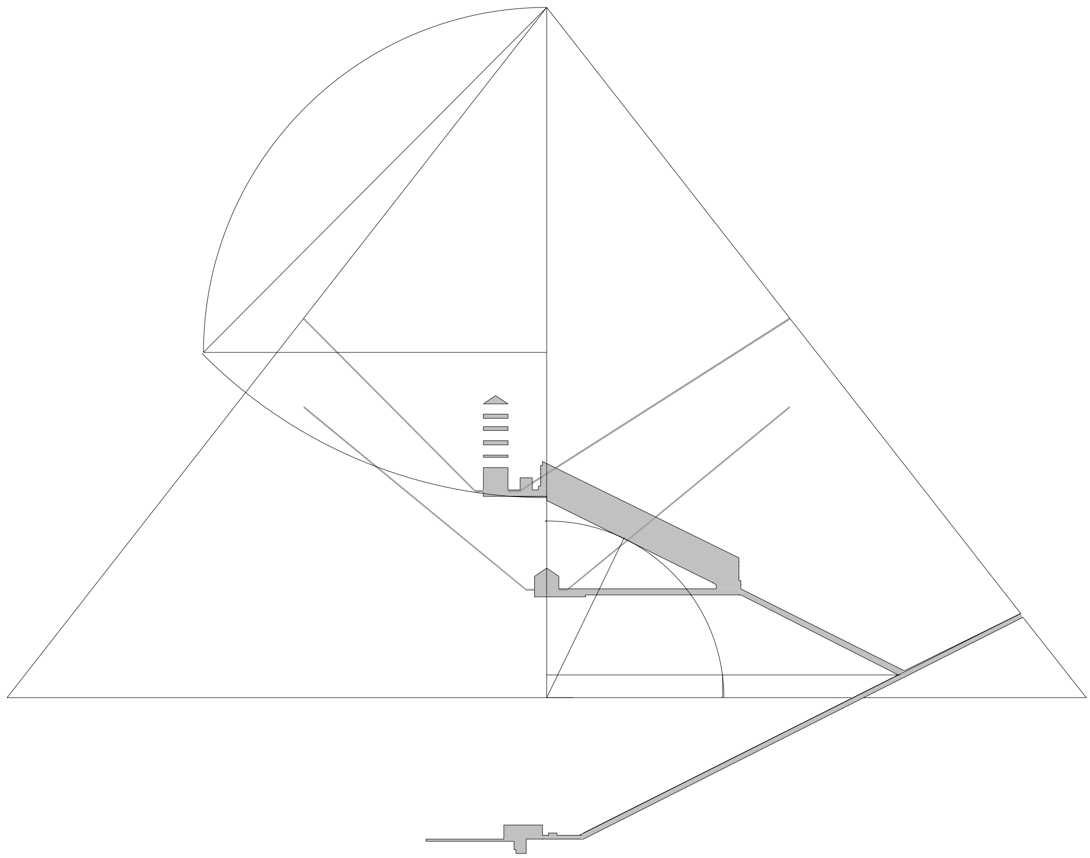


Diagram 15

Petrie concluded that "the floor of the King's Chamber was placed at the level where the vertical section of the Pyramid was halved, where the area of the horizontal section was half that of the base, where the diagonal from corner to corner was equal to the length of the base, and where the width of the face was equal to half the diagonal of the base." [4] All of these relationships are produced by the construction above. Given 279.96 cubits for the height of the pyramid, one half of the height is 139.98 cubits. From this height at the vertical midline, the apex is arced to a horizontal line at the height of 139.98 cubits above ground level. The length of a line from this intersection to the apex of the pyramid is 197.96 cubits (139.98 $\times\sqrt{2}$). This line is arced from the apex to the vertical midline of the pyramid. The 279.96 cubit height of the pyramid minus 197.96 cubits leaves 82.00 cubits above ground level for the height of the floor of the King's Chamber and the King's Chamber passage. The floor of the gallery is 80.29 cubits above ground level at the midline of the pyramid (diagram 12). The floor of the King's Chamber passage is 82 cubits above ground level. Petrie measured 34.9 to 35.8 inches from the floor of the gallery to the floor of the King's Chamber passage at the midline of the pyramid.[5] 1.71 cubits × 20.6 inches per cubit = 35.23 inches.

The horizontal distance from the axis point to the midline of the pyramid is 143.992 cubits. One half of this distance is 72.00 cubits. This point is extended to ground level, and from ground level at the midline of the pyramid, this point is arced to the midline, 72 cubits above ground level.

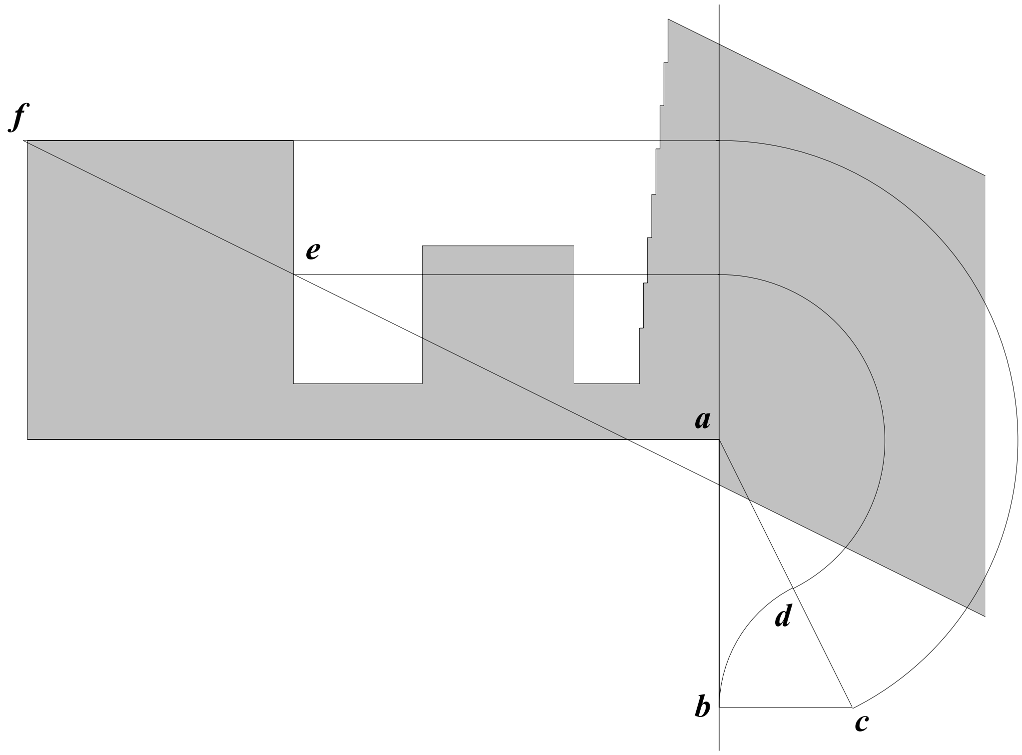


Diagram 16

b is 10 cubits below the floor of the King's Chamber passageway at the vertical midline of the pyramid. *bc* is one-half of the length of *ab*. *ad* is 6.18 cubits ($1/\varphi \times 10$). *ad* is arced to the vertical midline of the pyramid and extended horizontally to the projection of the floor of the gallery at *e*, 6.18 cubits above the floor of the King's Chamber passageway. The floor of the King's Chamber passageway is 1.71 cubits above the intersection of the floor of the gallery at the vertical midline of the pyramid. 1.71 + 6.18 = 7.89 cubits for the height of *e* above the intersection of the floor of the gallery at the vertical midline. The slope of the projection of the floor of the gallery is 1/2.031. *e* is 16.025 cubits south of the vertical midline of the pyramid (7.89 × 2.031 = 16.025). Petrie[6] and Smyth[7] both reported a length of 330.3 inches from the edge of the great step to the end of the King's Chamber passage. 330.3/16.025 = 20.61 inches per cubit.

ac is 11.18 cubits ($\sqrt{5} \times 5$). *ac* is arced to the vertical midline of the pyramid, marking the height of the ceiling of the King's Chamber, 11.18 cubits above the floor. Petrie reported a mean height of 230.09 inches from the floor to the ceiling of the King's Chamber.[8] 230.09 inches/11.18 cubits = 20.58 inches per cubit. *f* marks the intersection of the projection of the floor of the gallery with the height of the ceiling of the King's Chamber. *f* is 5 cubits above *e* (11.18 − 6.18 = 5). *f* is 10.155 cubits south of *e* (5 × 2.031 = 10.155). The horizontal distance from *f* to the vertical midline of the pyramid is 26.18 cubits, or $\varphi^2 \times 10$ (16.025 + 10.155 = 26.18).

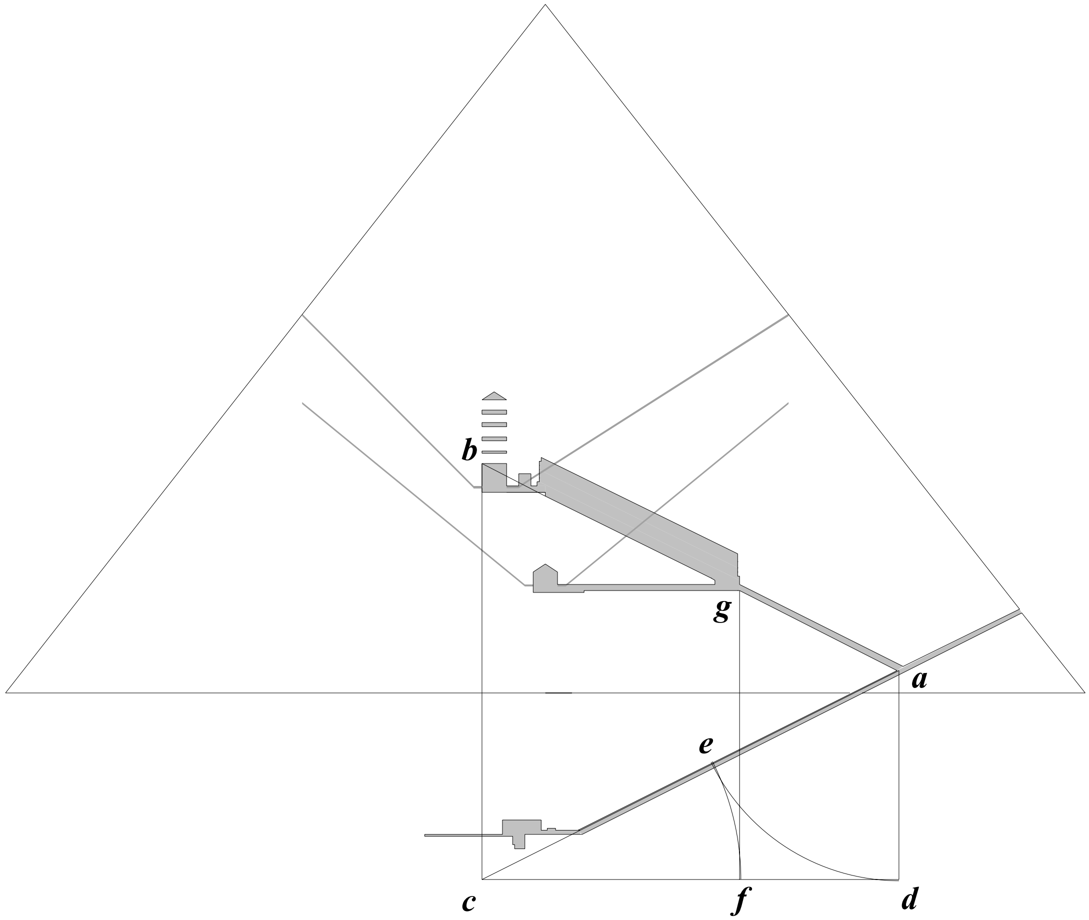


Diagram 17

The horizontal distance from the axis point to the vertical midline is 143.99 cubits. The horizontal distance from *b* (the intersection of the projection of the ascending passage with the height of the ceiling of the King's Chamber) to the vertical midline is 26.18 cubits. *cd* is 170.17 cubits (143.99 + 26.18). *ad* is arced to *e* and *ce* is arced to *f*. *f* divides *cd* into a golden section. *g* divides *ab* into a golden section. *g* marks the end of the ascending passage at the north wall of the gallery. The length of *df* is 65.00 cubits ($170.17/\varphi^2$). The horizontal distance from the midline of the pyramid to *g* is 78.99 cubits (143.99 − 65.00). Petrie calculated 1627 inches from the vertical midline to the end of the ascending passage.[9] 1627/78.99 = 20.6 inches per cubit.

The ceiling of the King's Chamber is 93.18 cubits above ground level (82 + 11.18) . *a* is 9.4 cubits above ground level (diagram 12). The height from *a* to *b* is 83.78 cubits (93.18 − 9.4). The division of the height from *a* to *b* into a golden section marks the end of the ascending passage at the north wall of the gallery. $83.78/\varphi^2 = 32.00$ cubits. 9.4 + 32 = 41.4 cubits above ground level. Petrie calculated a vertical distance of 852.6 inches from ground level to the end of the ascending passage at the north wall of the gallery.[10] 852.6/41.4 = 20.6 inches per cubit. The horizontal and vertical lengths from *a* to *g* give the slope of the ascending passage in whole cubits (65/32 = 2.031).

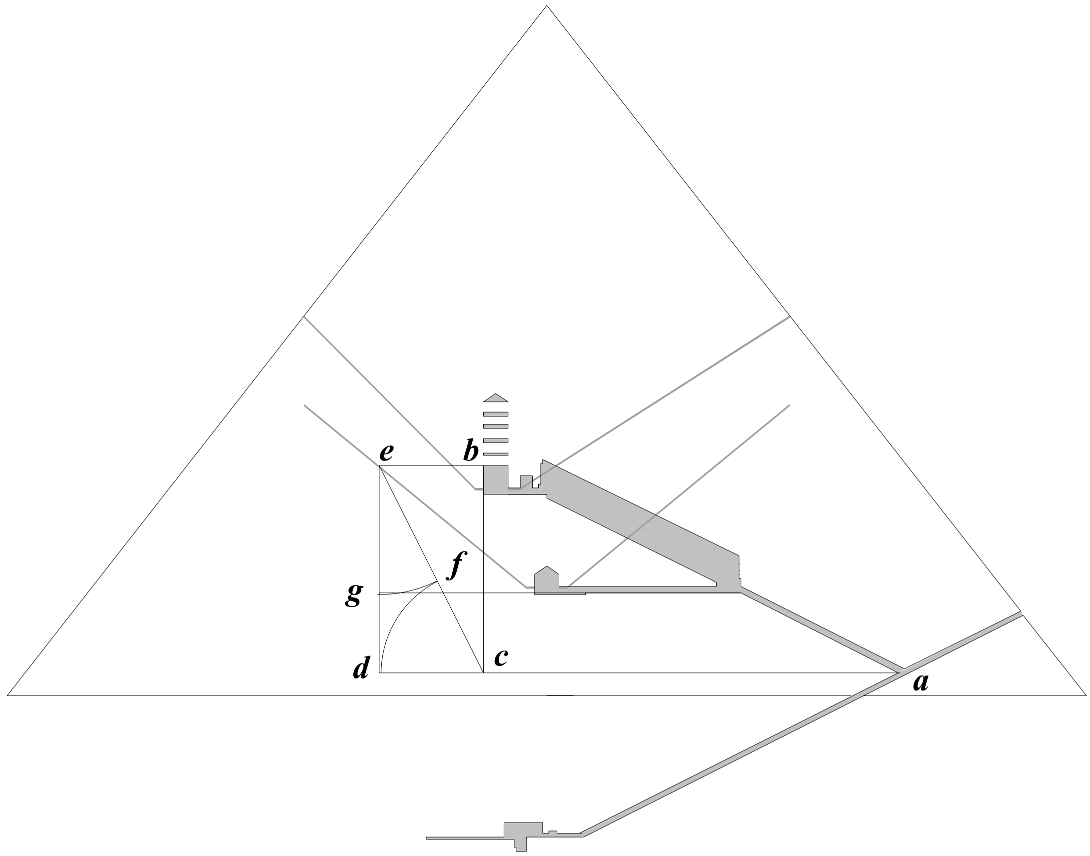


Diagram 18

The beginning of the floor of the gallery is 41.4 cubits above ground level and 78.99 cubits from the midline of the pyramid. The end of the floor is 80.29 cubits above ground level at the midline of the pyramid. The rise of the floor is 38.89 cubits (80.29 − 41.4). The length of the floor of the gallery is 88.04 cubits ($78.99^2 + 38.88^2 = 88.04^2$). Petrie measured 1815 inches for the length of the floor of the gallery, from the north wall to the vertical midline of the pyramid.[11] 1815/88.04 = 20.62 inches per cubit.

The intended height of the floor of the Queen's Chamber and the queen's chamber passage is uncertain because the floor blocks in the chamber and the last 15 cubits of the passage are missing and the floor blocks for the rest of the passage are rough and unfinished. The ceiling height varies from 903.8 to 901.0 inches above ground level and the floor height varies from 858.4 to 854.6 inches above ground level to the point where the flooring blocks are missing.[12] The height of the apex of the chamber is 1078.7 inches above ground level[13] (1078.7/20.6 = 52.36 or $\varphi^2 \times 20$). The top of the north and south walls of the chamber are 60.63 inches below the apex, or 1018.07 inches above ground level[14] (1018.07/20.6 = 49.4 cubits). If the intended height of the floor of the chamber was 41.4 cubits then the north and south walls would be 8 cubits high, the area of the north and south walls would be 88 square cubits, and the volume of the chamber to the top of the north and south walls would be 880 cubic cubits.

The below ground sections of the vertical sides of the double square are 55 cubits, the above ground sections are 55/φ and the length of the axis of the entrance passage from the lower end of the axis to the intersection of the axis with the ascending passage is $55 \times \varphi^2 = 143.99$. The horizontal distance from the midline of the pyramid to the intersection of the axis with the ascending passage is also $55 \times \varphi^2$. The horizontal distance from the south side of the double square to the intersection of the axis with the ascending passage is $55 \times \varphi^2 \times 2/\sqrt{5} = 128.79$. The horizontal distance from the south side of the double square to the vertical midline of the pyramid is 15.20 cubits (143.99 − 128.79).

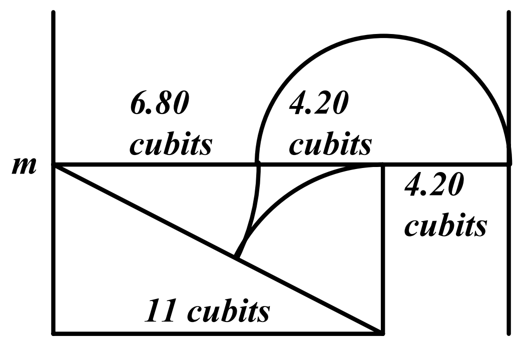


Diagram 19

m is the vertical midline of the pyramid at ground level. The length of 11 cubits is divided into golden sections: $11/\varphi = 6.80$ and $11/\varphi^2 = 4{,}20$. $11 + 4.20 = 15.20$ cubits, marking the south side of the double square.

$$1 + 1/\varphi^2 = 5\varphi^2 - (5\varphi^2 \times 2/\sqrt{5})$$

$$11 + 4.20 = 15.20$$

$$143.99 - 128.79 = 15.20$$

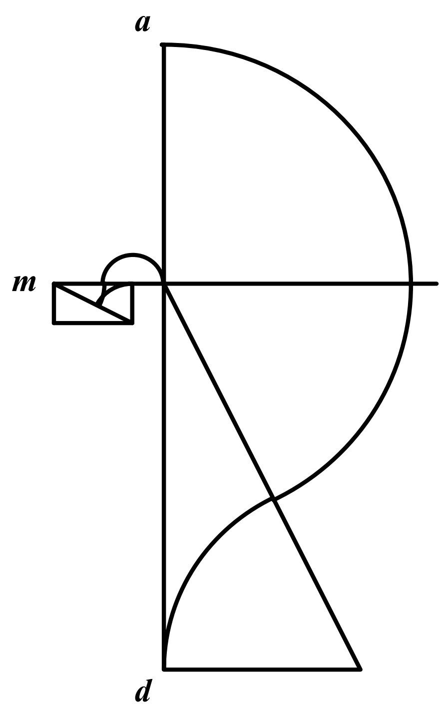


Diagram 20

The vertical distance from ground level to *d* is five times the unit length of 11 cubits. The horizontal section from *d* is one-half of the distance from ground level to *d*. The horizontal section is arced from *d* to the diagonal and the point at ground level vertically above *d* is arced from that point on the diagonal to *a*, 33.99 cubits above ground level.

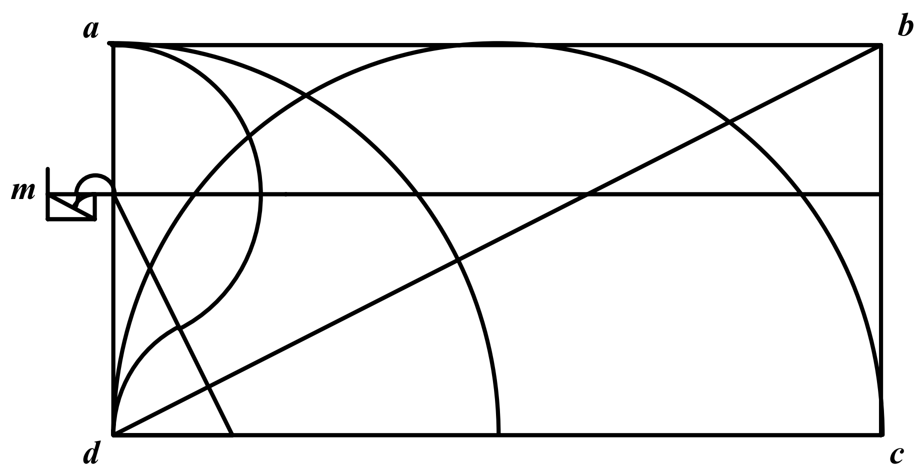


Diagram 21

da is arced to the horizontal line from *d* and this point is arced from *d* to *c*, marking the double square shown in diagrams 8 - 12 and 16. The ratio between the two double squares is 1/10φ.

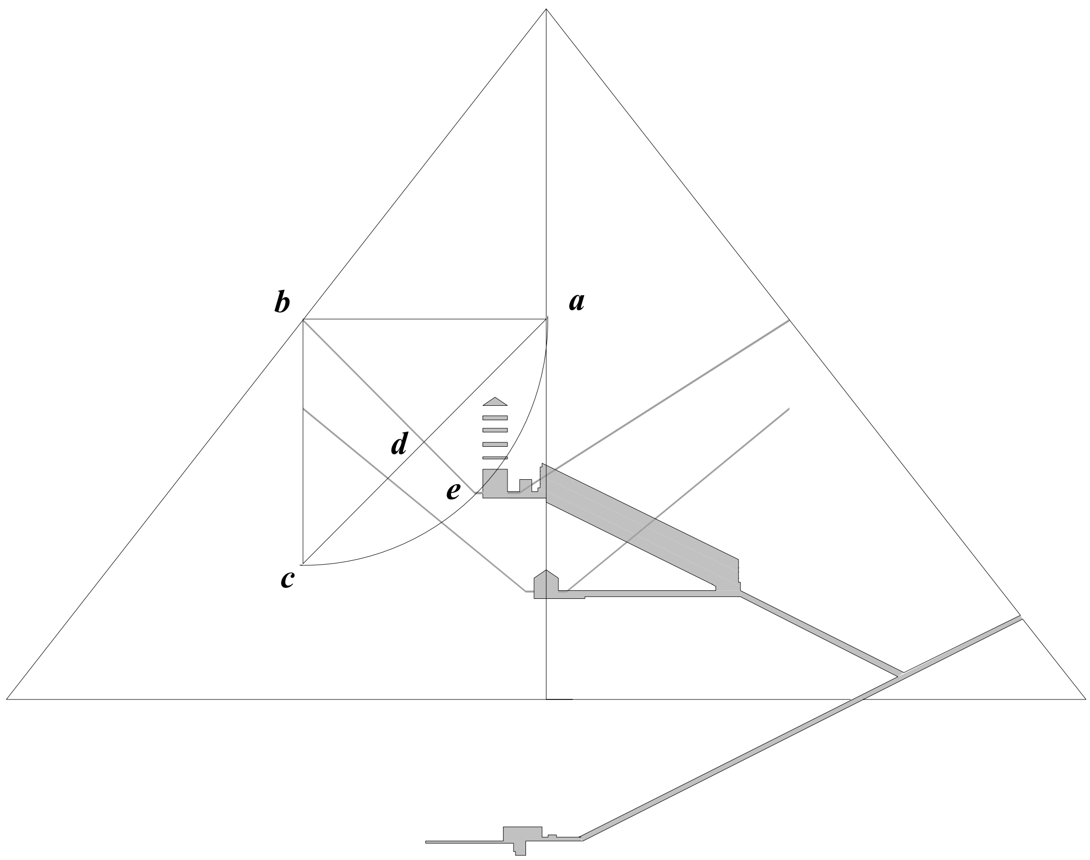


Diagram 22

a is 154 cubits above ground level (14 times the unit length of 11 cubits in Diagram 19). The precise height of the apex of the pyramid is $(4/\varphi + \varphi^2) \times 55 = 279.9593$ cubits. The height from *a* to the apex of the pyramid is 125.9593 cubits (279.9593 − 154). The Length of *ab* is 98.9282 cubits (125.9593 divided by $4/\pi$). *ba* is arced to *c*. The midpoint of *ac* is *d*. *b* is extended through *d* to the intersection of the arc at *e*. This produces a 45° angle for the southern King's Chamber shaft and a length of 98.9282 cubits for the length of the inclined part of the southern King's Chamber shaft.

In 1992, Rudolph Gantenbrink conducted a robotic survey of the diagonal shafts in the Great Pyramid.[15] Gantenbrink reported that the mean angle of the southern King's Chamber shaft is 45°. Based on his measurements, Gantenbrink concluded that both of the shafts exited the finished outer casing of the pyramid at the same height, 154 cubits above ground level. Petrie concluded that the southern King's Chamber shaft exited the finished outer casing of the pyramid at the 104th course of masonry,[16] and Petrie reported the top of the 104th course of masonry was 3175 inches above ground level.[17] The roofs of all four of the shafts are at joints in the masonry in the King's Chamber and the Queen's Chamber. The joint between courses104 and 105 is 3175 inches above ground level. (3175/154 = 20.62 inches/cubit).

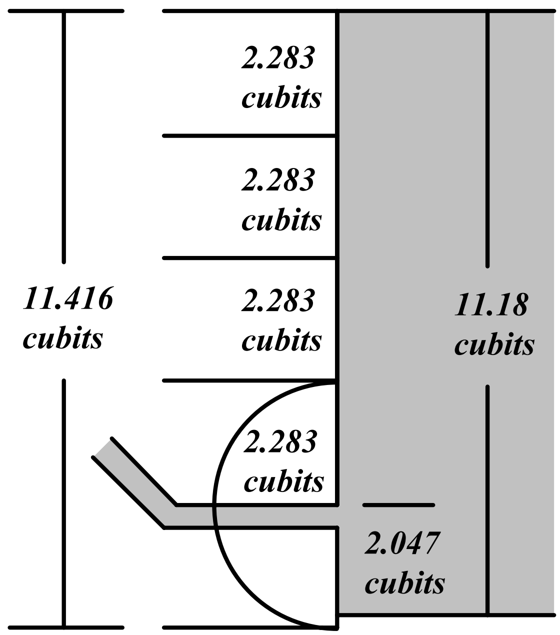


Diagram 23

Given the 98.9282 cubit length and the 45° angle of *be*, the vertical height from e to b is 69.9528 cubits (98.9282/√2). *e* marks the lower end of the sloping roof of the southern King's Chamber shaft. Given the 154 cubit height of the roof of the shaft at the outer casing, *e* is 84.0472 cubits above ground level (154 − 69.9528). The height of the floor of the King's Chamber is 82 cubits above ground level. In the King's Chamber, the height of the roof of the shaft is 2.0472 cubits above the floor (84.0472 − 82).

The height from the floor to the ceiling of the King's Chamber is 11.18 cubits (√5 × 5). The height from the roof of the shaft to the roof of the chamber is 9.1328 cubits (11.18 − 2.0472). The height of each of the four courses of the walls of the chamber from the roof of the shaft to the roof of the chamber is 2.2832 cubits (9.1328/4), giving a wall height for all five courses of 11.416 cubits (2.2832 × 5). The perimeter of the walls is 62.832 cubits (22.832 cubits for the height of all five courses on both sides of the wall, plus 40 cubits for the length of the floor and the roof). The perimeter divided by the floor equals π (62.832/20 = 3.1415).

In *The Histories* (440 B.C.), Herodotus reported that "the sides of the great pyramid are eight plethra and the height is the same." In ancient Greece, plethra denoted both a linear measure of 100 Greek feet and a square measure of one acre (100 × 100 feet). The height of 280 cubits squared is 78,400 cubits. The area of the sides of the pyramid is equal to half of the base length times the slant height. The slant height is 356 cubits and 78,400 divided by 220 (half base) equals 356. 78,400 cubits divided by eight is 9800 cubits and the square root of 9800 is 99 cubits. Livio Stecchini observed that the statement by Herodotus regarding the sides and the height of the pyramid being eight plethra is based on an ancient Egyptian acre with side lengths of 99 cubits.[18]

99 cubit side lengths allows the acre to be halved and doubled in whole numbers of cubits. One half acre is 4900 square cubits (side lengths of 70 cubits); one acre is 9800 square cubits (side lengths of 99 cubits); two acres is 19,600 square cubits (side lengths of 140 cubits); four acres is 39,200 square cubits (side lengths of 198 cubits); and eight acres is 78,400 square cubits (side lengths of 280 cubits).

The floor of the king's chamber and the antechamber is 82 cubits above ground level or 198 cubits below the apex of the pyramid. A height of 198 cubits produces a square with an area of four ancient Egyptian acres. The area of the sides of the pyramid from the height of the king's chamber to the apex of the pyramid is also four acres. The King's Chamber shafts exit the pyramid 154 cubits above ground level or 126 cubits below the apex of the pyramid. The side length of the pyramid at this height is 198 cubits. The horizontal area of the pyramid at this height is four acres.

The King's Chamber shafts begin their angle of ascent two cubits above the floor of the King's Chamber, or 84 cubits above ground level. The angle of the southern shaft is 45°. The rise/run of this shaft is 1/1. The rise of the shaft is 70 cubits (154 − 84 = 70) and the horizontal run of the shaft from the point it begins it's angle of ascent to the point it exits the pyramid is also 70 cubits. This gives two side lengths of a square with an area of one-half acre. The diagonal length of the shaft is 99 cubits, the side length of one acre.

The perimeter of the pyramid at ground level is 1760 cubits (440 × 4). The perimeter/height ratio is 1760/280 or 6.28 or 2π. The acreage of the vertical cross section of the pyramid is also 6.28 or 2π (280 × 220 = 61600 square cubits and 61600/9800 = 6.28 acres). The horizontal acreage of the pyramid at ground level is 19.75 or $\pi^2 \times 2$ (440 × 440 = 193600 square cubits; 193600/9800 = 19.75 acres).

The horizontal acreage of the pyramid at the height of the floor of the king's chamber is 9.87 or π^2 (311.13 × 311.13 = 96800 square cubits and 96800/9800 = 9.87 acres) . The side length of the pyramid at the height of the floor of the king's chamber is equal to the 99 cubit side length of the ancient Egyptian acre times π. The side length of the pyramid at ground level is equal to the 140 cubit side length of two acres times π.

The slant height of the pyramid divided by one half of the base length (356/220 = 1.618) equals φ. In the 16th century A.D. Johannes Kepler studied the unique properties of a right triangle with the short side equal to one and the hypotenuse equal to φ. Today this is known as the Kepler triangle. The long side of this triangle is equal to the square root of φ. The hypotenuse (slant height) of the triangle times the short side (½ of the base length) is equal to φ ($\varphi \times 1 = \varphi$). The long side (height) squared is the same ($\sqrt{\varphi}^2 = \varphi$). This is how Herodotus correctly described the area of the sides of the pyramid being the same as the area of the height squared.

The area of the base of the pyramid times φ is equal to the surface area of all four sides of the pyramid. Taking the height of the pyramid as the radius of a sphere, the surface area of all four sides of the pyramid times π is equal to the surface area of the sphere and the volume of the pyramid times π times φ is equal to the volume of the sphere. Taking the height of the pyramid as the radius of a circle, the area of the circle is equal to the area of the base times 4/π and the area of the circle times 4/π is equal to the surface area of all four sides of the pyramid. The area of the base of the pyramid plus the area of the circle equals 440,000 square cubits. The circumference of the circle is equal to the perimeter of the pyramid.

Taking the height of the pyramid as the diameter of a sphere, the surface area of all four sides of the pyramid is equal to the surface area of the sphere times 4/π and the volume of the pyramid is equal to the volume of the sphere times 2/π. Taking the height of the pyramid as the diameter of a circle, the circumference of the circle is equal to the base length of the pyramid times two and the area of the circle is equal to the area of the vertical cross section of the pyramid.

The diagonal of the base of the pyramid at ground level is 622.25 cubits. The slope of the slant edges of the pyramid is 311.13/280 or 10/9 (one half of the base length diagonal over the height). The slope of the sides of the pyramid is 220/280 or 11/14. The denominator in both cases is the height while the numerator of 10 is the diagonal half base and the numerator of 11 is the half base. Multiplying 14 by 9 produces a common denominator of 126. Multiplying the diagonal half base of 10 by 14 gives 140 for the diagonal half base. Multiplying the half base of 11 by 9 gives 99 for the half base. The height of 126 cubits with a diagonal half base of 140 cubits and a half base of 99 cubits are the dimensions of the pyramid from the height the king's chamber shafts exit the pyramid.

The perimeter of a square with a diagonal length of 1.111 equals π ($1.111/1.414 \times 4 = \pi$). Since 1.111/1.000 equals 10/9, the circumference of a circle with a diameter of nine is equal to the perimeter of a square with a diagonal length of ten. With a height for the pyramid of four, the diagonal half base is 4.444 (10/9 = 4.444/4), the half base is π and the slant height is π times φ. The diagonal half base divided by the square root of two gives a half base of π ($4.444/1.414 = \pi$). The square of the diagonal half base (4.444×4.444) is 19.75 or $\pi^2 \times 2$, the same figure as the ancient Egyptian acreage of the base of the pyramid.

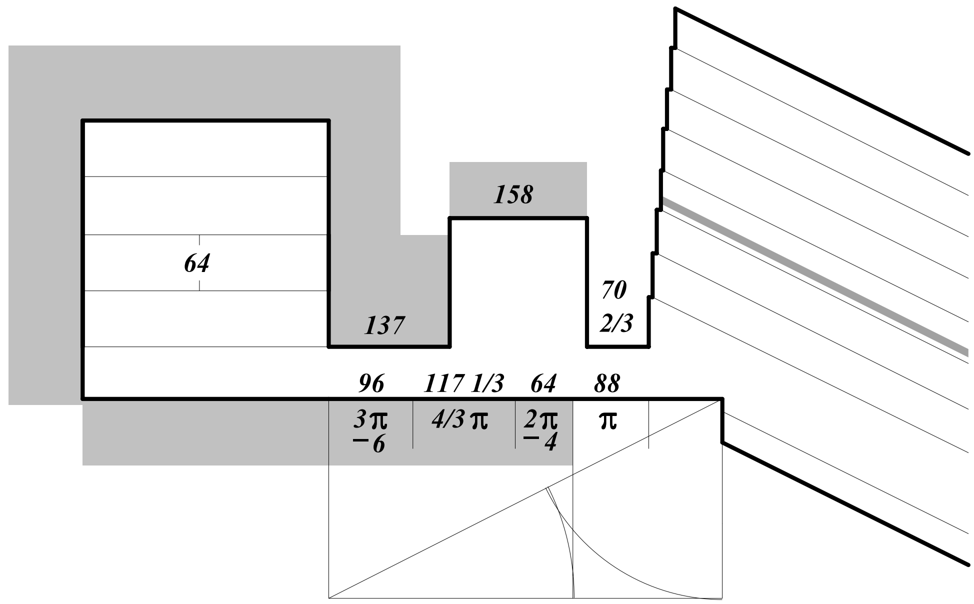


Diagram 24

Flinders Petrie and Piazzi Smyth gave the following measures in inches for the lengths of the blocks and passages, from the edge of the great step to the king's chamber:

	Petrie[1]	-	Total	Smyth[2]	-	Total
North edge of great step	0.00		0.00	0.0		0.0
South wall of gallery	61.32		61.32			
First floor joint				62.1		62.1
North end of antechamber			113.34			
Second floor joint (granite begins)	64.9		126.22	64.6		126.7
Third floor joint	47.25		173.47	47.2		173.9
South end of antechamber			229.42			
Fourth floor joint	86.26		259.73	85.8		259.7
End of passage floor	70.63		330.36	70.6		330.3

Part Three - The King's Chamber Passage

The horizontal passage system that leads to the King's Chamber begins at the great step. The edge of this step is at the north-south midline of the pyramid. The first and second floor blocks are limestone. The last three blocks in the floor of the passage are granite. In diagram 24 the granite blocks are shaded.

Petrie gives 61.32 inches from the edge of the step to the south wall of the gallery and Smyth gives 62.1 inches from the edge of the step to the first floor joint. In Smyth's diagram of the plan of the passage, he also indicates that the joint between the first and second floor blocks is slightly south of the south wall of the gallery.[3] Petrie also gives the distance from the edge of the step to the north and south walls of the antechamber.

The ancient Egyptian cubit that was used in the great pyramid was divided into seven palms of four digits each, or 28 digits per cubit. Smyth measured 64.6 inches for the second floor block. This converts to 88 digits, or 22 palms, or three cubits and one palm, or π cubits.

The first granite floor block is 47.2 inches long. This converts to 64 digits or 16 palms or $2\pi - 4$ cubits. This is the same length as the height of the courses of the King's Chamber and the same length as the perpendicular height of the descending and ascending passages in the pyramid. The second granite floor block is 85.8 to 86.2 inches long. This is 4/3 the length of the 88 digit floor block ($64.6 \times 4/3 = 86.1$). This converts to 117 1/3 digits, or 29 1/3 palms, or $4/3\pi$ cubits. The third granite floor block is 70.6 inches long. This is one and a half times the length of the first granite floor block ($47.05 \times 1.5 = 70.6$ inches). This converts to 96 digits, or 24 palms, or $3\pi - 6$ cubits.

In 2006 I wrote an article[4] describing the length of the floor blocks as π, $2\pi - 4$, $4/3\pi$ and $3\pi - 6$. In 2009, Clive Ross commented that the limestone section and the granite section of the passage are divided into a golden section.[5] This division is shown in diagram 24. $330.3/\varphi^2 = 126.2$ inches (the length of the limestone section of the passage). $204.1/126.2 = 330.3/204.1 = \varphi$.

The granite blocks are $(2\pi - 4) + 4/3\pi + (3\pi - 6) = 9.897$ cubits. 9.897 cubits $\times\ \varphi = 16.013$ cubits for the total length of the passage. 330.3 inches /16.013 cubits = 20.62 inches per cubit. Given the 22/7 palm/cubit measure of 3.1428 for π, $(2\pi - 4) + 4/3\pi + (3\pi - 6) = 9.905$ cubits. $9.905 \times \varphi = 16.025$ cubits. This is in precise agreement with length of the king's chamber passage that is produced by the construction in diagram 17. $330.3/16.025 = 20.61$ inches per cubit.

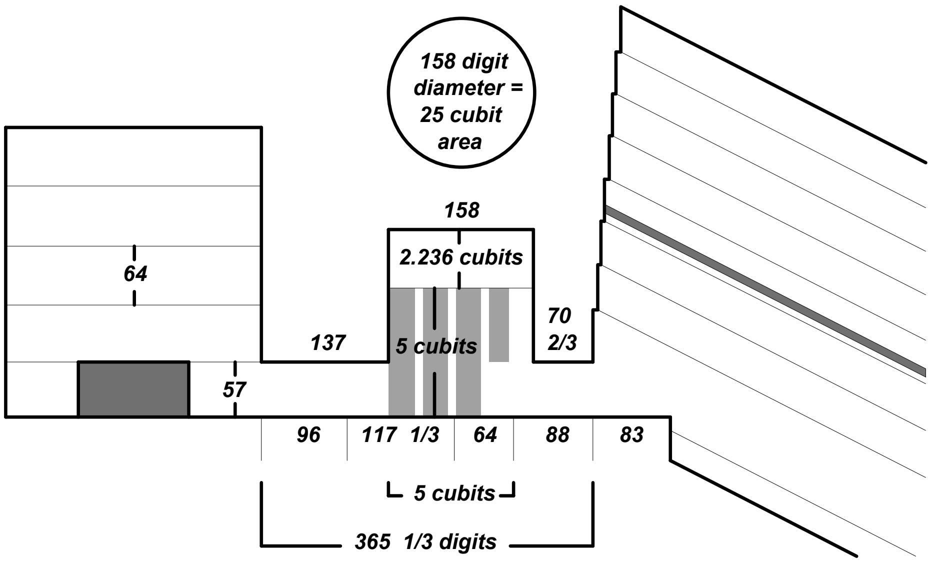


Diagram 25

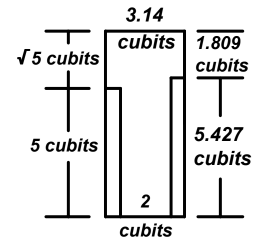


Diagram 26

The antechamber is 149.3 inches high.[6] This converts to $5 + \sqrt{5}$, or $(\varphi^2 + 1) \times 2$, or 7.236 cubits. The chamber is 116.3 inches long.[7] This converts to 158 digits, or 39.5 palms, or 5.64 cubits, or $(2.5 + \pi)$ cubits. Petrie gave four measurements of the upper width of the chamber, just below the ceiling, of 64.80, 64.48, 64.96, and 64.76 inches.[8] The mean of these measurements is 64.75 inches. This converts to 88 digits or 22 palms or 3.1428 cubits (64.75/3.1428 = 20.6 inches per cubit). Granite wainscots four palms wide line the lower portion of the east and west walls of the chamber, making the floor and the lower portion of the chamber 14 palms or two cubits wide.

Three vertical slots are carved from the top of the wainscots to the floor. These slots are believed to have held stones blocking entrance to the king's chamber. Three semicircular hollows, carved into the top of the wainscot on the west side of the chamber are believed to have held beams for lowering the blocking stones into place. The top of the wainscot on the east side of the antechamber is flat, five cubits (103.2 inches) above the floor of the chamber and 2.236 cubits (46.1 inches) below the ceiling.[9] The top of the wainscot on the west side of the antechamber is three quarters of the total height of the chamber, 5.427 cubits above the floor and 1.809 cubits below the ceiling.[10] 1.809 equals $\varphi^2 - \varphi/2$, or $\varphi/2 + 1$, or $(\varphi^2 + 1)/2$.

From the beginning of the first granite floor block to the south end of the antechamber is also 103.2 inches[11], the same as the height of the east wainscot above the floor of the chamber. The height of the wainscot above the length of the granite portion of the floor gives a square area of 25 cubits. A circle with an area of 25 cubits has a diameter of 5.64 cubits (116.3 inches), equal to the length of the chamber. A circle with a diameter of $2.5003 + \pi$ cubits has an area of 25 cubits and a circumference of 17.724 cubits ($\sqrt{\pi} \times 10$). The rectangular area of the ceiling of the antechamber, with a length of $2.5003 + \pi$ cubits and a width of π cubits, is also 17.724 cubits .

The length from the south wall of the grand gallery to the north wall of the antechamber is 52.02 inches, or 70 2/3 digits. The 70 2/3 digit length times 2.236 is equal to the length of the antechamber (70 2/3 × $\sqrt{5}$ = 158 digits, and 52.02 × $\sqrt{5}$ = 116.3 inches). The distance from the south wall of the antechamber to the north wall of the King's Chamber (137 digits) is 3/5 the distance from the south wall of the gallery to the south wall of the antechamber (228.66 digits).

The volume of a sphere with a diameter of 2.5 + π cubits is 94.0 cubic cubits. The volume of the antechamber approximates this volume. The lower part of the chamber, up to the top of the east wainscot has a width of 2 cubits, a height of 5 cubits, and a length of 2.5 + π cubits. This gives a volume of 56.416 cubits (10π + 25). The upper part of the chamber has a width of π cubits, a height of 1.809 cubits and a length of 2.5 + π cubits. This gives a volume of 32.06 cubits. The middle height of the chamber has a width of 2.57 cubits (π/2 + 1) a height of .427 cubits (2.236 − 1.809) and a length of 2.5 + π cubits. This gives a volume of 6.19 cubits. The combined volume of the three different widths of the chamber gives a total volume of 94.67 cubic cubits. If the upper height of the west side of the chamber was $\sqrt{\pi}$, the width of the west wainscot was $\sqrt{\pi} - 1$, and the width of the east wainscot was $\pi - (\sqrt{\pi} + 1)$, the volume of the chamber would be 94.0317 cubits. A sphere with a diameter of 2.5003 + π cubits has a volume of 94.0316 cubits.

"The roof stones (of the gallery) are set each at a steeper slope than the passage, in order that the lower edge of each stone should hitch like a paul into a ratchet-cut in the top of the walls; hence no stone can press on the one below it, so as to cause a cumulative pressure all down the roof; and each stone is separately upheld by the side walls across which it lies."[12] As a result, the vertical height of the gallery varies, depending on measurement from the higher or lower part of each individual ceiling block. Petrie gives a range of 334 to 344 inches for the height of the ceiling[13], while Smyth gives a range of 333.9 to 346 inches[14]. Neither survey indicates if any of the measurements were made from the extreme lower end of any of the blocks. 16.18 cubits × 20.6 inches = 333.3 inches. If this was the intended height of the gallery, the height of the King's Chamber is $\sqrt{5} \times 5$ cubits, the height of the antechamber is $\sqrt{5} + 5$ cubits and the height of the gallery is $(\sqrt{5} \times 5) + 5$ cubits.

The length of the passage from the beginning of the second floor block to the north wall of the King's Chamber is 365.33 digits. Petrie gives 269.04 inches from the south wall of the gallery to the north wall of the King's Chamber. 269.04/20.62 × 28 = 365.33 digits.

A mean earth diameter of 7920 miles, times 5280 equals 41817600 feet, times 12 equals 501811200 inches, divided by 20.6087 equals 24349483 ancient Egyptian cubits, times 1.5 equals 36,524,225 ancient Egyptian feet, divided by 100,000 = 365.24225. Applying the ratio of one and a half feet to the ancient Egyptian cubit, the number of feet in the diameter of the earth is equal to the number of days in 100,000 years.

The floor of the grand gallery from the north wall to the great step is 1815.5 inches[15], or 88 cubits, or 28 times the width of the antechamber. The floor is two cubits wide, but the base of the gallery is four cubits wide. Blocks one cubit wide and one cubit high line the length of the gallery on both sides of the base. The walls are comprised of seven overlapping courses. Each successive overlap is one palm narrower on each side, so the ceiling of the gallery is also two cubits wide. The width of the gallery at the third overlap is 22 palms or 3.14 cubits, the same as the upper width of the antechamber. The groove in the third overlap is half way between the floor and the ceiling of the gallery.[16] Diagram 27 shows the south wall of the gallery, the great step and the entrance to the antechamber passage.

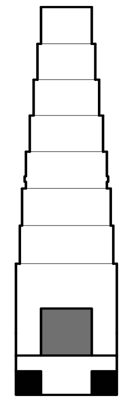


Diagram 27

The north and south walls of the King's Chamber are 20 cubits or140 palms, or 560 digits long, and the walls are 11.428 cubits, or 80 palms, or 320 digits high, from the base of the first course to the ceiling. The east and west walls are 10 cubits, or 70 palms, or 280 digits long. The perimeter of the north and south walls is 1760 digits (560 × 2 plus 320 × 2). The height of the pyramid is 280 cubits and the perimeter is 1760 cubits (440 × 4). The ratio of the height to the perimeter of the pyramid is the same as the ratio of the length of the short walls to the perimeter of the long walls of the King's Chamber, and the number of cubits in the height and the perimeter of the pyramid is the same as the number of digits in the short wall lengths and the long wall perimeters in the King's Chamber. The ratio between the pyramid proportion and the same proportion found in the King's Chamber is the cubit to digit ratio of 28 to one.

Petrie gives the following measures in inches for the dimensions of the coffer[17]:

	Outside	Inside
Length	89.62	78.06
Width	38.5	26.81
Height	41.31	34.42

These measures convert to ancient Egyptian Cubits as follows:

	Outside	Inside
Length	4.345	3.786
Width	1.867	1.301
Height	2	1.67

The diagonal of the inside base of the coffer is 4 cubits ($3.786^2 + 1.301^2 = 4^2$). This is twice the height of the coffer, just as the diagonal of the base of the King's Chamber is equal to twice the 11.18 cubit height of the King's Chamber ($20^2 + 10^2 = 22.36^2$).

A sphere with an area equal to the area of the inside base of the coffer has a volume equal to the volume of the inside of the coffer. The area of the inside base of the coffer is 4.925 square cubits (3.786 × 1.301). A circle with an area of 4.925 square cubits has a radius of 1.252 cubits. A sphere with a radius of 1.252 cubits has a volume of 8.224 cubic cubits. The inside volume of the coffer is 8.224 cubits (3.786 × 1.301 × 1.67 = 8.224).

A circle with a radius of φ (1.618 cubits) has a circumference of 10.17 cubits, equal to the perimeter of the inside base of the coffer (3.786 × 2) + (1.301 × 2) = 10.17. A circle with a radius of φ has an area of 8.224 square cubits, equal to the cubic measure of the inside volume of the coffer and a volume of 17.74 cubic cubits, approximating $\sqrt{\pi} \times 10$ (17.72).

The outside volume of the coffer is 16.224 cubic cubits (4.345 × 1.867 × 2 = 16.224). The inside volume of the coffer is 8.224 cubic cubits. The solid bulk of the coffer is 8 cubic cubits (16.224 − 8.224), equal to the bulk of a cube with side lengths of 2 cubits, the same as the height of the coffer.

A sphere with a diameter of 16.224 cubits has a volume of 2,236 cubic cubits, equal to the volume of the King's Chamber. The radius of this sphere is 8.112 cubits, equal to the area of the outside base of the coffer. After a comment I wrote in 2004 regarding these relationships, Michael Saunders commented that a sphere with a diameter of π cubits has a volume of 16.23 cubic cubits, closely approximating the outside volume of the coffer.

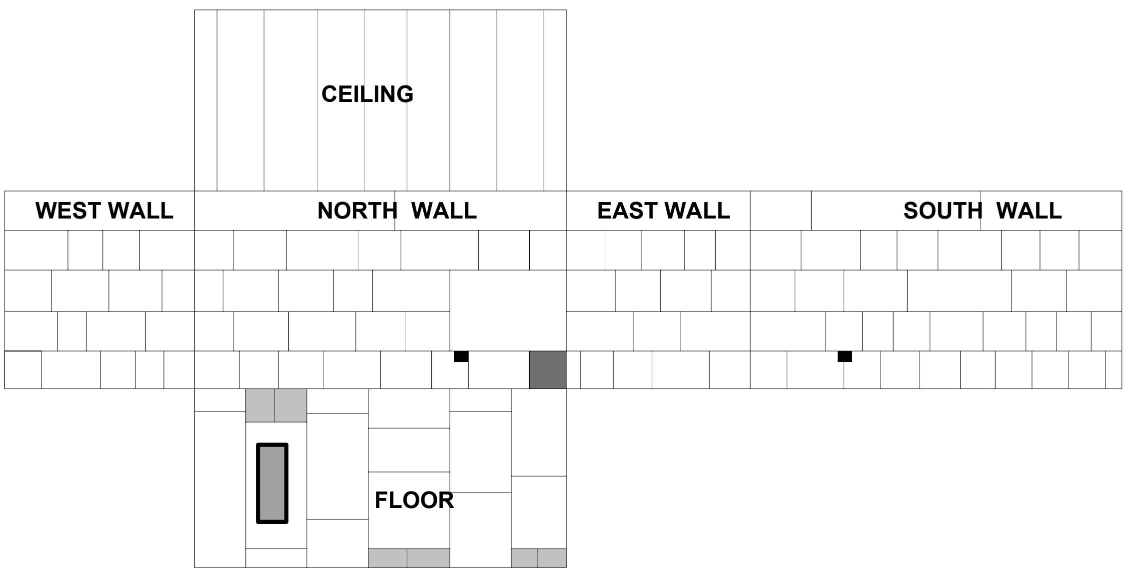


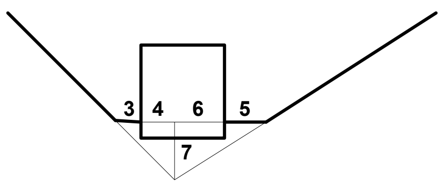
Diagram 28

Every block in the ceiling of the King's Chamber is cracked. There are damaged blocks in the south wall near the east side, and in the west wall near the south side. Several of the flooring blocks were removed and three of those have been replaced with six blocks (lightly shaded blocks in Diagram 28). Originally there were 18 blocks in the floor, 9 blocks in the ceiling and 99 blocks in all four walls.[18] The number of blocks in all six surfaces are multiples of nine: East wall, west wall and floor - 9 × 2; north wall - 9 × 3; south wall - 9 × 4; ceiling - 9. This is a total of 126 blocks in the chamber and 99 blocks in all four walls. The ratio between the total number of blocks in the chamber and the number of blocks in the walls is 126/99, or 14/11 or 4/π, the same ratio that is found in the external slope of the pyramid.

The angles of the shafts from Gantenbrink's 1993 robotic survey[19] are:

Upper southern shaft: 45°	Upper northern shaft: 32° 36'
Lower southern shaft: 39° 36'	Lower northern shaft: 39° 7'

The outer casing stones are missing where the upper shafts exit the pyramid, but Gantenbrink calculated that both of the upper shafts exited the outer casing 154 cubits above ground level, and concluded that the slope of the upper southern shaft is 1/1 (45°); the upper northern shaft is 7/11 (32° 28' 16"); and both lower shafts are 9/11 (39° 17' 22").



The north wall of the king's chamber is 16 cubits south of the vertical axis of the pyramid and the south wall is 26 cubits south of the vertical axis. The shaft outlets are two cubits above the floor of the chamber, or 84 cubits above ground level. The northern shaft runs horizontally for five cubits before beginning its ascent. The southern shaft runs horizontally for three cubits before beginning its ascent. Extended downward, the intersection point of the shafts is five cubits below the floor of the chamber, or 77 cubits above ground level. This height is also 77 cubits below the height the shafts exit the pyramid. The intersection point is 22 cubits south of the vertical axis of the pyramid. 22 cubits south of the vertical axis of the pyramid is 77 cubits north of the point the southern shaft exits the pyramid and 121 cubits south of the point the northern shaft exits the pyramid.

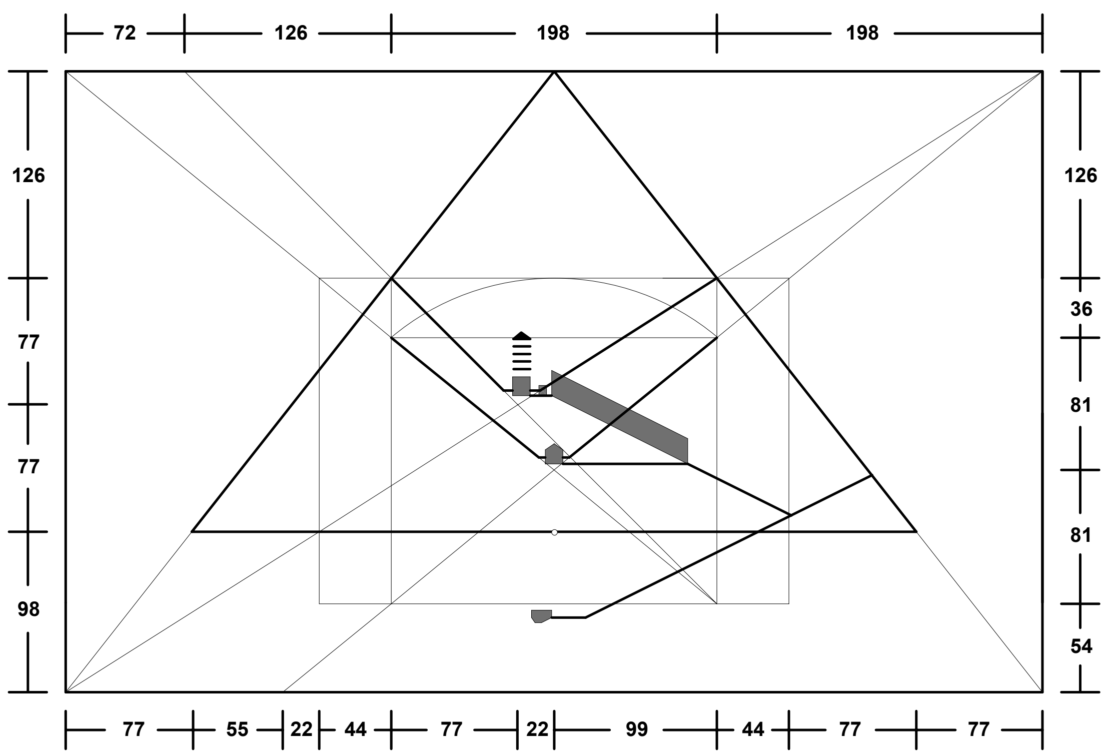


Diagram 29

Extending the upper northern shaft beyond the face of the pyramid, it reaches the height of the apex of the pyramid 198 cubits north of its exit point (126/198 = 7/11). Extended downward, the upper northern shaft intersects ground level 77 cubits north of the southern edge of the pyramid. The upper northern shaft intersects the southern face of the pyramid (as extended downward), 77 cubits south of the southern edge of the pyramid and 98 cubits below ground level, showing the slope of the pyramid (98/77 = 14/11) and the slope of the shaft (98/154 = 7/11). These points define a rectangle with a height equal to the height of the pyramid plus 98 cubits (below) and a width equal to the base length of the pyramid plus 77 cubits on each side. The 198 cubit square, centered just above the apex of the queen's chamber in diagram 29, has the same dimensions as the side lengths of the pyramid at the height of the exit points of the upper shafts. The upper southern shaft as extended downward forms a diagonal from the upper southern corner to the lower northern corner of the square. The length of this diagonal is 280 cubits, equal to the height of the pyramid.

Unlike the upper shafts, the lower shafts do not exit the pyramid. "They are exactly like the air channels in the King's Chamber in their appearance, but were covered over the mouth by a plate of stone, left not cut through in the Queen's Chamber walls, until they were discovered by Waymon Dixon."[20] The shaft outlets in the Queen's Chamber are 44 cubits above ground level. The chamber is 10 cubits wide and the shafts are horizontal for 3.75 cubits before beginning their angles of ascent. The two lower shafts (extended downward) intersect at the midline of the pyramid, 37 cubits above ground level and 81 cubits above the lower border of the square. This point is also 81 cubits below the point where the lower shafts intersect the sides of the square. The sides of the square are 99 cubits across from the intersection point of the lower shafts (81/99 = 9/11). Both of the lower shafts intersect the sides of the square 36 cubits below the upper shaft exit points and both of the lower shafts (extended upward) cross over the height of the upper shaft exit points 44 cubits across from the exit points (36/44 =9/11).

The lower northern shaft, intersects the square 36 cubits below the exit point of the upper northern shaft and 162 cubits below the apex of the pyramid. Extended upward, the lower northern shaft intersects the upper northern shaft at the height of the apex of the pyramid,[21] 198 cubits north of the exit point of the upper northern shaft (162/198 = 9/11). The lower southern shaft intersects the square 36 cubits below the exit point of the upper southern shaft and as extended downward, intersects the upper southern shaft at the lower northern corner of the square, 198 cubits north of the exit point of the upper southern shaft, (162/198 = 9/11).

The point on the midline of the pyramid at ground level is the origin point of the arc in the diagram. The height of the upper shaft exit points is 154 cubits above ground level. The points where the lower shafts cross directly under the upper shaft exit points are also 154 cubits from the midline of the pyramid at ground level. The distance of these diagonals may be calculated as the hypotenuse of the right triangle formed by the height above ground level where the lower shafts cross under the upper shaft exit points (118 cubits) and the distance across from this point to the centerline of the pyramid (99 cubits): $118^2 + 99^2 = 154^2$.

Petrie's survey produced the following base lengths, azimuths, angles, height and relative locations of the three main Giza pyramids[1]

	First pyramid:		Second pyramid:		Third pyramid:	
	Length	Azimuth	Length	Azimuth	Length	Azimuth
N	9069.4"	−3' 20"	8471.9"	−5' 31"	−−−−−	+16' 48"
E	9067.7	−3' 57"	8475.2	−6' 13"	4149.2"	+12' 23"
S	9069.5	−3' 41"	8476.9	−5' 40"	4157.8	+12' 57"
W	9068.6	−3' 54"	8475.5	−4' 21"	4153.9	−−−−−
Mean	9068.8"	−3' 43"	8474.9"	−5' 26"	4153.6"	+14' 3"
	440 cubits		411 cubits		201.5 cubits	

First pyramid angle:		Second pyramid:		Third pyramid:	
casing:	51° 46' 45"	casing:	53° 12'	granite:	50° 57'
top and base:	51° 49'	plumb line:	53° 2'	loose blocks:	51° 00'
north face:	51° 53' 20"	loose blocks:	53° 14'	limestone:	51° 58'
Conclusion:	51° 52'	Conclusion:	53° 10'	Conclusion:	51° 10' 30"

Height (calculated from base length and angle):

5776"	280 cubits	5664"	274 cubits	2580"	125 cubits

Distances from the centers of the pyramids:

Inches:	NS Distance:	EW Distance:	Total Distance and Angle West of Due South:
First to Second	13,931.6	13,165.8	19,168.4 at 43° 22' 52"
First to Third	29,102.0	22,616.0	36,857.7 at 37° 51' 6"
Second to Third	15,170.4	9,450.2	17,873.2 at 31° 55' 12"

Cubits:			
First to Second	675.5	638.5	929.5
First to Third	1,412	1,097	1,788
Second to Third	736.5	458.5	867.5

Part Four - The Siteplan

In addition to the azimuth of the sides of the pyramids, Petrie also surveyed the azimuths of the entrance passages and noted Smyth's survey of the azimuths of the entrance passages. Smyth reported an azimuth of − 5' 49" for the great pyramid entrance passage and − 5' 37" for the second pyramid entrance passage. Petrie also reported an azimuth of − 5' 49" for the built part of the great pyramid entrance passage and +13' 16" for the third pyramid entrance passage.[2]

Petrie believed that all three of the pyramids were intended to be oriented with the cardinal directions, but he measured the distances between the centers of all three pyramids on parallels inclined − 5' to true North, which he regarded as the mean azimuth of the first and second pyramids. He concluded that "the third and lesser pyramids are so inferior in work, that they ought not to interfere with the determination from the accurate remains."[3]

The NS distance from the center of the first pyramid to the center of the second pyramid is 675.5 cubits. Subtracting the half base of both pyramids gives 250 cubits from the south side of the first pyramid to the north side of the second pyramid (675.5 − 220 − 205.5 = 250). The EW distance is 638.5 cubits. Subtracting the half base of both pyramids gives 213 cubits from the West side of the first pyramid to the East side of the second pyramid (638.5 − 220 − 205.5 = 213). John Legon pointed out that the distance from the NS midline of the first pyramid to the East side of the second pyramid is equal to 250 times the square root of three ($250 \times \sqrt{3} = 433$) and the distance from the South side of the first pyramid to the South side of the second pyramid is equal to 250 times the square root of seven ($250 \times \sqrt{7} = 661$), as shown in diagram 30.[4] The unit length of one in diagram 30 equals 250 cubits. The 500 cubit segment equals 2, or the square root of 4, producing the right triangle $\sqrt{3}^2 + \sqrt{4}^2 = \sqrt{7}^2$.

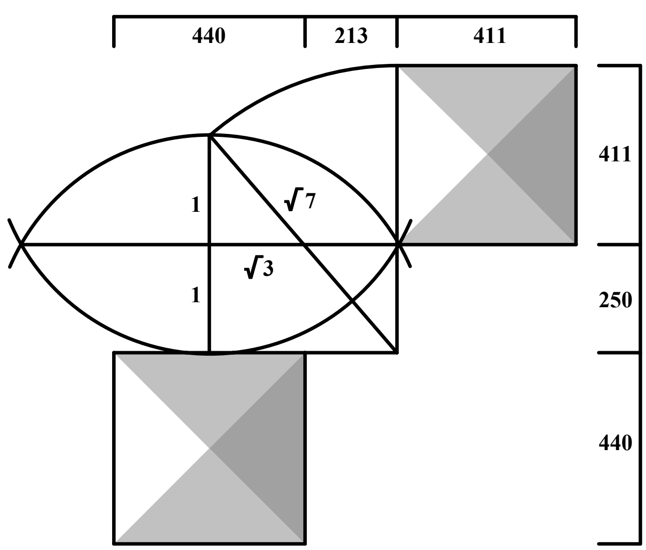


Diagram 30

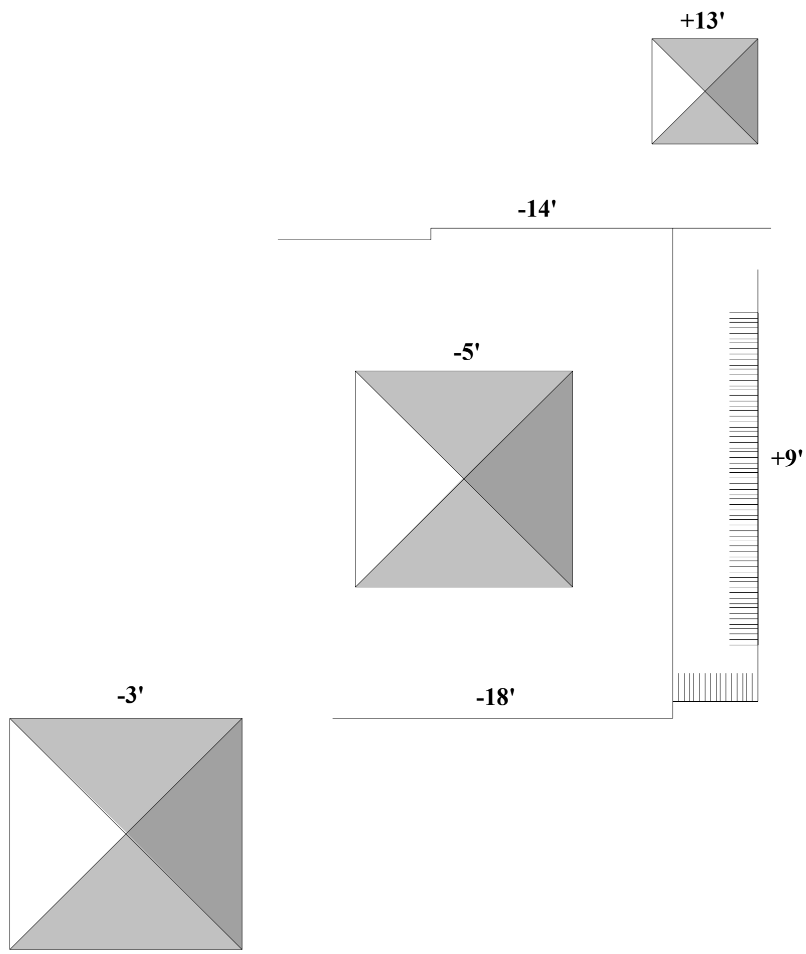


Diagram 31

Petrie used an orientation of − 5' to survey the NS and EW distances between the centers, and by extension, the sides and corners of the pyramids. If the sizes and relative locations of the pyramids were planned to begin with, this would have also required an orientation for surveying the NS and EW distances between the pyramids. The almost identical deviation from cardinality of the first and second pyramids suggests that one determination of cardinality was used to orient both pyramids and to survey the NS and EW distance between the two, but given their almost identical azimuths, even if the first two pyramids were oriented separately, and one or the other of these orientations was used to survey the distance between the two, the result would be the same.

Petrie surveyed the low walls around the North, South and West sides of the second pyramid, as well as the two structures shown on diagram 31, that he believed were workers barracks. The barracks were divided into 91 long galleries and Petrie estimated they could have housed up to 4,000 workers. Petrie reported that the azimuth of the West wall of the long barracks was oriented clockwise +9'. He also commented that "the wall at the head of the galleries, if prolonged, would pass but 29 inches within the W. side of the third pyramid and therefore those seem to be intended for the same line." [5]

The 29 inch discrepancy between the West wall of the barracks and the West side of the pyramid is based on the survey orientation of − 5'. The South wall of the barracks is 6,567 inches NS from the NW corner of the third pyramid.[6] If the third pyramid was intended to be aligned with the West wall of the barracks, and the ancient survey extended from the barracks to the NW corner of the pyramid, the +9' azimuth of the barracks varies from Petrie's survey by 14' clockwise. A circle with a radius of 6,567 inches has a circumference of 41,261 inches. 14' of arc, or 1/4.2 of one degree, is equal to 27.3 inches (41,261/360° = 114.6 and 114.6/4.2 = 27.3), along the circumference of this circle. Over the distance of 6,567 inches, the alignment is within two inches. This suggests that the orientation used to locate the third pyramid was the clockwise orientation of the barracks, rather than the counterclockwise orientation of the first two pyramids.

Based on the orientations of the first two pyramids, the South side of the barracks is also further West of the first two pyramids than the North side of the barracks. Petrie used the apexes of the pyramids in his survey of the relative locations of the pyramids. If the EW distance from the second pyramid to the West wall of the barracks was determined by a survey from the middle of the West side of the second pyramid, then a circle with a radius of 15,170.4 inches (NS distance from the apex of the second pyramid to the apex of the third pyramid) has a circumference of 95,318 inches. The 14' clockwise discrepancy would mean the third pyramid is 63 inches West of it's intended location, based on the orientation of the first two pyramids, and Petrie's survey. If the third pyramid was oriented independently, it would still be aligned with the West wall of the barracks based on the azimuth of the third pyramid or the azimuth of the barracks, and it would still be West of it's intended location in relation to the azimuth of the first two pyramids.

The lower part of the third pyramid was cased in granite blocks that were never dressed. The casing blocks on the first course indicate a formation like the second pyramid, with paving stones rising above the base of the first course, with the top of the paving stones marking ground level, but the first course could not receive paving stones without first being dressed, and there are no paving stones around the third pyramid. The upper courses of the third pyramid were cased in dressed and polished limestone blocks, but no limestone casing blocks remain on the pyramid. The third pyramid was surrounded by rubble that Petrie excavated on both sides of the NE, SE, and SW corners, but he was not able to reach the NW corner.[7]

In 1982 Gay Robins and Charles Shute surveyed the face angles of the Giza pyramids. They confirmed Petrie's findings for the first and second pyramid, but found 14/11 or 51° 52' for the face angle of the third pyramid,[8] the same as the first pyramid and close to Petrie's measures for the fragments of the limestone casing blocks recovered from the rubble around the third pyramid. In 1997, Miroslav Verner confirmed Petrie's findings for the base lengths of the first and second pyramids, but reported base lengths of 104.6 meters, or 4118 inches for the third pyramid, citing Maragioglio and Rinaldi.[9] Petrie commented that "it seems most probable that the third pyramid was designed to be 200 cubits long." [10] Given the base lengths of 4118 reported by Margioglio and Rinaldi, 4118/20.6 inches = 199.9 cubits.

Although Petrie gives precise base lengths for three sides of the third pyramid, these are not direct measurements. They are calculated from: Distances measured by triangulation from excavated points at the base of the first course and from points on higher courses; an estimate of the height of the missing paving stones; estimates of the length the granite blocks would have been reduced by dressing; and his calculation of the angle of the pyramid. Petrie reported a weighted mean angle of 51° 10', concluding that the designed angle was 51° 20', for a slope of 5/4, but he used a pyramid angle of 51° 0' to calculate the base lengths, producing a longer base length than the steeper angle reported by Robins and Shute, and longer than the base length reported by Margioglio and Rinaldi.

The NS distance of 29,102 inches between the apexes of the first and third pyramids, plus the half bases of the two pyramids (4534 + 2059) equals 35,695 inches. $\sqrt{3} \times 1000 = 1732$ and 35,695/1732 = 20.61 inches per cubit. John Legon suggested that $\sqrt{3} \times 1000$ cubits was the designed NS distance from the North side of the first pyramid to the South side of the third pyramid.[11] The EW distance of 22,616 inches between the apexes of the first and third pyramids, plus the half bases of the two pyramids equals 29,209 inches. 29,209/20.61 = 1417.2 cubits. $\sqrt{2} \times 1000 = 1414.2$. Despite the three cubit difference, it has also been suggested that the designed EW distance from the East side of the first pyramid to the West side of the third pyramid was $\sqrt{2} \times 1000$ cubits. Over the NS distance from the apex of the second pyramid to the third pyramid, the deviation from the counterclockwise orientation of the first two pyramids to the clockwise orientation of the workers barracks produces an additional 3 cubits of EW distance (63 inches/20.61 = 3 cubits).

If the distance between the North side of the first pyramid and the South side of the third pyramid is 1732 cubits, and the distance between the East side of the first pyramid and the West side of the third pyramid is 1414 cubits, and if the base length of the third pyramid is 200 cubits, then the EW distance between the EW midline of the third pyramid and the West side of the second pyramid is the same as the NS distance between the South side of the first pyramid and the North side of the second pyramid, and the EW distance between the EW midline of the third pyramid and the East side of the second pyramid is the same as the NS distance between the South side of the first pyramid and the South side of the second pyramid.

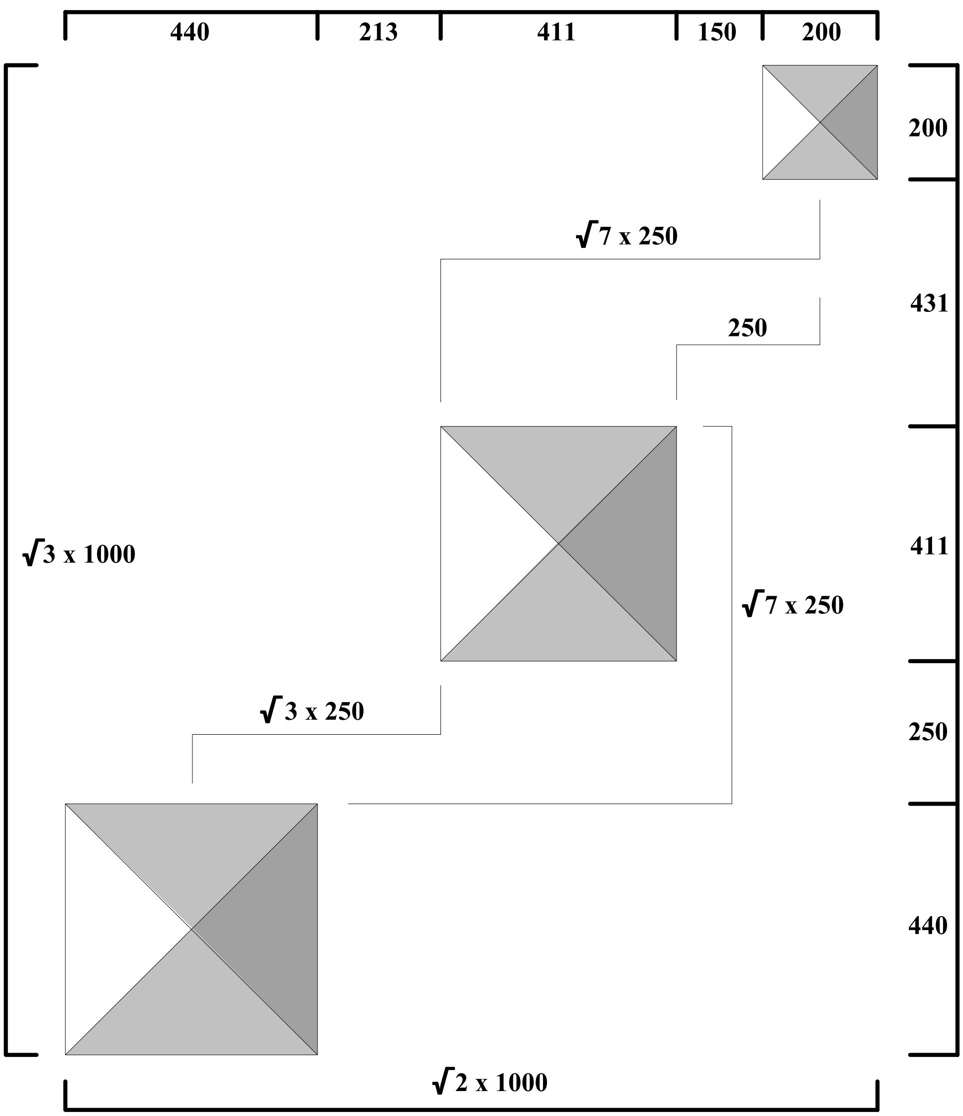


Diagram 32

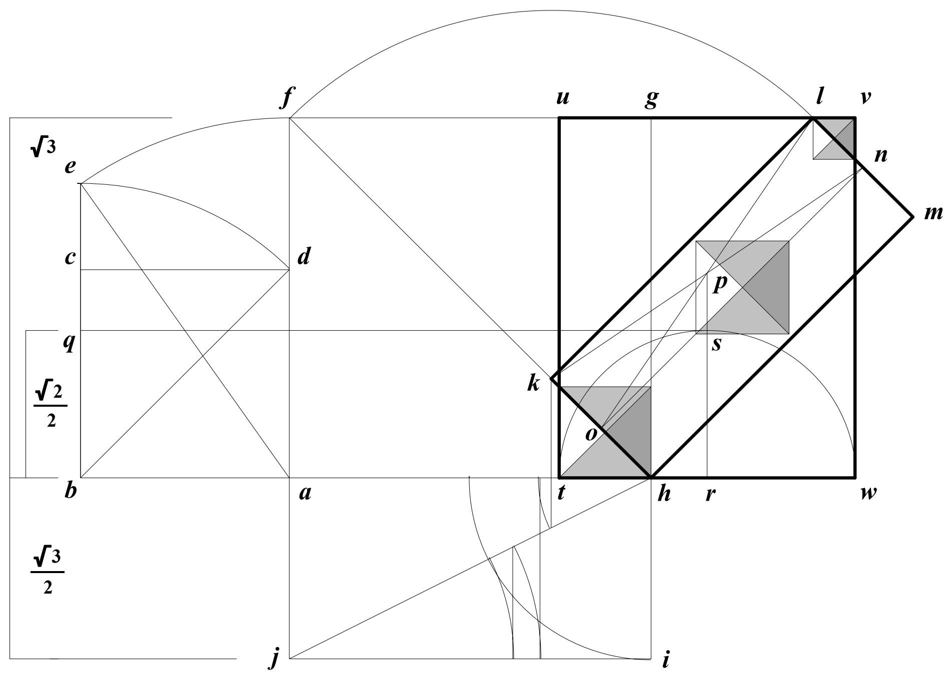


Diagram 33

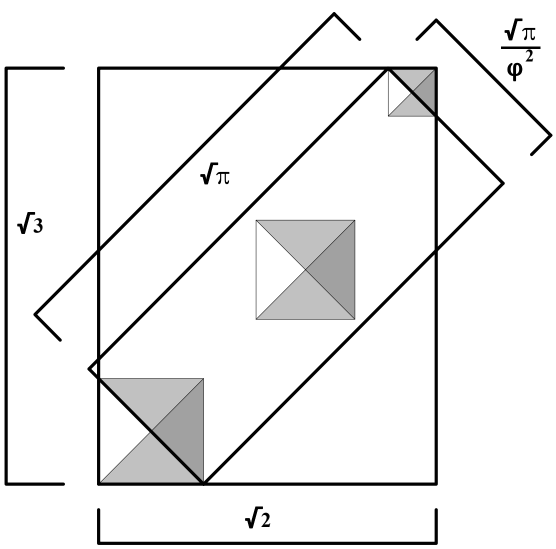
Square *abcd* side lengths = 1, eb = √2 and square *afgh* side lengths = √3. The diagonal *hf* is √6 ($\sqrt{3}^2 + \sqrt{3}^2 = \sqrt{6}^2$). The side lengths of the double square *ahij* are √3 and √3/2. The diagonal of the double square is divided into the ratio of φ^2 to one and projected to *k* to divide *fh* into the ratio of φ^2 to one. √6 = 2.44948974. The length of *fk* is $\sqrt{6} \times \varphi^2/(\varphi^2 + 1)$, or √6 × .72360679 = 1.77246742. The square of 1.77246742 is 3.14640786. This is exactly the same figure for π produced by 6/5 φ^2 and exactly the same figure for π produced by the angle of the first pyramid as constructed in Part II. According to Robins and Shute, the third pyramid has the same angle as the first pyramid.

kf is arced to *l*. The vertical length from *h* to *l* is √3. The horizontal length from *h* to *l* is *kl* − *hk*, divided by √2. *kl* = *kf* = 1.77246, and *hk* = *hf* − *kf* = 2.449489 − 1.77246 = .677022. *kl* − *hk* = 1.095445 (1.77246 −.677022). 1.095445/√2 = .774596. Diagram 32 gives integer values for the side lengths of the pyramids and for √2 × 1000 and √3 × 1000. The distance from the West side of the first pyramid to the East side of the third pyramid in Diagram 32 is 774 cubits. In Diagram 33, *h* marks the NW corner of the first pyramid, and *l* marks the SE corner of the third pyramid.

Rectangle *hklm* side lengths equal √π and $\sqrt{\pi/\varphi^2}$. *lm* is bisected at *n*, *p* is the midpoint of *kn*, *rs* = √2/2 and *tw* = √2. *p* is projected to *r* and *q* is projected to *s*, the midpoint of the √2 distance between the East side of the first pyramid and the West side of the third pyramid. The horizontal distance between the West side of the first pyramid and the East side of the third pyramid is 774.596 cubits and the distance between the East side of the first pyramid and the West side of the third pyramid is 1414.213 cubits. The base length of the first pyramid is longer than the base length of the third pyramid by ½ $kh/\sqrt{2}$. $kh = \sqrt{\pi}/\varphi^2 = 677.022$ cubits, $677.022/2 = 338.511$ cubits and $338.511/\sqrt{2} = 239.363$ cubits. $1414.213 - 774.596 - 239.363 = 400.254$ cubits. $400.254/2 = 200.127$ cubits (the base length of the third pyramid). $200.127 + 239.363 = 439.49$ cubits (the base length of the first pyramid). The length from the NE corner of the first pyramid to the SW corner of the third pyramid is $\sqrt{5} \times 1000$ cubits - $\sqrt{3}^2 + \sqrt{2}^2 = \sqrt{5}^2$. The length from the NW corner of the first pyramid to the SE corner of the third pyramid is $\sqrt{3.6} \times 1000$ cubits - $\sqrt{\pi}^2 + (\sqrt{\pi}/\varphi^2)^2 = \sqrt{3.6}^2$.

The second pyramid is located based on the constructions in diagrams 31 and 32. The half base of the first pyramid is 219.745 cubits. The distance from the EW midline of the first pyramid to the East side of the second pyramid is 433.013 cubits ($\sqrt{3} \times 250$). The half base of the third pyramid is 100.063 cubits. The distance from the EW midline of the third pyramid to the East side of the second pyramid is 661.437 cubits ($\sqrt{7} \times 250$). $219.745 + 433.013 + 100.063 + 661.437 = 1414.258$ cubits.

The base lengths of the pyramids are 439.49, 411.437 and 200.127. Given a slope of $4/\pi$ for the first and third pyramids and a slope of 4/3 for the second pyramid, the height of the first pyramid is 279.787 cubits ($219.745 \times 4/\pi$), the height of the second pyramid is 274.290 cubits ($205.718 \times 4/3$) and the height of the third pyramid is 127.404 cubits ($100.063 \times 4/\pi$). The combined base lengths of the three pyramids is 1051.054 cubits ($439.49 + 411.437 + 200.127$). The combined height of the three pyramids is 681.481 cubits ($279.787 + 274.290 + 127.404$). $1051.054 + 681.481 = 1732.535$ cubits.



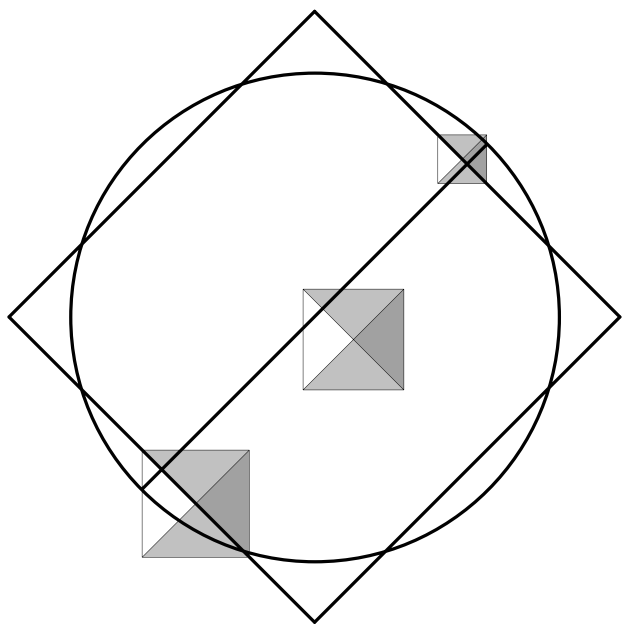
$\sqrt{6} \times \varphi^2/(\varphi^2 + 1) = \sqrt{\pi}$

Half perimeter - $\sqrt{3} + \sqrt{2} = 3.146$

Area - $\sqrt{3} \times \sqrt{2} = \sqrt{6}$

Half perimeter - $\sqrt{\pi} + \sqrt{\pi}/\varphi^2 = \sqrt{6}$

Area - $\sqrt{\pi} \times \sqrt{\pi}/\varphi^2 = \pi/\varphi^2 = 1.2 = 6/5$



Side lengths of square - $\sqrt{\pi} \times 1000$ cubits

Area of square - 3,141,604 square cubits

Radius of circle - 1000 cubits

Area of circle - 3,141,592 square cubits

Footnotes

Part One

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2. Petrie, *The Pyramids and Temples of Gizeh,* p. 55
3. Petrie, *The Pyramids and Temples of Gizeh,* p. 52
4. Petrie, *The Pyramids and Temples of Gizeh,* p. 221
5. Petrie, *The Pyramids and Temples of Gizeh,* p. 191
6. http://home.hiwaay.net/~jalison/entrance.html
7. http://www.grahamhancock.com/phorum/read.php?f=1&i=251501&t=250443#reply_251501
8. Petrie, *The Pyramids and Temples of Gizeh,* p. 39
9. Petrie, *The Pyramids and Temples of Gizeh,* p. 43
10. Petrie, *The Pyramids and Temples of Gizeh,* p. 220
11. Petrie, *The Pyramids and Temples of Gizeh,* p. 81
12. Petrie, *The Pyramids and Temples of Gizeh,* p. 190
13. Petrie, *The Pyramids and Temples of Gizeh,* p. 190
14. Petrie, *The Pyramids and Temples of Gizeh,* p. 189
15. Petrie, *The Pyramids and Temples of Gizeh,* p. 52
16. Petrie, *The Pyramids and Temples of Gizeh,* pp. 51-52
17. Petrie, *The Pyramids and Temples of Gizeh,* p. 189
18. Petrie, *The Pyramids and Temples of Gizeh,* p. 51
19. Petrie, *The Pyramids and Temples of Gizeh,* p. 52
20. Petrie, *The Pyramids and Temples of Gizeh,* p. 55
21. Petrie, *The Pyramids and Temples of Gizeh,* p. 52
22. Petrie, *The Pyramids and Temples of Gizeh,* p. 59
23. Adam Rutherford *Pyramidology Vol III* (1966) p. 1103
24. Petrie, *The Pyramids and Temples of Gizeh,* p. 59
25. Petrie, *The Pyramids and Temples of Gizeh,* p. 80
26. Piazzi Smyth, *Life and Work at the Great Pyramid Vol II* (1867) pp. 108-109 Note: The full text of *Life and Work at the Great Pyramid* is freely available online at books.google.com
27. Petrie, *The Pyramids and Temples of Gizeh,* p. 81
28. Petrie, *The Pyramids and Temples of Gizeh,* p. 189

Part Two

1. Petrie, *The Pyramids and Temples of Gizeh,* p. 65
2. Petrie, *The Pyramids and Temples of Gizeh,* p. 75
3. Petrie, *The Pyramids and Temples of Gizeh,* p. 62
4. Petrie, *The Pyramids and Temples of Gizeh,* p. 220

5. Petrie, *The Pyramids and Temples of Gizeh,* p. 75
6. Petrie, *The Pyramids and Temples of Gizeh,* p. 76
7. Smyth, *Life and Work at the Great Pyramid Vol II*, p. 74
8. Petrie, *The Pyramids and Temples of Gizeh,* p. 195
9. Petrie, *The Pyramids and Temples of Gizeh,* p. 65
10. Petrie, *The Pyramids and Temples of Gizeh,* p. 65
11. Petrie, *The Pyramids and Temples of Gizeh,* p. 71
12. Petrie, *The Pyramids and Temples of Gizeh,* p. 66
13. Petrie, *The Pyramids and Temples of Gizeh,* p. 95
14. Petrie, *The Pyramids and Temples of Gizeh,* p. 66
15. Rudolph Gantenbrink, www.cheops.org
16. Petrie, *The Pyramids and Temples of Gizeh,* p. 53
17. Petrie, *The Pyramids and Temples of Gizeh,* plate 8
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2. Smyth, *Life and Work at the Great Pyramid Vol II*, p. 92
3. Smyth, *Life and Work at the Great Pyramid Vol II*, plate 11
4. Jim Alison, article in *Best of Astraea* (2006), online at home.hiwaay.net/~jalison/gpkc.html
5. Clive Ross, at www.hallofmaat.com/read.php?6,512696,513035#msg-513035
6. Petrie, *The Pyramids and Temples of Gizeh,* p. 77
7. Petrie, *The Pyramids and Temples of Gizeh,* p. 76
8. Petrie, *The Pyramids and Temples of Gizeh,* p. 76
9. Petrie, *The Pyramids and Temples of Gizeh,* p. 194
10. Smyth, *The Great Pyramid*, 1880 (1978 reprint), p. 223
11. Petrie, *The Pyramids and Temples of Gizeh,* p. 193
12. Petrie, *The Pyramids and Temples of Gizeh,* p. 72
13. Petrie, *The Pyramids and Temples of Gizeh,* p. 73
14. Smyth, *Life and Work at the Great Pyramid Vol II*, p. 86
15. Petrie, *The Pyramids and Temples of Gizeh,* p. 71
16. Petrie, *The Pyramids and Temples of Gizeh,* p. 74
17. Petrie, *The Pyramids and Temples of Gizeh,* p. 86
18. Smyth *Life and Work at the Great Pyramid Vol II*, plate 13 and pp. 109-110; Smyth, *The Great Pyramid*, plate 17 and p. 170; Petrie, *The Pyramids and Temples of Gizeh,* p. 81; Maragioglio and Rinaldi, *L'architettura Delle Piramidi Menfite, Parte 4* (1965), p. 48 and plates 7 and 8; J. P. Lepre, The Egyptian Pyramids, 1990 (2006 reprint), p. 103
19. Gantenbrink, www.cheops.org
20. Petrie, *The Pyramids and Temples of Gizeh,* p. 70
21. Robin Cook, *The Horizon of Khufu*, http://sevenislands.tk

Part Four

1. Petrie, *The Pyramids and Temples of Gizeh,* p. 125
2. Petrie, *The Pyramids and Temples of Gizeh,* p. 125
3. Petrie, *The Pyramids and Temples of Gizeh,* p. 125
4. John Legon, www.legon.demon.co.uk/gizaplan.htm
5. Petrie, *The Pyramids and Temples of Gizeh,* p. 114
6. Petrie, *The Pyramids and Temples of Gizeh,* p. 114
7. Petrie, *The Pyramids and Temples of Gizeh,* p. 110
8. Robins and Shute, *Historia Mathematica*,12 (1985), p. 109
9. Miroslav Verner, *The Pyramids* (1997), Appendix 1, citing
Maragiolio and Rinaldi, *L'Architettura della Piramidi Menfite, Parte 6*
10. Petrie, *The Pyramids and Temples of Gizeh,* p. 204
11. Legon, www.legon.demon.co.uk/gizaplan.htm