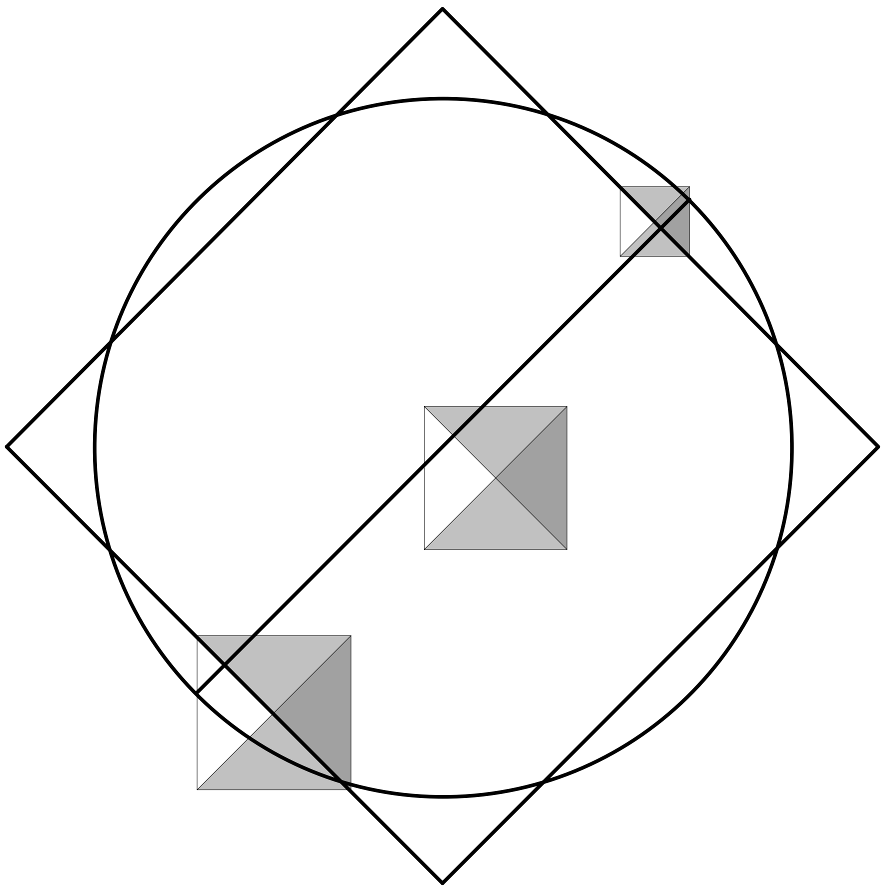
EARTH MEASURES



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The Measure of the Remen and the Royal Cubit;
and the Meridian of Egypt and the Earth

Jim Alison - February 2020
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The area of one square royal cubit is double the area of one square remen. The linear measure of one royal cubit is equal to the linear measure of the remen times the square root of two. Evidence suggests that the remen was the original measure, from which the royal cubit was derived. Given .5238 meters, or 20.62 inches, for the length of the royal cubit, the length of the remen is .3704 meters or 14.58 inches. The meridian circumference of the earth is 40,008,000 meters or 111,133 meters per average degree of latitude. 300,000 remen times .3704 equals111,120 meters. 5000 remen equals one minute of latitude and 500 remen equals one tenth of one minute of latitude. Given 20.62 inches for the length of the royal cubit, the remen expresses the length of an average degree of latitude with greater accuracy than the modern meter, that was fixed before the exact length of the meridian circumference was known to its creators, and unlike the meter, the remen is in unity with minutes and degrees of latitude. Archaeological and textual evidence from throughout ancient Egyptian history, as well as textual evidence from ancient Greek and Roman sources, support a conclusion that the correspondence between the length of the remen and the royal cubit, and the meridian length of Egypt and the earth, was known to their creators.

In 1883, William Flinders Petrie stated: "The digit, from about a dozen examples deduced from monuments, I had concluded to be .7276 inches; here, from three clear and certain examples of it, the conclusion is .727 ± .002 for its length in the fourth dynasty, practically identical with the mean value before found. As I have already pointed out, the cubit and digit have no integral relation one to the other, the connection of 28 digits with the cubit being certainly inexact, and merely adopted to avoid fractions. Now these earliest values of the cubit and digit entirely bear out this view; 28 of these digits of .727 is but 20.36 ± .06, in place of the actual cubit of 20.62 ± .01. Is there then any simple connection between the digit and cubit? Considering how in the Great Pyramid, so much of the design appears to be based on a relation of the squares of linear quantities to one another, or on diagonals of squares, it will not be impossible to entertain the theory of the cubit and digit being reciprocally connected by diagonals. A square cubit has a diagonal of 40 digits, or 20 digits squared has a diagonal of one cubit, thus, a square cubit is the double of a square of 20 digits, so that halves of areas can be readily stated. This relation is true to well within the small uncertainties of our knowledge of the standards; the diagonal of a square cubit of 20.62 being 40 digits of .729 and the actual mean digit being .727 ± .002. This is certainly the only simple connection that can be traced between the cubit and digit." [1]

In 1892, Francis Llewellyn Griffith stated: "The royal cubit of 20.6 inches with its subdivisions into 7 palms and 28 digits is the ordinary measure of length." [2] In 1893, Griffith stated: "This supplement is due to the author of 'The Pyramids and Temples of Gizeh,' for he has given me permission to use the metrological material which he discovered at Tell el-Amarna, and has added to this a sheet of brief but valuable criticisms upon my previous essay:

The *remen* is the side of a square of which the royal cubit is the diagonal and the standard digit is 1/20 of this *remen*...In measures of area, *remen* is the name of the half *arura* of 100 square cubits. If we assume, as I think we fairly may, that the word *remen* was given to this superficial measure because it was the square of 100 of the linear *remen* of the cubit rods, the whole argument is at once clinched by the mathematically correct agreement of the results, as follows:

arura	:	*remen*	::	2 : 1
(100 cubits)^2	:	(100 *remen*)^2	::	2 : 1
* cubit^2	:	*remen*^{2}	::	2 : 1

* The cubit is the diagonal of the square *remen*.

This coincidence of results from three probable hypotheses, two of which are absolutely independent of each other, shows that the assumptions are correct: -

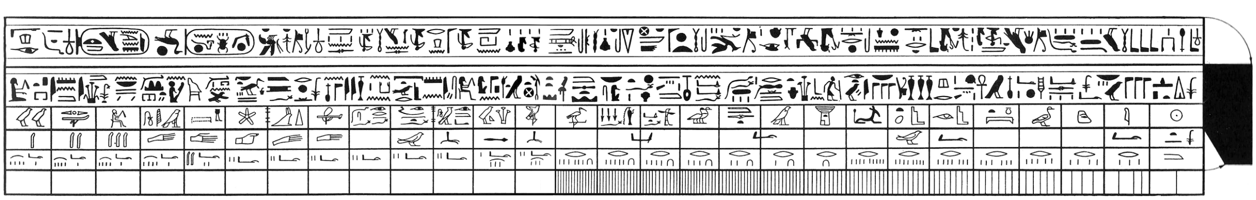
(1) That the standard digit is .729 inches when the cubit is 20.62 inches.

(2) That the *remen* of the cubit rods is 20 of these digits, or 14.58 inches.

(3) That the half *arura* is named *remen* owing to its being a square of 100 *remen* of the cubit rods (while the *arura* itself is a square of 100 cubits).

Also, it is clear from the relation of cubit to *remen* that the *arura* is a square of 100 royal cubits, and that it is the royal cubit and no other that forms the basis of the system of land measurement." [3]

In 1940, Petrie stated: "The unit of linear measure was the royal cubit of 20.6 inches. The half diagonal of this was the remen, a second unit of 14.6 inches, which was divided in 20 digits of .73 inches. Thus, by the use of the diagonal, the half of any square area could be readily formed and defined. That this was fully recognized is shown by the half of the area of 100 × 100 cubits being also called remen in land measure. The result of this system was that the royal cubit was 28.28 digits, commonly reckoned as 28 digits, and on one cubit rod the digit value is exactly retained, and the last digit lengthened, to make up 1.28 with the fraction." [4]



In 1865, Richard Lepsius published a drawing of an ancient Egyptian cubit rod that is presently located in the Turin Museum.[5] On the third register from the bottom, which is the upper register of the bevel: The remen of 20 digits is designated in the middle of the fifth palm, between the 18th and 19th digits; the small cubit of 24 digits is designated in the middle of the sixth palm, on the 22nd and 23rd digits; and the royal cubit is designated on the 27th and 28th digits. Like the cubit rod described by Petrie, the 28th digit is longer than the other digits.

In 1957, Schwaller de Lubicz stated: "The cubit that measures 52.36 centimeters divided by 28.2842, or $20\sqrt{2}$, instead of exactly 28 fingers, results in a very short digit measuring 1.85 centimeters; and 28 of these shorter digits would make a cubit of 51.85 centimeters, which has not yet been found as a cubit of 28 digits, but one often comes across the remen cubit measuring 20 of these digits marked on the large royal cubit...The study of the measures of the cubits demonstrates that the larger part of each royal cubit includes the remen cubit composed of 20 shorter digits, allowing, with two of these remen cubits, for the establishment of a surface exactly double that of the royal cubit." [6] Schwaller published several drawings of ancient Egyptian cubit rods, including the rod drawn by Lepsius, and stated:

> "(The cubit drawn by Lepsius) represents the typical standard cubit. Carved of wood, it is divided into 28 digits, and on its sloping face there is an enumeration of the different divisions: digits, palms, hand, fist, small span, large span, djezer cubit, remen cubit, small cubit, and royal cubit, reading from left to right. On the top face, twenty-eight neters can be seen, each attributed to a digit. Ra, the first of the Ennead, corresponds to the last digit; the neters are therefore enumerated from right to left. Also from right to left, the fractions of the digit can be read on the vertical face; the values are written above. Lepsius gives it a total length of .525 m.
>
> No. 61316, Cairo Museum: This cubit, found in the tomb of Tutankhamun, is made of wood and has the cartouche of the king on its back face. Its total length is .529 m ± .5 mm, maximum. It is divided into seven palms. Read on the five palms to the right, it contains a remen cubit of 20 digits. This cubit corresponds to the division of the length of the entire cubit by the square root of 2.
>
> No 61320, Cairo Museum: This cubit, like the preceding one, was found in the tomb of Tutankhamun and bears his cartouch. It is made of wood and is covered with blue paint. Its total length is .525 m. The 5 palms on the left measure .372 m, that is, 20 shorter digits that result from the division of the total length of this cubit by the square root of 2. In practice, the small remen can be the side of a square, the diagonal of which is equal to the entire cubit." [7]

In 1953 A.E. Berriman stated: "Griffith identified the diagonal of the square on the remen with the cubit Herodotus called royal. This relationship implies that the square on the royal cubit is twice the area of the square on the remen and was of practical convenience in land measurement. The royal cubit can be defined as $\sqrt{2}$ remen = $20\sqrt{2}$ digits = 20.62 inches, for a digit of .729 in." [8] In 1972, Richard Gillings stated: "A double remen was the length of the diagonal of a square whose side was one cubit. Using the royal cubit, a double-remen was therefore 29.1325 inches, and consequently the remen was 14.566 inches. It is thought that the double-remen was used in measuring land, because it enabled areas to be halved or doubled without altering their shapes." [9] In 2005, Dieter Lelgemann stated: "The development of the old Egyptian length units is connected to the Egyptian method to mark off a square such as the ground plan of the pyramid of Cheops; (Petrie 1934) found an old papyrus that described this method. Based on the remen of 370.4 mm, a new length unit was defined: The royal cubit $= \sqrt{2}$ remen = 523.8 mm = 20.62 inches." [10]

In his 1892 article, Griffith stated:

"For land, a measure of 100 cubits named *khet*, and for itinerary measures the *ater* or *schoenus* formed the units: The areas of fields were reckoned in squares of the *khet*, 100 royal cubits: such a square was called in Egyptian *set* and in Greek *arura*, and it was considered to be composed of 100 strips, each one cubit in breadth. The half *arura* was named *remen*, being the square of 100 of the linear *remen* of 5 palms in length (Griffith includes a table showing the *remen* as one-half of the *arura* in the Kahun papyrus, the Rhind papyrus, the Berlin papyrus and the Harris papyrus).

For very long distances there is the *ater*, which apparently corresponds to the *schoenus* of Herodotus and other Greek authors, who value it at 60, 40, 32 or 30 stades. A very interesting inscription published by Brugsch from the temple of Edfu, raises hopes, that are hardly realized, by giving an estimate of the length, breadth, and area of the Egyptian Nile valley: the first, from Elephantine, is 106 *ater*, 'the breadth in level land from the western barrier of Kemt to the eastern barrier likewise is 14 *ater*, comprising (?) 27,000,000 *aruras*.' If 14 *ater* is the average breadth of Egypt, then14 × 106 = 1484, so here we may have an equation between 27,000,000 *aruras* and 1484 square *ater*, giving 18,200 *aruras* to the square *ater*; the *arura* contained 10,000 square cubits, so the square *ater* would have contained 182,000,000 square cubits, and the *ater* of Edfu would be 14,000 cubits." [11]

In his 1893 article, Griffith stated:

"In the 18th dynasty the *atru* is found as a multiple of the *khet*, etc. Its value has not yet been ascertained. The Ptolemaic and Roman *ar*, called by the Greeks *schoenus*, is of uncertain value, although its name is but a later form of the same word: The evidence of the classical authors for the *schoenus* (see Hultsch), as well as that of the Ptolemaic texts for the *ar*, indicate a highly variable measure, from 30 to 120 stades in length; it is possible that the differences may be due to mistakes of ancients and moderns; but compare the vague league called *malakeh* in modern Egypt and Nubia, and the explanation by St. Jerome quoted in Hultsch.

The standard (?) *schoenus* of 12,000 cubits, mentioned by an Alexandrine metrologist, is found marked upon a road of unknown date (Ptolemaic or Roman?). For the *atru* and *ar* see some further instances quoted by Brugsch, *Die sieben jahre der hungersnoth, p. 70 ff*. The intermediate form is especially interesting in both age and orthography between the 18th dynasty *atru* and the late Ptolemaic *ar*. This completing link in the chain of forms of the word is found in an inscription of Darius, and on the stela of the seven years of famine at Sehel.

The stelae of Tell el-Amarna, recently discovered by Petrie, record that, as fixed by Akhuenaten: 'Akhut-aten, from the south stela to the north stela, when measured from stela to stela on the eastern hill, amounts to 6 *atru*, 1 *khet*, 1 *remen*, 1/4 *khet*, 4 cubits.' and further that 'likewise on the western hill from stela to stela

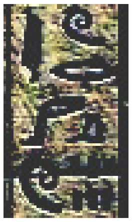
> it is 6 *atru*, 1 *khet*, 1 *remen*, 1/4 *khet*, 4 cubits.' This text gives an early hieroglyphic notation of the *khet* measures; it associates the *atru* with the *khet*, etc., treating it as one of a series and as a measure of fixed length; and further it affords a hope of determining the value of this standard *atru*.
>
> On the Eastern and Western hills of Akhutaten exist a large number of stelae, nearly all of which show fragments of one and the same text. The Eastern hill is the one that was most closely examined by Mr. Petrie, who detected upon it remains of no less than nine rock tablets. On his map, the northernmost is exactly 6 3/4 miles from each of two at the extreme south. Since 6 3/4 miles = 20760 cubits, the *atru* would according to this be about 3460 cubits." [12]

Another boundary stela was discovered by N. de G. Davies in 1901, approximately two miles north of the northernmost stela found by Petrie. In 2006 another boundary stela was discovered by Helen Fenwick. The boundary stelae specified that the royal tombs would be within the boundaries of Akhentaten, but some of the royal tombs are outside of the boundaries of presently known boundary stelae.[13] It is possible that additional boundary stelae remain undiscovered and it is also possible that one or more of the actual boundary stelae were destroyed during or after the destruction of Akhentaten that followed the reign of Akhenaten. After the discovery by Davies of the additional stela, Ludwig Borchardt proposed a special length of 5000 cubits for the *itr* of the Amarna boundary stelae, but he believed that the standard length of the *itr* (from the Edfu inscription and elsewhere) was longer.[14]

In 1944, Alan Gardiner stated: "A fascinatingly interesting inscription at Edfu, of which the first and most important lines were published and translated by Brugsch, *Thesaurus*, pp. 604 ff., gives detailed statistics of the dimensions of Egypt, and indicates as its total length 106 *itr*. This figure is repeated in the charred geographical papyrus from Tanis, with a further dimension of 20 *itr*. The complete elucidation of these data was afforded by some votive cubits found at Karnak, the gist of which was announced by Borchardt in the afore-mentioned note (Attention was called to the decisive evidence by Borchardt as early as 1906, but for more or less comprehensible reasons was overlooked by Sethe and Kees), though Borchardt did not publish the actual inscriptions until much later. It will suffice to reproduce here the crucial words from the best preserved of the three; this dates from the reign of Nekhtharhebe: Sum total of *itr*, 106 complete. Mode of calculating it: Elephantine to Pi-Ha'py, 86 *itr*; from upstream at Pi-Ha'py to the hinterland of Behdet, 20 *itr*. At some very early moment Behdet became known as the northernmost town or village of Egypt. Obviously the compiler of these figures set before himself the task of stating the lengths of the upper and lower Egyptian Niles respectively...The language of the cubits seemed Middle Egyptian, but statistics of such precision appeared to demand an advanced and sophisticated state of society." [15]

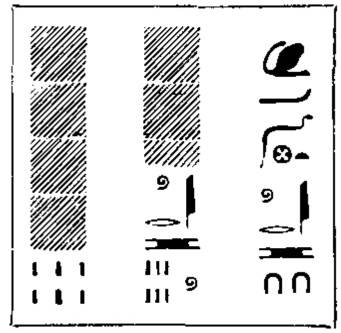
Gardiner also commented on unpublished inscriptions from the White Chapel of Sesostris at Karnak, sent to him by Pierre Lacau: "The division of the Nile from Elephantine to the Mediterranean is the same, and again we find 'the hinterland of Behdet.' Below these figures, however, are others not available to me for publication, and there cannot be any doubt but that all these statistics belong to the same series. Nor is it to be supposed that with the Karnak chapel we are at the beginning of the story; that presumably belongs to the Old Kingdom."

In hieroglyphic notation, the coil is 100, the hobbles are 10 and the single strokes are one.



From the Edfu inscription

Lacau hand copy from the Tanis papyrus



 – Lacau hand copy

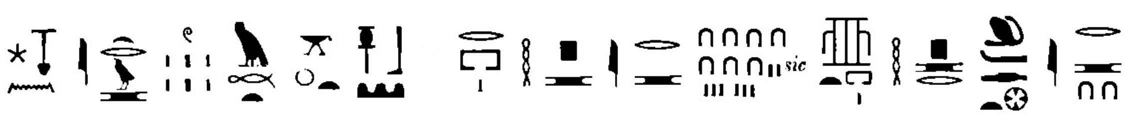
In 1956, Lacau published hand copies of the part of the Edfu inscription that gives 106 *itr* for the length of Egypt, and the part of the Tanis papyrus that gives 106 *itr* for the length of Egypt and 20 *itr* for the length of lower Egypt, and that may have given 86 *itr* for the length of upper Egypt, but the section above the six single strokes is lost.[16]

1. Aus Karnak. Kairoer Museum.

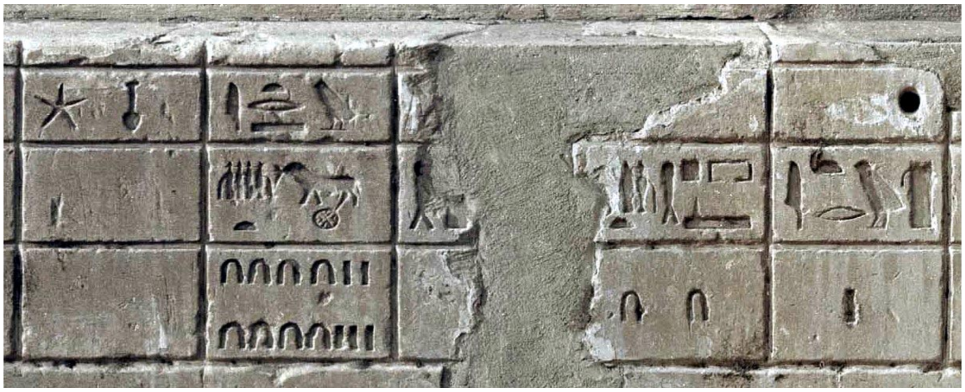
2. Aus Karnak. Kairoer Museum.

Source of Image: Ludwig Borchardt, Altagyptische Zeitmessung

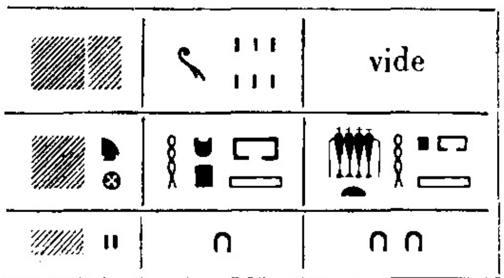
In 1920, Borchardt published photographs of the inscribed cubit rods from Karnak. Both of these rods read from right to left. Gardiner and Lacau both published hand copies (from left to right) of the geographical inscription from the cubit rods:



Gardiner described the presence of the star in combination with the lung and windpipe in the twelfth dynasty as most astonishing.

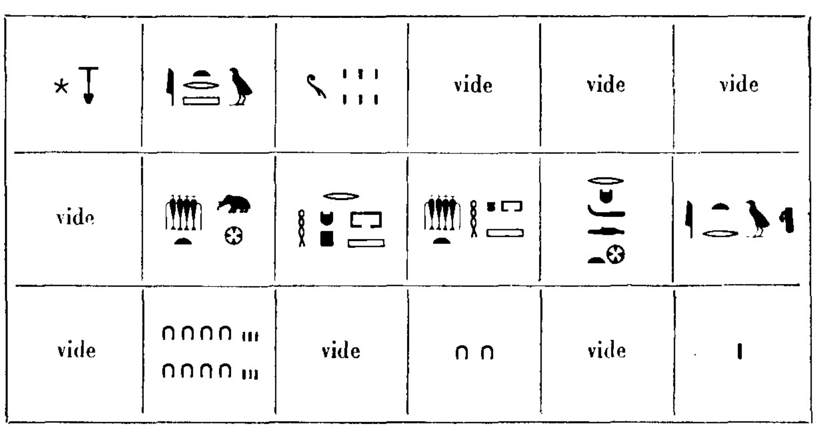


White Chapel - Karnak ***(Source of Image: sith.huma-num.fr/karnak/1027 © cnrs usr3172/cfeetk)***



Inscription from the block found in the Temple of Montu - Karnak

The White Chapel was dismantled and the blocks from the chapel were used as core blocks in the third pylon of the Temple of Amun at Karnak. During modern restorations of the third pylon, the inscribed blocks from the White Chapel were discovered and reassembled. As a result, many of the inscriptions are in very good condition, but some parts of the inscriptions are damaged, destroyed, or missing. Lacau published photographs and drawings of the White Chapel inscription that is similar to the inscription on the cubit rods, but the middle section of the inscription is missing. Lacau also published a drawing of an inscription found by Alexandre Varille on a block in the Temple of Montu at Karnak, that Lacau believed was a double of the inscription from the White Chapel, filling in the missing portion of the inscription. Considering the cubit rods and the other inscriptions, Lacau believed that 85 *itr* for upper Egypt in the White Chapel inscription was a scribal error, and published a drawing that he regarded as the correct restoration of the inscription:



According to Cleomedes, Eratosthenes demonstrated the meridian circumference of the earth by giving the difference in latitudes between Alexandria and Syene of 7°12' (one-fiftieth of the 360° circumference) and the length between Alexandria and Syene of 5000 stadia. Eratosthenes stated that Alexandria and Syene are on the same meridian and that the length of 5000 stadia is a north-south meridian measure of the difference in the degrees of latitude, which it would have to be, to calculate the length of the full 360° meridian circumference of the earth. [17] According to STRABO 17.1.2, Eratosthenes gave a length of 5,300 stadia for the NS length of Egypt from Syene to the sea.

Eratosthenes measured 7°12' as the angle of the shadow of the sun in Alexandria at noon on the summer solstice and he believed that the northern edge of direct vertical sunlight made Syene shadowless at noon on the summer solstice, meaning the angle at Syene was zero. The obliquity of the ecliptic has been slowly decreasing for the past 10,000 years. During the early period of ancient Egypt, Syene was shadowless at the summer solstice, but by Eratosthenes' time, the tropic had moved too far south for the sun to cast no shadow in Syene. Alexandria and Syene are not on the same meridian. They are separated by three degrees of longitude.

Given a measure of 300 royal cubits for the length of Eratosthenes' stadia, 5000 stadia for the meridian distance from Alexandria to Syene is correct. The NS length between Alexandria and Syene is 786 km. 786/5000 =157.2 m. 157.2/300 =.524 m = 20.63 inches. The given distance of 5300 stadia for the NS length of Egypt from Syene to the sea is also correct. The latitude of the northern limit of Egypt is 31°35'. The distance from 31°35' to 24°05' is 832 km. 832/5300 = 157m. 157/300 = .523m = 20.6 inches. 5300 stadia × 300 royal cubits = 1,590,000 royal cubits.

Pierre-Simon Laplace stated: "The celestial arc between Syene and Alexandria, as determined by Eratosthenes, differs little from the results of modern observations. Eratosthenes erred in supposing that Syene and Alexandria existed under the same meridian; he also erroneously supposed that the distance between the two cities was only 5000 stadia, if the stadium which he most probably employed contained three hundred cubits of the nilometer of Elephantine. Then the two errors of Eratosthenes would be very nearly compensated, which would lead us to conclude that this astronomer only employed a measure of the earth, formerly executed with great care, the origin of which was lost." [18]

HERODOTUS 2.6 stated the Egyptian schoinos contained sixty furlongs. HERODOTUS 2.9 gave a distance of 7920 furlongs, or 132 schoinos, for the total length of Egypt from Elephantine to the sea. HERODOTUS 2.7 gave a distance of 1500 furlongs, or 25 schoinos, from Heliopolis to the sea. HERODOTUS 2.18: "The answer given by the Oracle of Ammon bears witness in support of my opinion that Egypt is of the extent which I declare it to be in my own account." HERODOTUS 2.15: "Reckoning its sea coast to be from the watch-tower called of Perseus to the fish-curing houses of Pelusion, a distance of forty schoinos."

Edme-Francois Jomard stated: "There are distances along routes that conform very closely to the latest observations, and the number of stades that the Egyptians reported to Herodotus, to Diodorus, and to Strabo, when these travelers asked them about the distances to places, was very exact...the distances one finds in these authors works are not of the traveled routes; rather, they are straight-line measurements." [19]

Petrie stated: "This Fayum road is a most interesting, and so far unique, example of an ancient Egyptian road, with its way-marks. Probably it may be assigned to the Ptolemaic period, when Arsinoe, Bacchis, and the temple of Kasr-Kerun, all show the flourishing state of the Fayum. The distances of the way-marks show unmistakably what was the itinerary system of the time; the decimal cubit lengths, ended by a schoinos measure of 12,000 royal cubits." [20] PLINY 12:30: "The length of the schoinos, according to the estimate of Eratosthenes, is forty stadia." In 1882, Friedrich Hultsch stated: "Eratosthenes set the stadium at 300 royal cubits, thus equal to the fortieth part of the schoinos." [21] In 2010, Duane Roller stated: "One suspects that Eratosthenes' sources provided data in schoinoi and that he converted these, obviously at 40 stadia to a schoinos." [22]

Elephantine and Syene have the same latitude of 24°05'N. If the lengths given by Herodotus were based on the late period Egyptian schoinos of 12,000 royal cubits, converted by Herodotus into sixty furlongs per schoinos, then the total distance of 132 schoinos times 12,000 equals 1,584,000 cubits, a correct measure of the total meridian length of Egypt and virtually the same as the distance of 1,590,000 cubits reported by Eratosthenes. 12,000 royal cubits times 25 schoinos equals 300,000 royal cubits. The latitude of Heliopolis is 30°08'N. The NS distance from 30°08' to 31°35' is 160 km. 160/300,000 = .533 m = 21 inches per cubit.

5,300 stadia, given by Eratosthenes as the length of Egypt from Syene to the sea, is a 50x multiple of 106 *itr*. 300 cubits × 50 = 15,000 cubits and 15,000 cubits × 106 = 1,590,000 cubits. The distance given by Herodotus of 25 schoinos from Heliopolis to the sea, based on the schoinos of 12,000 cubits, is equal to 20 *itr* of 15,000 cubits, which is the number of *itr* given on the Karnak cubit rods for the distance from the hinterland of Behdet (northern limit of Egypt) to Pi-Ha'py.

In Gardiner's article about Behdet and the cubit rods, he stated: "Pi-Ha'py is now known to be, not the island of Rodah opposite Old Cairo, but Atar en-Naby on the east bank 2 km. farther south." [23] In 2015, following a multi year geological and archeological study in the area of Rodah, Old Cairo and Atar en-Naby, Peter Sheehan stated: "The archaeological evidence noted during the groundwater lowering project clearly indicates the existence of a settlement predating the Roman period. This settlement was probably founded during the Saite or Persian periods in the seventh or sixth century BC. None of the ceramic material recovered from the shafts excavated to the natural rock in the eastern part of the fortress area appears to be earlier in date than the Third Intermediate Period...The remains noted to the south of Old Cairo at Athar al-Nabi appear to date from the Late Period and the Ptolemaic-Roman period, notwithstanding the *ex situ* find of a monumental statue of the New Kingdom king Merenptah. We may conclude that in the second millennium BC the wetter prevailing conditions meant that the future Babylon lay within the flood plain or perhaps even within a still functioning paleo-channel. Only with the lower floods from the beginning of the first millennium BC did it become feasible to settle there...The location of Babylon at the southern limits of the city of Heliopolis (ancient On) has in more recent times suggested a derivation from the hypothetical *pr-H'pi n iwnw* (the house [that is the temple] of [the river god] Hapi in [the city of] On). The first century BC accounts of Siculus and Strabo of the foundation of Babylon have thus been considered to reflect ancient attempts to explain the etymology of Babylon, as having come about through the occidentalizing of an Egyptian toponym. However, both of these authors' accounts show clearly that by 50 BC a view existed of the foundation which could very plausibly represent the establishment of a trading colony or military base by the Persian kings. ('they named

Babylon from their native land' -- Siculus -- 1:56:3) ('Babylon was built by some Babylonians who had taken refuge there' -- Strabo -- 17:1:30)...The tremendous symbolic significance of the junction between the Nile Valley and the Delta was recognized and celebrated by the ancient Egyptians. The area was called Kher-Aha, 'the battlefield,' and was identified as the site of the mythical battle between Horus and Seth, perhaps a reflection of real battles between the peoples of Upper and Lower Egypt in the Predynastic period, such as those shown on the Narmer Palette." [24]

In 2003, S. el-Kholi stated: "The name of Kheraha according to PT 550 and PT 1272 refers to the field of the battle in which the fight between Horus and Seth took place, and in whose vicinity (Pi-Hapy) the dividing line between Upper and Lower Egypt is supposed to lie." In Faulkner's footnotes to § 1272, he stated: "Location unknown." el-Kholi also stated: "Gardiner has shown that Nilopolis, in which Pi-Hapy was supposed to be found, can not be identified with Kheraha, namely Babylon, but that it should be two kilometers to the south in Athar al-Nabi. Moreover, in a later study of the location of Pi-Hapy, it was proved that it should be a little south from where Gardiner suggested (four kilometers northwest of Helwan)." [25]

1n 1953, G. A. Wainwright stated: "But more important at the moment is the division, such as we get in a Ptolemaic papyrus of the river into 'the Nile of Upper Egypt which is in Bigah' and 'the Nile of Lower Egypt which cometh forth from Heliopolis'. This, surely gives the clue, for it is here said that the Nile of Upper Egypt begins at Bigah and the Nile of Lower Egypt starts at Heliopolis. Heliopolis is at the apex of the Delta near Cairo, and Bigah is one of the islands at the southern end of the First Cataract, and it, rather than Elephantine at the northern end, was sometimes considered as the frontier of Egypt." In the footnotes to this section, Wainwright stated: "This division is not a late idea of Ptolemaic times, but without being as expressly stated goes back at least to the fourteenth century BC. At that time it appears in Nu's statement just referred to. The whole passage reads: 'There is this serpent belonging to them in the Two Caverns of Elephantine at the gate of the Inundation-god. He cometh with water; he riseth up at that district of Kher-aha with his company of gods; Head of the inundation.' Here we have Elephantine opposed to Kher-aha, which was a town a little up-river from Heliopolis and Cairo at the apex of the Delta." [26]

In 1905, Wallis Budge translated from the papyrus of Nu: "The mountainous region of Kher-Aha, the god of which was Hap, the Nile." Budge Stated: "Kher-Aha represents a region quite close to Heliopolis." [27] In 1895, Budge translated from the Papyrus of Ani: "Thou createst in Kher-Aha in Heliopolis," and interpreted this translation as: "Thou createst [what is] in Kher-Aha [and what is] in Heliopolis." Budge also translated from the Papyrus of Ani: "He (Osiris) goeth round about heaven robed in the flame of his mouth, commanding Hapi, but remaining himself unseen." [28]

In 1894, Gaston Maspero stated: "The Canopic branch flowed westward, and fell into the Mediterranean near Cape Abukir, at the western extremity of the arc described by the coast-line. The Pelusiac branch followed the length of the Arabian chain, and flowed forth at the other extremity; and the Sebennytic stream almost bisected the triangle contained between the Canopic and Pelusiac channels. Two thousand years ago, these branches separated from the main river at the city of Cerkasoros, nearly four miles north of the site where Cairo now stands. But after the Pelusiac branch had ceased to exist, the fork of the river gradually wore away the land from age to age, and is now some nine miles lower down." In the footnote to Cerkasoros, Maspero stated: "According to

Brugsch, the name of Kerkasoros (Herodotus, II 15, 17, 97), or Kerkesoura (Strabo, xvii p. 806), has its Egyptian origin in *Kerk-osiri*. Herr Wilcken proposes to correct the text of Herodotus and Strabo, and to introduce the reading Kerkeusiris in place of Kerkasoros or Kerkesoura. Professor Erman considers that Kerkeusiris means *The Habitation of Osiris*."[29] Maspero translated a hymn to Hapi from the Sallier papyrus and Anastasi papyrus that said "The place of his dwelling is unknown."[30]

HERODOTUS 2.15: "If we desire to follow the opinions of the Ionians as regards Egypt, who say the Delta alone is Egypt, reckoning its sea coast to be from the watch-tower called of Perseus to the fish-curing houses of Pelusion, a distance of forty schoinos, and counting it to extend inland as far as the city of Kercasoros, where the Nile divides and runs to Pelusion and Canobos." STRABO 17.1.30: "After Heliopolis is the Nile above the Delta. The country on the right, as you go up the Nile, is called Libya; the country on the left is called Arabia. The territory belonging to Heliopolis is in Arabia, but Kerkesoura is in Libya, situated opposite the observatory of Eudoxus, an observing station in front of Heliopolis, where Eudoxus marked certain motions of heavenly bodies."

Source of Image: Wikimedia Commons: Sematawy Photo Credit: Olaf Tausch, Own work, CC by 3.0. http://commons.wikimedia.org/w/index.php?curid=19533856

Hapi, the androgynous Nile god, was also a dual god of the upper Nile and the lower Nile. The carved relief from the temple of Luxor shows Hapi on both sides of the lung and windpipe, uniting the single stream of upper Egypt with the multiple streams of lower Egypt.

Unlike the locations of Pi-Hapy and Kher-Aha, the location of Kerkasoros is known. It is located at 30°05'N, on the western bank of the Nile at the apex of the Delta, where the Pelusiac channel diverged from the Nile and turned to the north-east, towards Heliopolis. As cited by Maspero, Kerkasoros could be regarded as the habitation of Osiris. It could also be regarded as the battlefield in the sense that it is the place where the Nile (Osiris?) is cut into pieces by the divergence. Livio Stecchini proposed that Kerkasoros was an occidentalization of Kher-Aha and that Pi-Hapy was in the immediate vicinity. As also suggested by Wainwright, Stecchini believed that the ancient Egyptians measured the southern border from both Syene-Elephantine (24°05'N) and from Biga-Philae (24°00'N). Stecchini believed that the ancient Egyptians measured by meridian degrees and he believed that the north-south extent of Egypt was regarded as 7°30', or 1/48th of the total circumference of the earth, regardless of which southern boundary was being used. He believed that the location of these boundaries at significant ratios in relation to the full circumference of the earth was intentional.[31] Given a southern boundary of 24°00'N, the NS distance to 31°30'N is 106 *itr* of 15,000 cubits. The distance from 24°00'N to Kerkasoros at 30°05'N is 86 *itr* and the distance from 30°05'N to 31°30'N is 20 *itr*.

Given Elephantine as the southern boundary at 24°05'N, and the northern boundary at 31°35'N, the NS distance is 106 *itr* of 15,000 cubits, and the division into 20 and 86 *itr* occurs at 30°10'N. The divergence of the Canopic and Sebennytic channels is presently located at 30°10'N and this may have been the largest divergence in ancient times, considering the eventual extinction of the Pelusiac channel. The latitude of the center of Heliopolis is 30°08'N, about half-way between the latitudes of the two divergences, and the latitude of the district of Heliopolis may have extended to either or both.

After he found the inscribed cubit rods, Borchardt proposed that the length of the *itr* was 20,000 royal cubits, based on what Gardiner called: "good grounds." [32] According to Stecchini and Peter Tompkins, Borchardt's grounds were: "One must absolutely exclude the possibility that the ancients may have measured by degrees." [33] Since the meridian distance of 106 *itr* of 15,000 cubits is the precise NS length of Egypt, the *itr* had to be longer. Since 20 *itr* of 15,000 cubits only equals 15 *itr* of 20,000 cubits, the location of Pi-Hapy had to be south of the apex of the delta for the proposed distance of 20 *itr* of 20,000 cubits from Pi-Hapy to the northern limit of Egypt.

In 1973, Hans Goedicke stated: "I tend to understand *ab3* as 'survey, calculation, estimation,' i.e. a projected amount rather than a factual sum. Thus the first part of the statement could be read 'total estimation 106 *itrw* in full.' The course of the Nile is calculated in two sections: the southern one reaches from Elephantine to *Pr-H'pi*, in which the Nilometer of Memphis has to be recognized, the northern one from there to *ph(w) Bhdt*. The opinions voiced concerning the latter remain highly unconvincing, which also applies to the discussion of the term *phw-phwy* rendered as 'northern limit.' A more specific point would seem necessary for establishing measurements, for which a rendering 'harbor' for *phw* would recommend itself. The southern limit is assumed to have been politically, administratively, and geographically in Elephantine throughout Egyptian history, which is an unfortunate oversimplification of an important problem. Urk. I 110-11 indicates the political frontier in the Old Kingdom was somewhat south of Elephantine. The story of the Shipwrecked Sailor suggests that Semneh was the actual border at the beginning of the Middle Kingdom. The discussion of *itrw*, simultaneously investigated by Oertel, *Herodots Agyptischer Logos und die*

Glaubwurdigkeit Herodots (Bonn. 1970), pp. 39 ff. (Which provides a more complete synopsis of previous treatments of this question) after carefully weighing the available evidence, supports the view that *itrw* is a road measure of approximate length (10.5 km.) rather than a specific length." [34]

According to Cleomedes, Eratosthenes used a sundial to measure the angle of the shadow of the sun in Alexandria on the summer solstice, compared to his given angle of zero for Syene at the summer solstice, to obtain the difference in latitude.[35] Petrie stated: "The Egyptians regularly worked with a dial for measuring the altitude of the sun, by a shadow on a horizontal,-or later a sloping,-surface, with scales for the variation in each month." [36] Cleomedes also stated: "At winter solstices sundials are positioned in each of these cities, and when each sundial casts shadows, the shadow at Alexandria is necessarily determined as the longer because this city is at a greater distance from the winter tropic. So by taking the amount by which the shadow at Syene is exceeded by that at Alexandria, they also determine this amount as 1/50 part of the great circle in the sundial." [37] The shadow from the summer solstice, compared with the shadow from the winter solstice, will also give the latitudes of the tropics in relation to the equator. In Alexandria at the summer solstice the shadow was 7°12' and at the winter solstice it was 55°12', a difference of 48°. One-half of 48° marks the tropics in relation to the equator and 24° plus 7°12' marks the latitude of Alexandria at 31°12', in relation to the equator.

Citing Eratosthenes, STRABO 2.5.7 stated: "The space from the equator to the pole contains fifteen of the sixty divisions into which the equator itself is divided. There are four divisions between the equator and the summer tropic or parallel passing through Syene. 15/60 = 1/4 = 90° from the equator to the pole and 4/60 = 1/15 = 24° from the equator to the latitude of the tropics. The distances for each locality are calculated by the astronomical observations." Strabo cites Eratosthenes for 5000 stadia from Syene to Meroe. The latitude of Meroe is 16°56', the latitude of Syene is 24°05' and the latitude of Alexandria is 31°12'. Eratosthenes gave 5000 stadia as the meridian distance from Alexandria to Syene and 5000 stadia as the meridian distance from Syene to Meroe because the difference in latitude is the same. This is what Strabo means by 'distances calculated by astronomical observations' and what Borchardt means by 'measured by degrees'.

Jaromir Malek stated: "The area south of the first cataract was from the earliest times regarded as belonging to Egypt by right." [38] The ancient Egyptians maintained their southern border at 24°00'N, even though significant cities, temples, cemeteries and monuments extended far south of the first cataract throughout ancient Egyptian history. The distance from the equator to 24°00'N is 1/15th of the circumference of the earth. The distance from the equator to Thebes, located at 25°42'N, is 1/14th of the circumference of the earth.

From the equator to 27°41'N is 1/13th of the circumference of the earth. The latitude of the main ruins of Amarna is 27°38'N, although several surviving boundary stelae indicate that the NS dimension of Amarna was six *itr*, plus a small additional fraction of uncertain length. The actual boundaries are not marked by most, if any, of the surviving stelae. Sixteen boundary stelae have been discovered at Amarna. Based on the assumption that the most northern and southern of the surviving boundary stelae marked the actual boundaries, it has been suggested that the *itr* in the boundary stelae may have been as short as 5000 cubits.[39] Even if this were true, six *itr* of 5000 cubits would be approximately 10 minutes of latitude, more than enough to include the latitude of

27°41'N for Amarna. Amarna is also in the NS center of Egypt. The midpoint from 24°00'N to 31°30'N is 27°45'. This is also within the boundaries for Amarna, even with a shorter *itr*. Assuming that the *itr* in the boundary stelae was 15,000 cubits, Stecchini pointed out that since Amarna is at the NS center of Egypt, 106 *itr* for the total length of Egypt would be reduced by the six *itr* of the Amarna district to 50 *itr* north and 50 *itr* south of Amarna.[40]

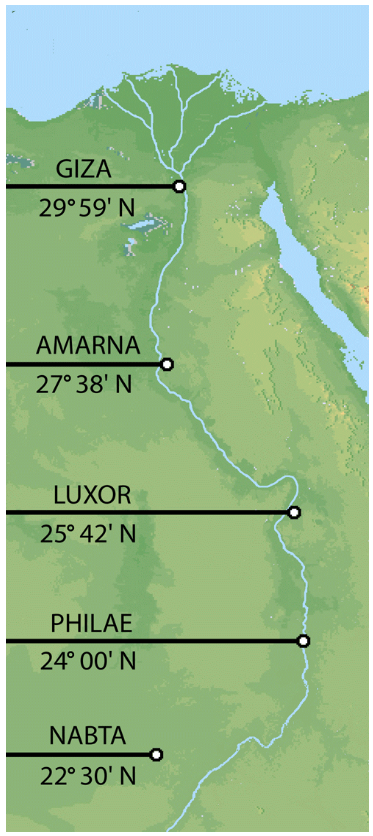


Diagram 1 ***(Map by Vector Globe © Cartografx)***

The distance from the equator to 30°00'N is 1/12th of the circumference of the earth. The distance from the southern boundary at 24°00'N to 30°00'N is 1/15th of the distance from the equator to the pole and 1/60th of the circumference of the earth. The latitude of the great pyramid, on the northern edge of the Giza Plateau, is 29°59'N, in the immediate vicinity of 30°00'N. The distance from 24°00'N to the northern border of Egypt at 31°30'N is 1/12th of the distance from the equator to the pole and 1/48th of the circumference of the earth.

Astronomically aligned stone circles at Nabta Playa have been dated to 4,000 B.C. Located at 22°30' N, Nabta is 1/4th of the distance from the equator to the pole. The distance from Nabta to the equator is 1/16th of the circumference of the earth. Nabta is 1°30' south of Biga-Philae, or 1/5th of the length of Egypt from 31°30'N to 24°00'N. The 9° from 31°30'N to 22°30'N is 1/10th of the distance from the equator to the pole and 1/40th of the circumference of the earth. The fort and the boundary stela at Semneh, located at 21°30'N, is 2°30' south of Biga-Philae, or 1/3rd of the length of Egypt from 31°30'N to 24°00'N. The 10° from 31°30'N to 21°30'N is 1/9th of the distance from the equator to the pole and 1/36th of the circumference of the earth.

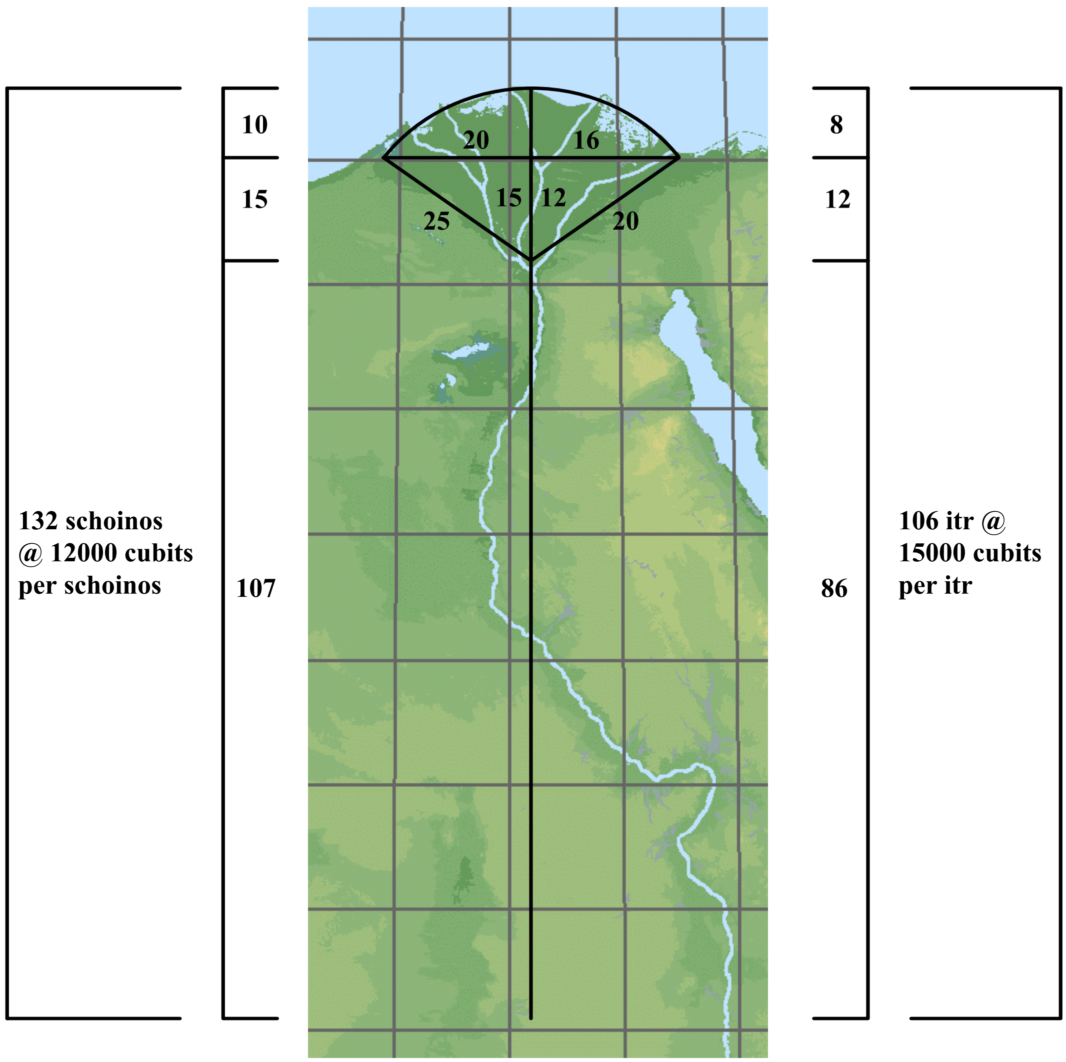


Diagram 2 ***(Map by Vector Globe © Cartografx)***

The NS distance from Elephantine to the northern limit of the Delta is 7.5°, from 24°05'N to 31°35'N. 7.5°/132 schoinos = .0568° per schoinos and .0568° × 25 = 1.42° or 1°25' for the NS distance of 25 schoinos from the northern limit of the Delta to Heliopolis. 31°35' − 1°25' = 30°10' for the latitude of Heliopolis. Every point along the arc marking the coast of the Delta is also 25 schoinos from the apex. Herodotus' measure of 40 schoinos from Pelusium to the other side of the Delta as a straight-line measure forms a right triangle with a hypotenuse of 25 schoinos and one side of 20 schoinos. Perforce, the other side of the triangle is 15 schoinos. .0568° × 15 = .852° = 51'. The latitude of Pelusium is 31°02'N. The 30°10' latitude of Heliopolis plus 51' equals 31°01'. The longitude of Pelusium is 32°32'E. 40 schoinos times .0568° of latitude per schoinos equals 2.272°. Degrees of longitude at 31°N equal 6/7 of a degree of latitude. 2.272° × 7/6 = 2.65° of longitude. One-half of 2.65° is 1.325° or 1°20', for 20 schoinos of longitude at 31°N. 32°32'E minus 1°20' equals 31°12'E. The longitude of Kerkasoros at the apex of the Delta is 31°13'E. Diagram 2 shows the schoinos measures of Herodotus, alongside the *itr* measures of the inscribed cubit rods.

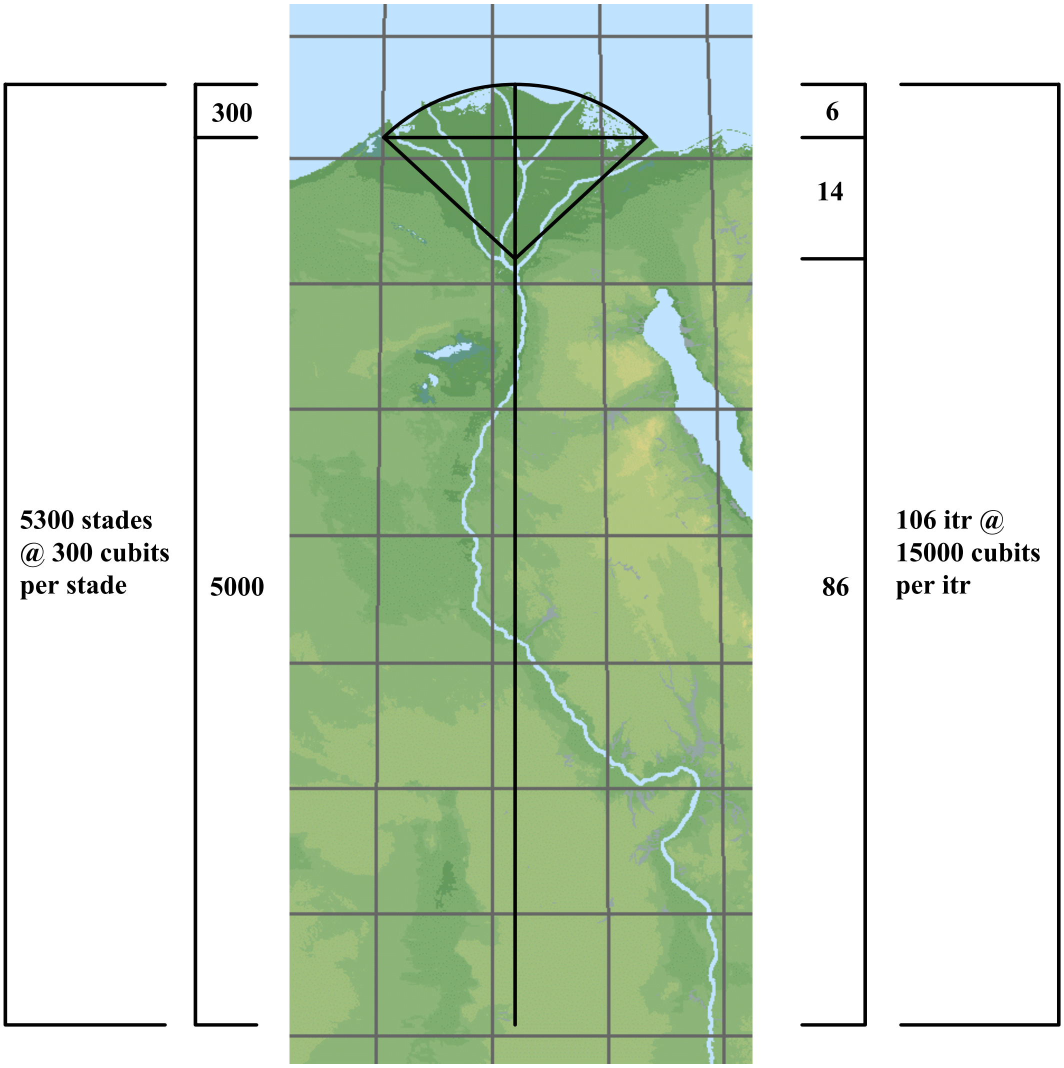


Diagram 3 ***(Map by Vector Globe © Cartografx)***

The NS distance from Syene to the northern limit of the Delta is 7.5°, from 24°05'N to 31°35'N. 300 stadia times 300 cubits per stadia equals 90,000 cubits. Six *itr* times 15,000 cubits per *itr* equals 90,000 cubits. 5,000 stadia times 300 cubits per stadia equals 1.5 million cubits. 100 *itr* times 15,000 cubits per *itr* equals 1.5 million cubits. Dividing 7.5° by 106 *itr* equals .07075° per *itr* and .07075° times six equals .424° or 25'. 31°35'N minus 25' equals 31°10' for the latitude of Alexandria. The latitude of the center of Alexandria is 31°12'N. The Egyptian name for the site of Alexandria was Raqote. The Egyptian quarter of Alexandria, called Rhakotis, is south of the center of Alexandria. The Serapeum was located in Rhakotis and maintained a library sometimes referred to as the daughter of the great library of Alexandria. Eratosthenes was the head librarian. The latitude of the Serapeum was 31°10'55"N. The ancient Egyptian city of Buto is due east of Alexandria, located at 31°11'47"N. Diagram 3 shows Eratosthenes' measures in stadia, alongside the *itr* measures of the inscribed cubit rods.

In 1949, Alton Moody stated:

> "The Greek stadium was 600 Greek feet long, but as the length of the foot varied, so did the stadium. In Attica it was 607.9 U.S. feet, or almost exactly one-tenth of a modern nautical mile. The Romans adopted this unit and extended its use to nautical and even astronomical measurements. The Roman stadium was 625 Roman feet long, or 606.3 U.S. feet.
>
> Eratosthenes of Alexandria attempted to measure the size of the earth during the third century before Christ and determined the circumference as 250,000 stadia, which he rounded off to 252,000 stadia so that each degree would have 700 stadia. The Almagest, still being published in the 15th century, 1,300 years after its first edition, considered 62 Roman miles equivalent to one degree. An edition appearing in 1466 contained a chart of southern Asia on which 60 Roman miles were shown to a degree. Whether the shift from 62 to 60 miles to a degree was considered a correction or an adaptation to provide a more convenient relationship between the mile and degree is not clear, but this is the first known use of the relationship that has gradually replaced all others. The modern minute-mile was thus born quite naturally and unpretentiously. In 1735 an expedition from the Paris Academy was sent to 'Peru' (within the present boundaries of Equador) to measure an arc of the meridian, to provide a more accurate determination of the size of the earth. Pierre Bouguer, a member of the expedition, stated: 'The Italians use miles, which count as 1000 geometrical or double paces, each of five feet; and they suppose that 60 of these miles make one degree. This method of counting distances is very convenient, but it is therefore necessary to modify its length, and increase it by one-seventh.
>
> As for the nautical mile, Norwood, after his measurement of an arc of the meridian, proposed that the length be established at 6120 feet. He later changed this to 6,000 feet. His mile was gradually accepted by seamen, but it was not known by the distinctive name nautical mile until a century later, the expression first appearing in 1730. Although the nautical mile as a minute of a great circle of the earth was now well established, there remained the problem of determining accurately the size and shape of the earth, so that the length of the nautical mile might be precisely defined. The metric system was legalized in the U.S. on July 28, 1866, when the length of the meter was defined as 39.37 inches. Since there was no standard U.S. unit of length, this, in effect, defined the U.S. system of length measurements in terms of the meter. Having selected the figure of the earth upon which to base our standard nautical mile (the Clark spheroid of 1866), it becomes necessary to define which great circle shall be used. In the U.S. the official value is that of one minute of arc of a great circle of a sphere having an area equal to that of the earth. The official U.S. value is 6080.20 feet. Neither the U.S. definition nor the U.S. standard length is universally accepted." [41]

In 1954 the U.S. adopted the international nautical mile of 1,852 meters, or 6,076.1 feet. Britain adopted it in 1970.

The remen value of .3704 m (20.622 inches/$\sqrt{2}$) × 5000 = 1852 meters, or 6076.1 U.S. feet, is an exact expression of one nautical mile. Moody's value of 607.9 U.S. feet for 600 Attica feet is almost exactly one-tenth of a nautical mile and almost exactly 5/6 of the length of one remen for the length of one Attica foot. Attica was a part of the Mycenaean civilization from 1600-1100 BC, prior to the Greek dark ages that preceded classical Greece. Maritime activity was a mainstay of the Mycenaean civilization and significant Egyptian influence is shown by Mycenaean remains.[42] However, the early period of Egyptian civilization was considerably older than the Mycenaean civilization. Charles Treat stated: "Although theories concerning the geographical origins of systems of weights and measures abound, ancient Babylon and Egypt usually share the credit." [43]

When the United States considered adopting the metric system, Thomas Jefferson's report to congress in 1790 stated: "To obtain uniformity in measures...the globe of the earth itself indeed might be considered as invariable in all it's dimensions, and that it's circumference would furnish an invariable measure. But no one of it's circles great or small is accessible to admeasurement through all it's parts: and the various trials to measure definite portions of them have been of such various result, as to shew there is no dependance on that operation for certainty. The motion of the earth round it's axis, tho' not absolutely uniform and invariable, may be considered as such for every human purpose. It is measured obviously but unequally, by the departure of a given meridian from the sun, and it's return to it, constituting a solar day. Throwing together the inequalities of Solar days, a mean interval, or day, has been found, and divided by very general consent into 86,400 equal parts. A pendulum, vibrating freely in small and equal arcs, may be so adjusted in it's length as by it's vibrations, to make this division of the earth's motion into 86,400 equal parts called seconds of mean time. Such a pendulum then becomes itself a measure of determinate length, to which all others may be referred, as to a standard." [44]

The U.S. rejected the metric system and the pendulum proposal, but Jefferson was correct that the proposed meter was not quite 1/40,000,000th of the meridian circumference of the earth. After widespread adoption of the metric system, Friedrich Struve conducted a survey from Hammerfest, Norway to the Black Sea between 1816 and 1855, establishing that the meridian circumference was approximately 40,008,000 meters. A number of the station points of the Struve geodetic arc, consisting of drilled holes in rock, iron crosses, cairns, or built obelisks, have been designated as UNESCO world heritage sites.

The approval of the Struve geodetic arc to the world heritage list was accompanied by the following comments: "The original arc consisted of 258 main triangles with 265 main station points. The listed site includes 34 of the original station points. The arc covers 25°20'8" of latitude equaling 2,820 km. The selection and preservation of the original points actually used is essential if some day future researchers want to re-measure the arc and to be able to compare results and interpret the evolution of the values." [45]

After the equatorial survey was completed, buildings of pyramidal form were constructed to memorialize the end points of the upper baseline and to commemorate the survey.[46] These surveys were not conducted along the same NS meridian. The Struve survey extended for 25° of latitude, over 3° of longitude. However, a continuous north-south measure was maintained to determine the distance of the degrees of latitude.

In describing his survey of the Fayum Road, Petrie stated: “As the pyramids are valuable survey signals, it is just as well to state here their positions as approximately fixed for this survey. N. to S. pyramids, Dashur, 6702 feet at 171°13'. N. to Step pyramid, Sakkara, 23,000 feet at 8°10'. S. to Step pyramid, 29,476 feet at 4°22'. N. to 2nd pyramid, Gizeh, 66,173 at 158°33'. Levels above highest Nile deposit in plain; top of S. pyramid 450 feet, top of N. pyramid 456 feet, top of Step pyramid 338 feet.” [47]

Miloslav Verner stated: “At Seila, on the edge of the Fayyum oasis some ten kilometers west of the Meidum pyramid, stands the remains of a small pyramid that has long been known to Egyptologists, but whose date and builder were unknown. In the course of the excavations conducted during the second half of the 1990's the Egyptian archaeologist Swelem, working with an American expedition from Brigham Young University, discovered written evidence that supported the view that Snefru had also built this pyramid. In contrast to the three other, much larger pyramids, this one had no underground chambers and could therefore not have been used as a tomb. What was its true meaning? The pyramid is one of a group of seven small but, in many respects, enigmatic step pyramids that are scattered all over Egypt. The northernmost of them is in Seila, while the southernmost is near the first cataract of the Nile, on the small island of Elephantine.” [48]

Gregory Marouard and Hratch Papzian stated: “Determined to find a funerary chamber under these monuments, early Egyptologists had cut large trenches or deep tunnels through the faces to no avail, save for contributing to the irreversible degradation of most of the monuments. The location of small step pyramids in considerable distance to the established Old Kingdom royal cemeteries bestows upon these structures the character of non-funerary monuments that did not nor were intended to serve as a burial place of any kind. Based on their shared design, similar dimensions, and construction techniques, the small step pyramids are contemporaneous to one another and date to the very beginning of the Fourth Dynasty, although an earlier date at the end of the Third Dynasty might also be very likely and should not be discarded. They are traditionally attributed to the reign of Huni or even his successor Snofru (2600-2575 BC). The seven examples actually known are not quite numerous enough to draw any firm conclusions about the exact date and function. The Edfu pyramid is located 5 km south of Tell Edfu, and at 25 km south of the pyramid of al-Kula, which is linked to the major Predynastic site of Hierakonpolis. The pyramid is situated between the edge of the desert and the cultivated areas of the Nile Valley.” [49]

Verner stated: “It is not impossible that similar small pyramids were also built in other places. For example, in the nineteenth century one could still be seen near Benha (ancient Athribis) in the central delta. Opinions regarding these pyramids vary widely. Lauer considers them to be queens’ centotaphs in the provinces where they were born. Maragioglio and Rinaldi thought they were shrines connected with the myth of Horus and Seth. According to Arnold, they embodied the memory of the ‘high sand’ or primeval mound on which life was created. Swelim believes they were sites of the sun cult, and thus in a sense predecessors of the later sun temple. Kaiser and Dreyer offer an interesting interpretation: that they are symbols of palatinates that were established near provincial centers and royal residences, and were intended as reminders of the ruler’s presence and authority in places far from the capital. Clearly the last word in the debate regarding these structures has not yet been spoken.” [50]

If the small pyramids were intended to be used for, or to memorialize and/or to commemorate a meridian survey of Egypt, it would not be necessary for the pyramids to be at particular locations or latitudes, but only that their locations be fixed, to allow re-measure of the same arc in the future. The latitudes of the seven pyramids are: Seila -29°23'; Zawiyet El-Meiyitin -28°03'; Sinki -26°09'; Naqada -25°56'; Kula - 25°07'; Edfu - 24°53'; and Elephantine - 24°05'.

The 7.5° meridian length of Egypt, divided by 106 *itr* = .07075° per *itr*. 30 ×.07075 = 2.12°, or 2°07'. From Zawiyet El-Meiyitin to Naqada is 2°07', or 30 *itr*. 26 × .07075 = 1.84° or 1°50'. From Naqada to Elephantine is 1°51', or 26 *itr*. The meridian distance of 56 *itr* from Zawiyet El-Meiyitin to Elephantine leaves 50 *itr* from Zawiyet El-Meiyitin to the 31°35'N limit of Egypt. 50 ×.07075 = 3.53° or 3°32'. At 28°03'N, Zawiyet El-Meiyitin is 3°32' south of 31°35'N. The meridian distance from Zawiyet El-Meiyitin to 30°10'N in the district of Heliopolis and the apex of the Delta is also 2°07', or 30 *itr*, and the distance from 30°10'N to the northern limit of Egypt is 20 *itr*. Amarna is located at 27°38'N, or 6 *itr* south of Zawiyet El Meiyitin. If the 6 *itr* boundary of Amarna was from Amarna to the pyramid at Zawiyet El-Meiyitin, this would give 50 *itr* south and 50 *itr* north of the district of Amarna for the boundaries of Egypt from the northern limit to Elephantine.

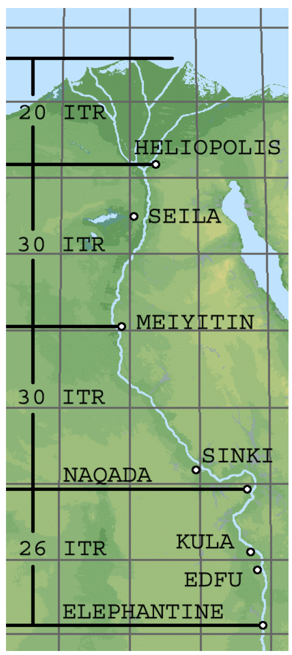


Diagram 4 ***(Map by Vector Globe © Cartografx)***

Amarna (6 *itr* south of Zawiyet El-Meiyitin) is 50 *itr* north of Elephantine. Amarna is also 50 *itr* south of the latitude of Alexandria (Alexandria is 6 *itr* south of the northern limit). The city of Buto is at the same latitude, due east of Alexandria. Malek stated: “Buto was the home of the cobra goddess Wadjet, the tutelary goddess of lower Egypt. The falcon headed figures connected with Buto may have represented the early local rulers ‘Lower Egyptian Kings’ of the area. The remains correspond to the expected layout of Buto, but the results of excavations carried out so far do not suggest that the town’s size was commensurate with its ideological importance throughout Egyptian history.” [51] Like Alexandria, the meridian distance from Buto to Elephantine is 5000 Egyptian stadia, or 100 *itr*, and Amarna is half-way between the two.

Joel Paulson stated: “Egypt was the home of the first known surveyors.” [52] Mark Lehner stated: “Draining and sowing needed to be closely co-ordinated and the basin administrators must have rapidly surveyed and identified field boundaries.” [53] Gardiner stated: “The great Wilbour papyrus in the Brooklyn Museum dated in year 4 of Ramesses V, is a genuine official document of unique interest. Its main text records in four consecutive batches covering a few days apiece the measurement and assessment of fields [the localization and acreage are given in every case] extending from near Crocodilonpolis southwards to a little short of the modern town of El-Minya, a distance of some 90 miles.” [54]

Translating from the Rhind Papyrus, Petrie stated: “Begin to do a triangle in a field. Say thou a triangle of 10 khat [1000 cubits] in height, 4 khat in base, what is its area? ” and “Trapezium - the portion of an acute triangle, after cutting off the acute angle, was called by the same name as the side produced by the cut. ‘Thus say thou a cut of a field, 20 khat is its height, 6 khat at its base, and 4 khat at its cut. What is its area?’” [55]

Lehner stated: “The sacred or Pythagorean triangle - three units on one side, four on the other and five on the hypotenuse - gives a right angle. Such triangles seem to be present in the design of Old Kingdom mortuary temples, though the evidence is inconclusive.” [56] According to Gay Robins and Charles Shute, the 3-4-5 right triangle is also demonstrated by the angle of “the pyramid of Khephren of the 4th dynasty; pyramids of Userkaf, Neferikare and Izezi of the 5th dynasty; and all the 6th dynasty pyramids.” [57] Petrie stated: “The angle of the second pyramid at Gizah is 53°10' ± 4' and the angle of the triangle 3:4:5 is 53°08'. The agreement is so exact that we must suppose that it was intended; if so, it suggests that the principle of the square of the hypotenuse being equal to the sum of the squares on the sides, was recognized, as 3:4:5 is the simplest right-angled triangle of whole numbers. This brings up the question of relations of squares of dimensions. In this connection it must be remembered that the Egyptian used two standards. One of these was the royal cubit. The other was the digit, and 40 digits went to the diagonal of the cubit, or the cubit may be regarded as the diagonal of 20 digits. Thus it was always easy to construct a square half the area of another. This system must have made the squares of lengths, and their relations, well known.” [58]

Translating from the Rhind Papyrus, Petrie stated: “The slope of a pyramid comes next as being connected with triangles. The technical terms used are ‘seeking the sole’ for the base; ‘giving the extent’ for the height. This latter, *peremus*, has usually been taken to be the slant height, but Egyptians always set out slopes by the proportion of the vertical to horizontal. There are also given sums of base and proportion, to find height; and height and base, to find proportion.” [59] The problems use the same expression of height and base for vertical pyramid triangles and horizontal field triangles. In both cases the height and base indicate the sides of a right (90°) triangle.

I.E.S. Edwards stated: “The last of the preliminaries to be accomplished on the site was the execution of an accurate survey in order to ensure that the base of the pyramid should form as nearly as possible a perfect square, each side of which would be so oriented as to face directly one of the four cardinal points. The unit of measurement was the royal cubit of 20.62 inches in length.” [60] John Wilson stated: “The Great Pyramid, near the beginning of the Fourth Dynasty, is a tremendous mass of stone finished with the most delicate precision. Here were six and a quarter million tons of stone, with casing blocks averaging as much as two and a half tons each; yet those casing blocks were dressed and fitted with a joint of one-fiftieth of an inch, a scrupulous nicety worthy of the jeweler's craft. Here the margin of error in the squareness of the north and south sides were 0.09 per cent and of the east and west sides, 0.03 per cent. This mighty mass of stone was set upon a dressed-rock pavement which, from opposite corners, had a deviation from true plane of only 0.004 per cent. The craftsman's conscience could not humanly have done better. Such cold statistics reveal to us an almost superhuman fidelity and devotion to the physical task at hand." [61]

Edwards stated: "To judge from the instrumental and representational evidence so far found, it seems more likely that the high degree of accuracy was achieved by astral than by solar observations. According to ancient Egyptian texts, "the king marked out the line of the four outer walls after observing the position of the stars in the Great Bear. In conducting the observation he was assisted by a priest impersonating the god Thoth, who was in charge of an instrument called the merkhet. The word merkhet means literally 'instrument of knowing.' It was used, as the Czech Egyptologist Z. Zaba was able to show, to express 'shadow-clock', 'water-clock' and 'astral-clock', an instrument for measuring nocturnal hours by determining the height of a star above the horizon. All three instruments also possessed individual names, the general term merkhet would refer to their common function as indicators of time by means of measurement. The merkhet used in astral observations and the merkhet used in sunlight for registering the hours of the day by measuring the length of shadow cast by the block on the horizontal bar were, however, identical apart from the addition of a plumb-line to the former and the calibration of the bar in the latter." [62]

Otto Neugebauer stated: "These [astronomical representations on the coffin lids] represent the sky with the decanal constellations inscribed on them, arranged in their ten-day intervals throughout the year, forming 36 columns with 12 lines each for the 12 hours of the night. The name of a specific decan moves from column to column, each time one line higher. Thus there originated a diagonal pattern which is the motivation for the name 'diagonal calendars' in these texts. In fact, we have here not a calendar but a star clock. The user of this list would know the 'hour' of night by the rising of the decan which is listed in the proper decade of the month." [63]

Gardiner stated: "The Egyptian year was divided into 12 months of 30 days, completed to 365 days by the addition of the five so-called epagomenal or 'added' days. Though for dating and calendrical purposes generally the year of 365 days perforce served as the basis, there was clearly a tendency to regard the year as of only 360 days...Reference must be made to the 'decans', the 36 constellations, or parts of such, which rise at particular hours of the night during the 36 different periods of ten days constituting the year. These periods or 'decades' are named according to the calendar months in which they occur, with the addition - first decade, middle decade, and last decade".[64]

Charles Treat stated: "The solar year of 365 days was the product of Egyptian astronomy, and our modern calendar is the result of that plus later refinement and calendar reform." [65] Gardiner stated: "The Egyptians were the first to divide the day into 24 hours." [66] Petrie stated: "A list of constellations (Brugsch, Thesaurus 137-152) more closely defined, is that of the decans; these being named as within 10°, and near the ecliptic." [67] "We only have whole hours defined, of 15° each (360°/24 hours = 15°)." [68]

STRABO 17.1.29: "Eudoxus went to Heliopolis with Plato and they both passed 13 years with the priests. These men taught them the fractions of the day and night which running over and above the 365 days, fill out the time of the true year. At that time the true year was unknown among the Greeks, as also many other things, until the later astrologers learned from the men who had translated into Greek the records of the priests; and even to this day they learn their teachings."

J. A. Belmonte described some of the textual and archeological evidence, as well as his own proposals, and proposals by Kate Spence and others, to show how cardinal alignments were obtained by stellar observations.[69] Belmonte and Spence concentrate on the north circumpolar region, as do ancient Egyptian texts. "You will give me satiety at the Pole, at that which is the foremost of its flagstaffs." (PT 519, 1218) Belmonte also discusses the theory that the upper northern shaft of the great pyramid was intentionally pointed at the upper culmination of Thuban, the closest polestar during the pyramid age, which would have required measuring the height of Thuban above the horizon. Measuring the height of the pole above the horizon would also give the latitude of the observer in relation to the equator and the pole.

In 1942, Nora Scott described ceremonial cubit rods including fragments of cubit rods in the collection of the Metropolitan Museum of Art: "Inscribed on the upper surface of the rod is a prayer that is addressed to 'all the gods and the royal cubit.' An inscription on the end states that the cubit is 'life, prosperity, and health.' The topmost register is divided into two parts. The first begins, 'The hour according to the cubit.' The rest of the line-measurements in cubits and palms listed according to the months of the year-has been interpreted by Borchardt as possibly a table by which readings of a sundial might be interpreted." [70] "The digits were connected in some way with the nomes, and their names are inscribed on the bevel. Some of the spaces were inscribed with numbers or measurements of astronomical proportions, at least to the ancient Egyptian, such as a million cubits, or one million five hundred thousand cubits." [71]

The distance of one million five hundred thousand cubits is equal to 5000 of the stadia of Eratosthenes or 100 *itr*. This is the meridian distance from Alexandria or Buto to Elephantine. This is also the meridian distance from Biga-Philae (24°00'N) to Xois (31°05'N). Xois was a major city in the Delta throughout ancient Egyptian history. At the end of the Middle Kingdom and the beginning of the Second Intermediate Period, Xois was the capital of Egypt, or possibly the capital of part of a divided Egypt. John Wilkinson stated: "From what Plutarch says of the proportionate rise of the inundation in different parts of Egypt, it is probable that a Nilometer stood at Xois; where as at Mendes, the river rose 7 cubits. At Memphis it reached 14, and at Elephantine 28." [72]

When Jefferson objected to adoption of the metric system, it was because he was concerned about the accuracy of then current surveys, but also because the metric system does not relate distance to time. The nautical mile relates distance to time, but it is not a system of measurement. The units that define the nautical mile, either in meters or feet, are not based on the nautical mile, resulting in uneven fractions of 1852 meters or 6076.1 feet. 100,000 digits, or 5,000 remen, equals one nautical mile, or one minute of latitude. 300,000 remen equals one degree of latitude. Given 15,000 remen, equal to the number of royal cubits in an *itr*, 20 remen *itr* equals one degree of latitude. The definition of a maritime league is three nautical miles, or three minutes of latitude, or one remen *itr*.

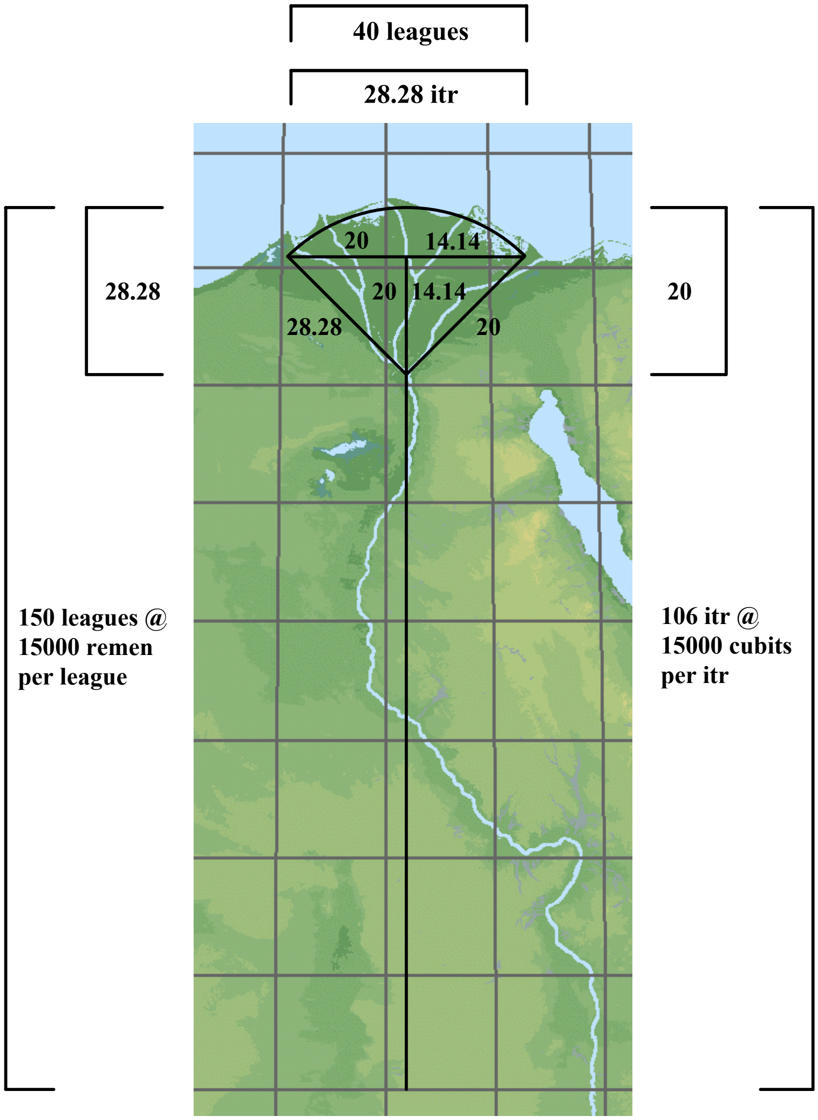


Diagram 5 ***(Map by Vector Globe © Cartografx)***

The NS distance from Biga (24°00N) to 31°30'N is 7°30', or 150 leagues, or 106 *itr*. The distance from 31°30'N to the apex of the Delta at 30°05'N is 28.28 leagues, or 20 *itr*. In diagram 5, the right isosceles triangles have sides of one degree of latitude, or 20 leagues, or 14.14 *itr*.

The meridian circumference is 7200 leagues. The polar diameter is 1000 × φ, or 1618 *itr*. 1618 *itr* × √2 = 2288 leagues. The proportion between the meridian circumference and the polar diameter is 7200/2288, or 3.1468. The meridian circumference of 7200 × 3.1468/π = 7212 leagues for the equatorial circumference. 7212/7200 = 601/600. π × 601/600 = 3.1468. The polar circumference of 40,008 km × 601/600 = 40,075 km for the equatorial circumference. The meridian circumference includes the flattening of the poles and the bulge of the equator. The polar diameter only includes flattening of the poles, giving a difference between the polar and equatorial diameters of 300/301, or twice the 600/601 difference between the meridian and equatorial circumferences.

The equatorial circumference of 7212 leagues/π = 2296 leagues for the equatorial diameter. 2296 × 300/301 = 2288 leagues, or 1618 *itr*, for the polar diameter. 7212 leagues/$\sqrt{2}$ = 5100 *itr* for the equatorial circumference. The proportion between the equatorial circumference and the polar diameter is 5100/1618 = 3.1520. In statute miles, the 24,901 mile equatorial circumference, divided by the 7900 mile polar diameter, equals 3.1520. π × 301/300 = 3.1520.

In determinating the standard for the metric system, Laplace stated: "The length of the pendulum, and that of the meridian, are the two principal means furnished by nature itself to fix the unity of linear measures. Both being independent of moral revolutions, they cannot experience a sensible alteration except by very great changes in the physical constitution of the earth. The first means, though easily applied, is notwithstanding subject to the inconvenience of making the measure of distance to depend on two elements which are heterogeneous to it, namely, gravity and time, the measure of which last is arbitrary; and as it is divided sexagesimally, it cannot be admitted as the foundation of a system of decimal measures. The second means was therefore selected." [73]

Petrie stated: "Whether the Egyptian treated the well-known plumb-line as a pendulum is not indicated by any remains, though the plumb-line was commonly in use from very early times. But the notable fact is that 29.157 inches, the diagonal of the 20.62 inch cubit, is the length which would swing 100,000 times in 24 hours, exactly true at Memphis latitude. This is so remarkable that it suggests that it may have been derived from that observed length, and the source entirely forgotten after the scientific age of the pyramid builders." [74]

Laplace stated: "A remarkable phenomenon is the variation of gravity at the surface of the earth. A very precise way of determining this is furnished by observations of the pendulum; for it is obvious that its oscillations must be slower in those places where the gravity is less. The length of the pendulum, which at the observatory of Paris makes one hundred thousand vibrations in a day, being assumed equal to unity, its length at the equator and at the level of the sea is equal to 0.99669, and in Lapland at an elevation of the pole equal to 74.22, it is observed to be 1.00137. Borda found by very exact and numerous experiments, that the length at the observatory at Paris which represented unity, was when reduced to a vacuum equal to 0.741887m. From a repetition of these experiments by Biot and Mathieu, this length came out equal to 0.7419076m, which differs very little from the preceding result. " [75]

For the equator, the factor of .99669 times unity of .7419m, gives a length of .7394m for the pendulum at the equator. For 74.22°, the factor of 1.00137 × .7419 = .7429m for the pendulum at Lapland. The difference in length between the equator and Lapland is .0035m. Dividing .0035 by 74.22° gives .000047m for the difference in the length of the pendulum per degree. The latitude of Memphis is 29.85°. 29.85 × .000047 = .0014. The length at the equator of .7394 + .0014 = .7408. The remen length of .3704 gives .7408 for the double remen. The measures given by Laplace are in precise accord with Petrie's statement that the length of the pendulum making 100,000 vibrations a day at the latitude of Memphis is equal to the length of the double remen. Like the meter, the remen is in unity with the measure of the meridian circumference of the earth. Unlike the meter, the remen is in unity with the length of the pendulum making 100,000 vibrations a day at the latitude of Memphis, the remen is in unity with minutes and hours as a measure of the distance of axial rotation, and the remen is in unity with minutes and degrees of the arc of the meridian circumference.

Laplace stated: “From the moment that man recognized the spherical form of the globe which he inhabits, he must have been anxious to measure its dimensions; it is therefore extremely probable that the first attempts to attain this object were made at a period long anterior to those of which history has preserved the record, and that they have been lost in the moral and physical changes which the earth has undergone. The relation which several measures of the most remote antiquity have to each other, and to the terrestrial circumference, gives countenance to this conjecture, and seems to indicate not only that this length was very exactly known at a very ancient period, but that it has also served as the base of a complete system of measures, the vestiges of which have been found in Asia and in Egypt.” [76]

“Thales, born at Miletus, 640 years before our era, went to Egypt for instruction; on his return to Greece, he founded the Ionian school, and there taught the sphericity of the Earth, the obliquity of the ecliptic, and the true causes of the eclipses of the sun and moon; he even went so far as to predict them, employing no doubt the periods which had been communicated to him by the priests of Egypt. Thales had for his successors-Anaximander, Anaximenes, and Anaxagoras; to the first is attributed the invention of the gnomon and geographical charts, which the Egyptians appear to have been already acquainted with. Anaxagoras was persecuted by the Athenians for having taught these truths of the Ionian school. They reproached him with having destroyed the influence of the gods on nature, by endeavoring to reduce phenomena to immutable laws. Proscribed with his children, he only owed his life to the protection of Pericles, his disciple and his friend, who succeeded in procuring a mitigation of his sentence, from death to banishment. Thus, truth, to establish itself on earth, has almost always had to combat established prejudices, and has more than once been fatal to those who have discovered it.” [77]

“Pythagoras, born at Samos, about 590 years before Christ, was at first the disciple of Thales. This philosopher advised him to travel into Egypt, where he consented to be initiated into the mysteries of the priests, that he might obtain a knowledge of all their doctrines. On his return to his own country, the despotism under which it groaned, obliged him again to quit it, and he retired to Italy, where he founded his school. All the astronomical truths of the Ionian-school, were taught on a more extended scale in that of Pythagoras; but what principally distinguished it, was the knowledge of the two motions of the earth, on its axis, and about the sun. Pythagoras carefully concealed this from the vulgar, in imitation of the Egyptian priests, from whom, most probably, he derived his knowledge. According to Pythagoreans, not only the planets, but the comets themselves, are in motion around the sun. They taught that the planets were inhabited, and that the stars were suns, disseminated in space, being themselves centers of planetary systems. These views should, from their grandeur and justness, have obtained the suffrages of antiquity; but having been taught with systematic opinions, such as the harmony of the heavenly spheres, and wanting, moreover, that proof which has since been obtained, by the agreement with observations, it is not surprising that their truth, when opposed to the illusions of the senses, should not have been admitted.” [78]

“It was the school of Alexandria that gave birth to the first system of astronomy that ever comprehended an entire series of celestial phenomena. This system was, it must be allowed, very inferior to that of the school of Pythagoras, but being founded on a comparison of observations, it afforded, by this very comparison, the means of rectifying itself, and of ascending to the true system of nature, of which it was an imperfect sketch.” [79]

Copernicus stated: "It is credible that Philolaus believed in the mobility of the Earth and some even say that Aristarchus of Samos was of that opinion." [80] Kepler referred to Pythagoras as 'the grandfather of Copernicans.' Galileo stated that the Papal Edict of 1616 imposed 'a seasonable silence upon the Pythagorean opinions of the mobility of the earth.' [81] In *The System of the World*, published posthumously, Isaac Newton stated:

> "It was the ancient opinion of not a few, in the earliest ages of philosophy, that the fixed stars stood immoveable in the highest parts of the world; that under the fixed stars the planets were carried about the sun; that the earth, as one of the planets, described an annual course about the sun, while by a diurnal motion it was in the mean time revolved about its own axis; and that the sun, as the common fire which served to warm the whole, was fixed in the center of the universe.
>
> This was the philosophy taught of old by Philolaus, Aristarchus of Samos, Plato, and the whole sect of the Pythagoreans; and this was the judgment of Anaximander, more ancient than any of them; and of that wise king of the Romans, Numa Pompilius, who, as a symbol of the figure of the world with the sun in the center, erected a temple in honor of Vesta, of a round form, and ordained perpetual fire to be kept in the middle of it.
>
> The Egyptians were early observers of the heavens; and from them, probably, this philosophy was spread abroad among other nations; for from them it was, and the nations about them, that the Greeks derived their first, as well as soundest, notions of philosophy; and in the vestal ceremonies we may yet trace the ancient spirit of the Egyptians; for it was their way to deliver their mysteries, that is, their philosophy of things above the vulgar way of thinking, under the veil of religious rites and hieroglyphic symbols" [82]

The inverse square law of gravity described by Newton in the *Principia* has only a minor effect in relation to the variation of gravity on the earth's surface because the surface at the equator is only slightly further away from the center of mass than the surface at the poles. In a part of his manuscript that was not included in the publication of the *Principia*, Newton suggested that the Pythagorean 'harmony of the spheres' expressed an understanding of the force of gravity and the inverse square law. Newton stated:

> "By what proportion gravity decreases by receding from the planets the ancients have not sufficiently explained. Yet they appear to have adumbrated it by the harmony of the celestial spheres, designating the sun and the remaining six planets, Mercury, Venus, Earth, Mars, Jupiter and Saturn, by means of Apollo with the lyre of seven strings, and measuring the intervals of the spheres by the intervals of the tones. Thus they alleged that seven tones are brought into being, which they called the harmony diapason, and that Saturn moved by the Dorian phthong, that is, the heavy one, and the rest of the planets by the sharper ones (as Pliny, bk. I, ch. 22 relates, by the mind of Pythagoras) and that the sun strikes the strings. Hence Macrobius, bk. I, ch. 19, says: 'Apollo's lyre of seven strings provides understanding

of the motions of all the celestial spheres over which nature has set the sun as moderator.' And Proclus on Plato's *Timaeus*, bk. 3, page 200, 'The number seven they have dedicated to Apollo as to him who embraces all symphonies whatsoever, and therefore they used to call him the God of the Hebdomagetes', that is the prince of the number seven. Likewise in Eusebius' *Preparation of the Gospel*, bk. 5, ch. 14, the sun is called by the oracle of Apollo the king of the seven sounding harmony. But by this symbol they indicated that the sun by his own force acts upon the planets in that harmonic ratio of distances by which the force of tension acts upon strings of different lengths, that is reciprocally in the duplicate ratio of the distances. For the force by which the same tension acts on the same string of different lengths is reciprocally as the square of the length of the string.

The same tension upon a string half as long acts four times as powerfully, for it generates the octave, and the octave is produced by a force four times as great. For if a string of given length stretched by a given weight produces a given tone, the same tension upon a string thrice as short acts nine times as much. For it produces the twelfth, and a string which stretched by a given weight produces a given tone needs to be stretched by nine times as much weight so as to produce the twelfth. And in general terms, if two strings equal in thickness are stretched by weights appended, these strings will be in unison when the weights are reciprocally as the squares of the lengths of the strings.

Now this argument is subtle, yet became known to the ancients. For Pythagoras, as Macrobius avows, stretched the intestines of sheep or the sinews of oxen by attaching various weights, and from this learned the ratio of the celestial harmony. Therefore, by means of such experiments he ascertained that the weights by which all tones on equal strings...were reciprocally as the squares of the lengths of the string by which the musical instrument emits the same tones. But the proportion discovered by these experiments, on the evidence of Macrobius, he applied to the heavens and consequently by comparing those weights with the weights of the planets and the lengths of the strings with the distance of the planets, he understood by means of the harmony of the heavens that the weights of the planets towards the sun were reciprocally as the squares of their distances from the sun.

But the philosophers loved so to mitigate their mystical discourses that in the presence of the vulgar they foolishly propounded vulgar matters for the sake of ridicule, and hid the truth beneath discourses of this kind. In this sense Pythagoras numbered his musical tones from the earth, as though from here to the moon were a tone, and thence to Mercury a semitone, and from thence to the rest of the planets other musical intervals. But he taught that the sounds were emitted by the motion and attrition of the solid spheres, as though a greater sphere emitted a heavier tone as happens when iron hammers are smitten. And from this, it seems, was born the Ptolemaic system of solid orbs, when meanwhile Pythagoras beneath parables of this sort was hiding his own system and the true harmony of the heavens." [83]

Source of Image: antikforever.com - Egypte - Divinites - Seshat

The sky goddess Nephthys-Sephekt-Seshat was one of the original gods from pre-dynastic Egypt. Seshat was the ancient Egyptian goddess of the number seven. She was related to Thoth, the god of wisdom, and along with Thoth, she was credited with inventing written language. She was responsible for surveying, for determining and orienting the ground plans of pyramids and temples, and for the measure of distance and time. An inscription from Edfu (I, 291) states that she 'reckons all things on earth.' [84] The carved relief from Queen Hatshepsut's Red Chapel in Karnak shows Hatshepsut and Seshat stretching the cord to establish the ground plan for the temple. In an inscription appearing under the title of 'stretching the cord' at the temple of Edfu, the king stated: "I have taken the stake and the mallet by the handle, I have grasped the rope with the goddess Sephekt; my gaze has followed the course of the stars; my eye has turned toward the Great Bear; I have measured time and counted the hour with the clepsydra, then I established the four corners that define the temple." [85]

Following the discovery of Kepler's third law, that the squares of the period of revolution of any two planets are proportional to the cubes of their mean distances from the sun, Kepler stated: "Since the first light eight months ago, since broad day three months ago, and since the sun of my wonderful speculation has shone fully a very few days ago: nothing holds me back. I am free to give my self up to the sacred madness, I am free to taunt mortals with the frank confession that I am stealing the golden vessels of the Egyptians, in order to build of them a temple for my God, far from the territory of Egypt." [86] Based on Kepler's second and third laws of planetary motion, for any given period of time, the areas swept by any two planets are proportional to the square root of their mean distances from the sun.

The Kepler triangle and the right triangle of the great pyramid have the same φ proportion. The ancient Egyptian funerary text known as the AmDuat is divided into 12 sections, regarded as the 12 hours of the night. The second and third sections are the first two sections of the underworld and the length of each is 309 *itr*. The combined length of these two sections is 1000/φ = 618 *itr*. The lengths of the remaining sections are unspecified. Chapter 110 of the Book of the Dead gives the length of 1000 *itr*. The proportion between these lengths is1000/618 = 1.618. The combined length is 1000 *itr* + 618 *itr* = 1618 *itr*. The φ proportion of the Kepler triangle and the great pyramid is the same as the proportion between 1000 *itr* and 618 *itr*.

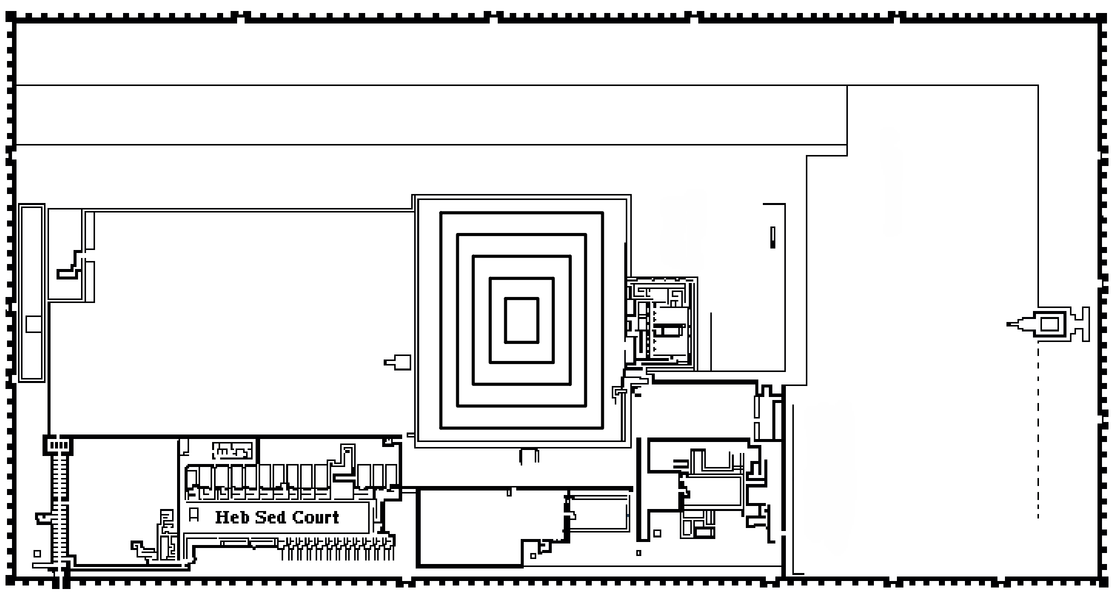
According to Budge: The first hour in the Amduat was a western distance of 120 *itr* before 'entering into the earth through the hall of the horizon.' The second and third hours are spent sailing north, 309 *itr* + 309 *itr* = 618 *itr*. Hours four through nine were spent crossing the sandy island realm of Sekar-Osiris. "It is pretty certain that the tenth and eleventh hours comprised a part of the eastern Delta, and that it extended some considerable distance south of Memphis. It follows that when the boat entered this division the god would have to alter his course from east to south. As the kingdom of Osiris marked the limit of his journey northwards, and the boat then turned eastwards, so the northern end of the kingdom of Heliopolis marked the limit of his journey eastwards, and the boat then turned southwards." [87]

The southern boundary of ancient Egypt was the northern boundary of direct vertical sunlight on the summer solstice. During the early period of ancient Egypt, the solstice also marked the reappearance of Sirius in the eastern sky just before dawn. These events also marked the beginning of the new year in ancient Egypt. If the east-west distance of 120 *itr* during the first hour also applied to the width of the island of Sekar-Osiris, and the north-south distance of 618 *itr* also applied to the distance traveled in the tenth and eleventh hours, the journey may have begun on the southern border of Egypt, under the path of the setting sun -- and ended in the twelfth hour, on the southern border of Egypt, under the path of the rising sun.

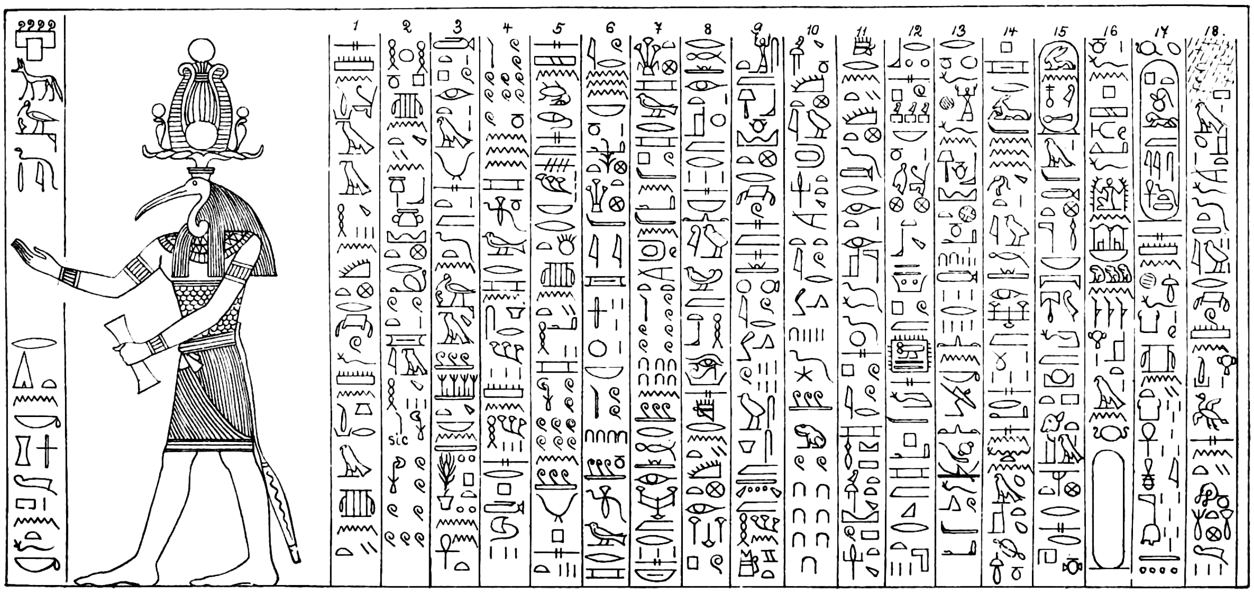
The ancient Egyptian Heb-Sed festival was also celebrated at the beginning of the New Year. Eric Uphill stated: "One of the first rites to be performed was the raising of the Djed-pillar at dawn, which is not shown at Bubastis but appears elsewhere, to symbolize the king's triumph over death, the victory of the new Horus over Set. Also at this time the king would need to perform the four running ceremonies in order to demonstrate his returned vigor and thus ensure continued prosperity for the country. These rites seem to have been performed in an enclosed area not far from the Sed festival palace." [88]

Gardiner stated. “Some attention must be paid to the word *itrt*. My rendering as ‘conclave’ confessedly sacrifices accuracy to intelligibility. The accepted translation is ‘sanctuary’ but this I hold to be altogether wide of the mark. In its religious application the term seems so much bound up with the great royal Sed festival or Jubilee that no single English word could possibly convey an adequate notion of its signification. I believe *itrt* to be related to ‘river’, ‘river channel’ and to mean fundamentally something like ‘line’ or ‘row’. Occurring frequently in the dual, it there means ‘the two sides’ or ‘rows’. Now in the Sed festival, which was normally celebrated in Memphis, all the deities of the two halves of the country were summoned thither, their statues or emblems housed in two rows of shrines on opposite sides of a jubilee court, the lower Egyptian shrines with the appearance of the primitive sanctuaries of the cobra-goddess of Buto, while the upper Egyptian shrines resembled that of the vulture-goddess of El-Kab. In effect, the expression *itrt smrt* would thus mean ‘the company or conclave of Egyptian deities’, though simultaneously it would conjure up the image of rows of Egyptian shrines such as could be seen at Memphis on the occasion of the great national festival. Excavation has actually unearthed imitations of them in the marvelous temple attached to King Djoser’s step pyramid.” [89] In ancient Egypt, ‘river’ was associated with the country itself and was also associated with the journey through the underworld. Given *itr* as a unit of measurement, *itrt* could be rendered as ‘area’, or ‘the area of Egyptian deities’, or the area of the underworld, or the area of Egypt.

Christine Schnusenberg stated: “The new year was often set apart for great reenactments of the acts of Re at the times of creation...Sometimes the Sed festival was celebrated at the beginning of the new year and the two became interwoven with each other...The Sed festival intertwined the visible and the invisible, of life here and now and of life after death.” [90]



The shrines that surrounded and defined the area of the Heb-Sed court were typically installed for the festival and removed afterwards, but the step pyramid complex of King Djoser included a permanent Heb-Sed court, with dummy shrines constructed of solid masonry. The size of Djoser’s court is 95 meters NS × 18 meters EW.[91] The proportion of Djoser’s court is approximately the same as the proportions of the NS and EW distances in the Amduat: 618/120 = 95/18.4.



Dieter Kurth translated the Edfu geographical inscription referred to by Griffith and shown above from the hand copy by Brugsch.[92] "The cultivable lands of all Egypt, from Elephantine to the marshes, 27,000,000 setat of land. All the fields with barley and wheat, 9,000,000. The waters of the Delta with papyrus, lotus, and all the vegetation grown by the inundation: 18,000,000[(a)] setat. In specification: All the rivers of upper and lower Egypt, 2,400,000 setat, the marshland of lower Egypt, that is Lake Moeris and its canal, 6,600,000 setat. Length from Elephantine in its entirety is 106 *itr*, its width on the land from the western river branch of Egypt to the eastern branch is 14 *itr*. [(a)] The number here is written 1800 or 1000 × 800 = 800,000, but the number 10,000 was omitted through oversight and the intended number was 1800 × 10,000 = 18,000,000." [93]

The bottom of the tenth column shows two coils of 100 each and seven hobbles of 10 each, for a total of 270. The frog is 100,000, and 270 × 100,000 = 27,000,000 setat. The setat, or Greek aroura, is a square measure with sides of 100 royal cubits and an area of 10,000 square cubits. Griffith stated: "27,000,000 arouras mean about 73,918 square kilometers or 28,420 square miles, while Schweinfurth, who gives the length of Egypt proper as 550 miles, makes the cultivable area only 11,342 square miles." [94]

Although parts of the Delta are much wider than the 14 *itr* distance given between branches of the Nile; and the cultivable area of the Nile valley is much narrower; and the Nile valley drifts east of the Delta, Griffith calculated a rectangular area of 106 *itr* × 14 *itr* = 1484 square *itr*. Assuming 27,000,000 aroura equals 1484 square *itr*, the square *itr* would contain 18,194 aroura, or 181,940,000 square cubits, and $\sqrt{181{,}940{,}000} = 13{,}487$ royal cubits for the length of the itr. Given the length of 15,000 cubits for the *itr,* $15000^2 = 225{,}000{,}000$ royal cubits. 225,000,000/10,000 = 22,500 setat per square itr. 27,000,000 setat/22,500 = 1200 square *itr*.

Kurth's translation gives a measure of 18,000,000 setat for the Delta. 18,000,000 setat is twice the entire area of the Delta. The figure of 9,000,000 setat is an accurate measure of the area of the Delta, and depending on the copying and punctuation of the inscription and the translation, 9,000,000 setat may have been the intended expression of the area of the Delta.

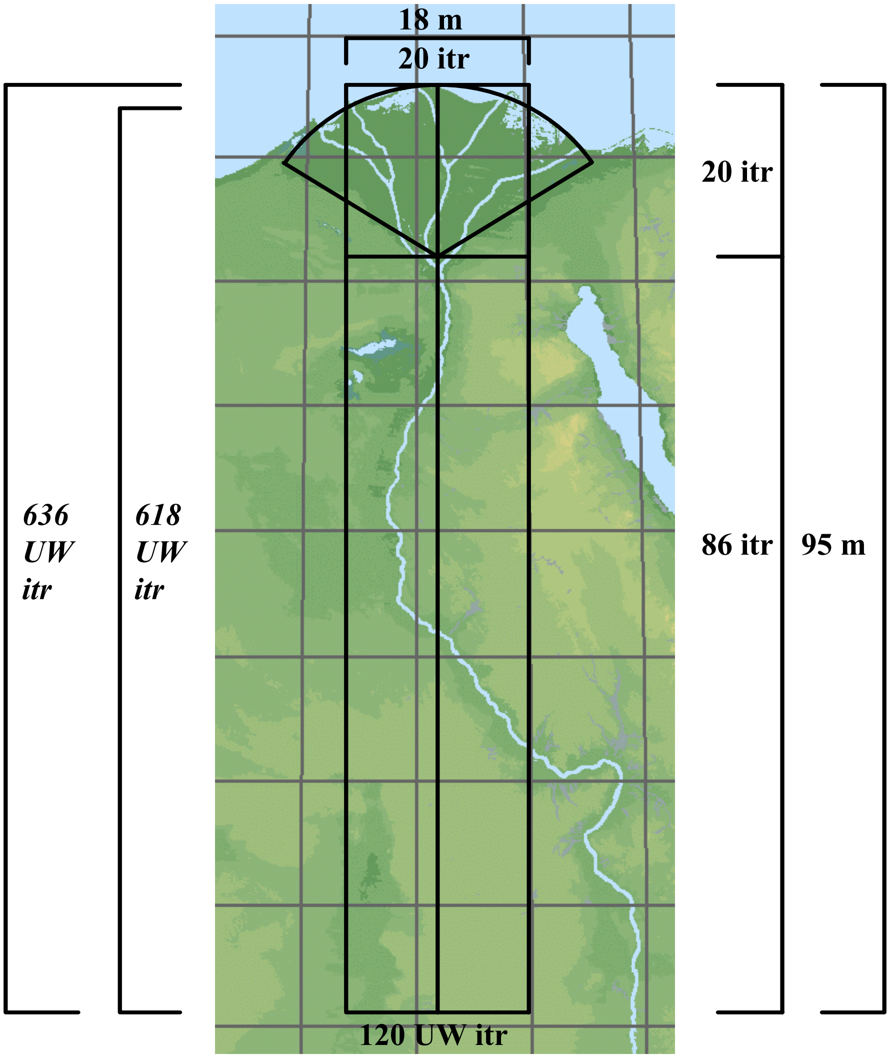


Diagram 6 ***(Map by Vector Globe © Cartografx)***

The 95/18 meter proportion of the Heb-Sed court is equal to the proportion of 106/20 *itr*. The inscriptions on the cubit rods and in the White Chapel divide the length of Egypt into 86 *itr* for upper Egypt and 20 *itr* for lower Egypt. $20^2 = 400$ square *itr*. Given 15,000 cubits for the length of the *itr*, 400 square *itr* = 9,000,000 setat. The length of the perimeter of the arc is 2 radians, or 40 *itr*. The area contained in the arc of 2 radians is $\pi r^2/\pi = r^2 = 20^2 = 400$ square *itr* = 9,000,000 setat.

$106 \times 20 = 2120$ square *itr*, or 47,700,000 setat. If the 18,000,000 setat of land was intended to be added to rather than included in the 27,000,000 setat of land, then the total would be 45,000,000 setat of land. 2,400,000 setat is the area given for the rivers of upper and lower Egypt.

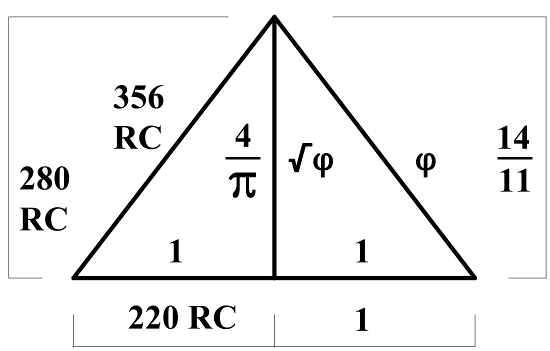
The *itr* measures from the Amduat are a 6x multiple of the *itr* measures from the cubit rods. The arc of the Delta intersects the EW boundaries 3 *itr*, or 18 underworld *itr*, south of the northern boundary. 106/20 = 636/120 and 103/20 = 618/120. The proportions of the Heb-Sed Court express the given proportions of Egypt and the underworld as shown in diagram 6.

Berriman observed that the length of one meter is equal to 54 ancient Egyptian digits. Schwaller observed that a circle with a diameter of one meter has a circumference of six royal cubits. Combining these observations, a diameter of 54 digits gives a circumference of six royal cubits and 54/6 = 9. A diameter of 9 digits gives a circumference of one royal cubit. The $20\sqrt{2}$ relation between the digit and the royal cubit gives 28.28 digits for the royal cubit and 28.28/9 = 3.142...

Petrie stated: "At the pyramid of Khufu, the variation of a course of casing is .04 inches on 20 feet, and .00 to a course of the core 40 feet distant. The sides of the pyramid varied 2.3 inches on an average in a length of 9069.4 inches, or 1 in 4,000. If this were laid out with copper measuring rods, this error would result from 15° C. difference of temperature. The angular accuracy of building is also a test; on the same pyramid (where diagonals could not be laid out, owing to rock) the average error is 1' 12" from a right angle, or 2 inches on the length. Considering the accuracy, which must have needed extraordinary care to carry them out in a gigantic mass of masonry, it is reasonable to look for accuracy of proportion. It happens that five different relations of the parts would each give the same angle as that of the Khufu pyramid, within 2' or 3'. A help in judging which of these is likely is given by the Meydum pyramid of Sneferu, next in age before that of Khufu, and of the same angle. If supposed numbers of proportion will not result in a recognized unit, when applied to both these pyramids, there is little probability that they were intended. There remain two possible designs: (1) The base circuit equal to a circle struck by the height (or a modification in height to base as 7:11); (2) the face area = the height squared (or height a mean proportional between half base and slope). The first design is supported by the dimensions in cubits being multiples of 7 and 11; and this is made almost certain by the Meydum pyramid dimensions also being multiples of 7 and 11:

Pyramid of Sneferu: 25 × 7 high, 25 × 11 base in cubits.
Pyramid of Khufu: 40 × 7 high, 40 × 11 base in cubits.

The angle of 7 high on 11 base is 51° 50' (strict π ratio 51° 51'); observed at Meydum pyramid 51° 48' ± 7' and Khufu's pyramid 51° 52' ± 2'. We conclude therefore that the approximation of 7 to 22 as the ratio of diameter to circumference was recognized." [95]



The 7/11 proportion for the height over the base is 14/11 for the height over the half base. Given a half base of one: 14/11 = 1.2727; $4/\pi = 1.2732$; and $\sqrt{\varphi} = 1.272$. The angle of the triangle $1 : \sqrt{\varphi} : \varphi$ is 51°49'36". This is the pyramid referred to by Petrie as 'face area = height squared' because the height of $\sqrt{\varphi}$, squared, equals φ; and for the face area, the slant height of φ, times the half base of 1, also equals φ. This triangle was described by Kepler and bears his name.

HERODOTUS 2.124 gives the sides of Khufu's pyramid as eight plethra and states that the height is the same. The ancient Greek plethron denoted both a linear measure of 100 Greek feet and a square measure of one acre (100×100 feet). The linear measures of the sides and the height of the pyramid are not the same and they are not 800 feet or 800 cubits. However, the area of each side of the pyramid is equal to the square area of the height.

The square area of the pyramid height of 280^2 is 78,400 square cubits. The area of the sides, with a slant height of 356 cubits and a half base of 220 cubits, is the same. 78,400/8 equals 9800 cubits and the square root of 9800 is 99 cubits. Approximate values for $\sqrt{2}$ are 7/5 and 10/7, but much more accurate approximations are 99/70 and 140/99. Given an acre with sides of 100 royal cubits, an approximate value for the side of a half acre, in royal cubits, is 70: $100/\sqrt{2} = 70.7$. Given acre sides of 99 royal cubits, the side of the half acre is precisely 70 cubits, and the side of the double acre is precisely 140 cubits. 70 cubits and 140 cubits are also factors of the 280 cubit height of the pyramid. Given a half acre with sides of 70 cubits, $70 \times 4 = 280$ cubits, and $4^2 = 16$ half acres, or 8 acres for the square of the height, and the same for the area of the sides, as stated by Herodotus.

φ is mathematically defined as $(\sqrt{5} + 1)/2 = 1.618...$ Fibonacci (c. 1170 - 1240 AD), described the sequence known as Fibonacci numbers. Each succeeding number is the sum of the two preceding ones, starting with $1 + 1 = 2$; $1 + 2 = 3$, etc:

1	1	2	3	5	8	13	21	34	55	89	144
	1	2	1.5	1.66	1.6	1.625	1.615	1.619	1.617	1.618	1.618

With each successive iteration, the proportion between the numbers moves ever closer to φ. 55×4 is equal to the half base of Khufu's pyramid. 89×4 is equal to the slant height of the pyramid. $55 \times \sqrt{\varphi} = 70$, or one fourth of the height of the pyramid. The presence of the proposed numbers of proportion, in recognized units, increases the probability that they were intended.

Plato alluded to φ in TIMEUS 31-32: "Two things can not be rightly put together without a third; there must be some bond of union between them. And the fairest bond is that which makes the most complete fusion of itself and the things which it combines; and proportion is best adapted to effect such a union. When the mean is to the first term as the last term is to the mean-then the mean becoming first and last, and the first and last both becoming means, they will all of them of necessity come to be the same; and having become the same with one another will be all one." EUCLID 6.3 stated: "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less."

Schwaller observed that φ^2, or 2.618, times 6/5, equals 3.1416, or π. Problem 38 of the Rhind mathematical papyrus (c. 1550 BC) asks for the reciprocal of $3 + 1/7$. The given solution is $1/6 + 1/11 + 1/22 + 1/66$. Adding these fractions with the lowest common denominator gives 21/66. Given 21 as unity, $21 \times \varphi = 34$ and $21 \times \varphi^2 = 55$. Because 66 is 6/5 of 55, this gives the sequence: $21/21 = 1$ :: $34/21 = \varphi$ :: $55/21 = \varphi^2$ :: $66/21 = \pi$. Based on his studies in and around the temple of Amun, Schwaller concluded that the π and φ proportions, and the relations between the two, were well known and frequently employed in ancient Egypt.

φ and π proportions are also found in the dimensions of the Khufu's burial chamber. The chamber is 20 cubits long and 10 cubits wide. The horizontal diagonal is 22.36 cubits ($\sqrt{5} \times 10$). The walls consist of five courses of equal height, but the floor of the chamber is inserted between the walls, and the top of the floor is approximately one-quarter of a cubit above the base of the first course of the walls. As a result, there are two wall heights for the chamber: The height from the top of the floor and the height from the base of the first course of the walls. The height from the top of the floor to the ceiling is 11.18 cubits, equal to one- half of the diagonal of the chamber, or $\sqrt{5} \times 5$, or $10\varphi - 5$. $(11.18 + 5)/10 = \varphi$. The diagonal of the short side of the chamber from the floor to the ceiling is 15 cubits and the cubic diagonal from the floor to the ceiling is 25 cubits (the hypotenuse of a 3-4-5 right triangle with sides of 15 and 20 cubits). The volume of the chamber from the floor to the ceiling is 2236 cubits or $\sqrt{5} \times 1000$.

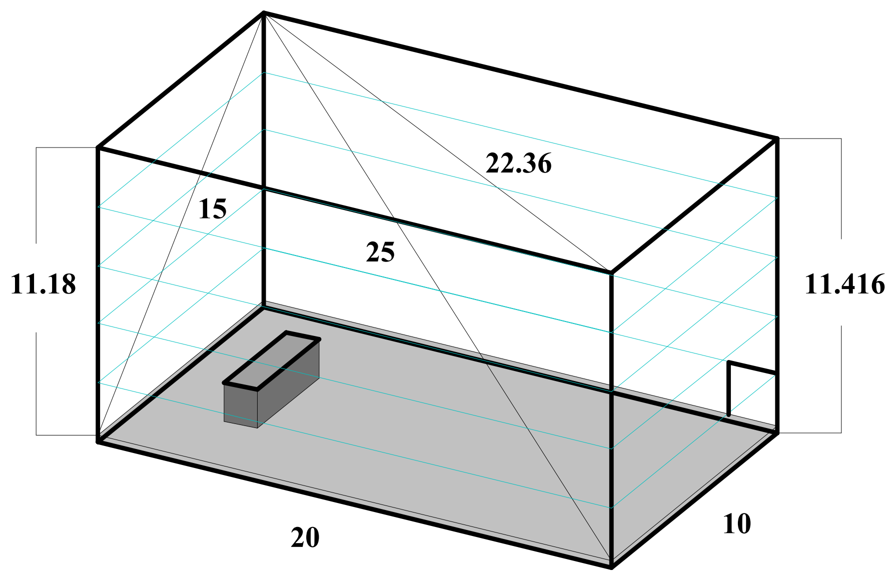
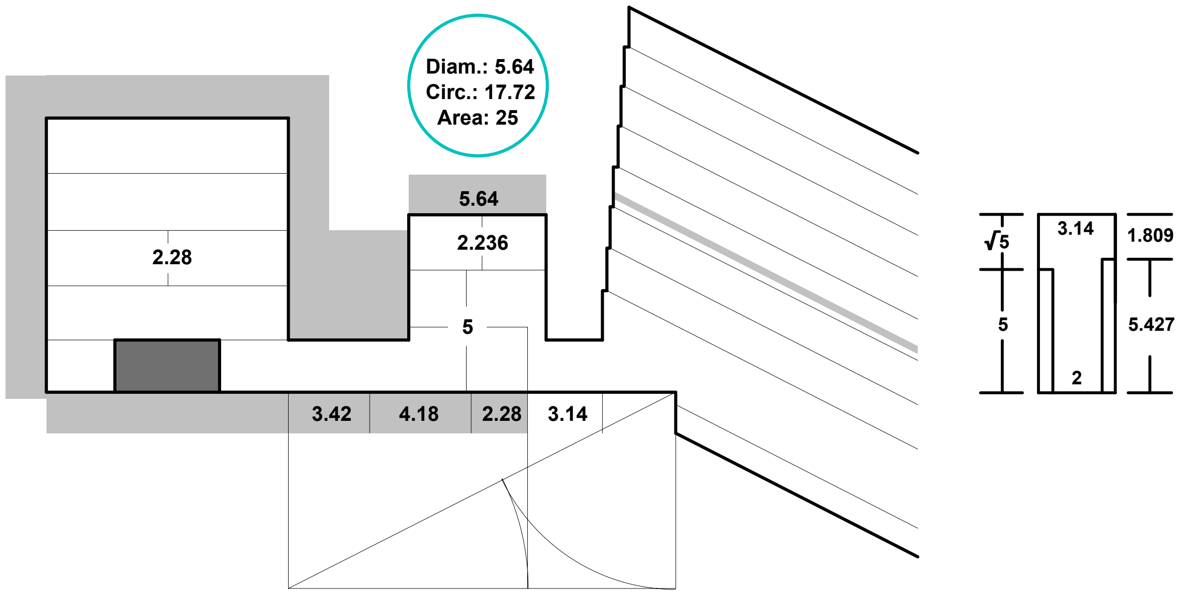


Diagram 7

According to Petrie's survey "the average variation of the thickness of the courses from their mean is .051 inches, the mean being 47.045 inches between similar joints. The total height of the five courses is 235.2 ± .06. The theory of the height of the walls is similar to one of the best theories of the outside of the Pyramid; it asserts that taking the circuit of the N. or S. walls, that will be equal to the circumference of a circle whose radius is the breadth of the chamber at right angles to those walls, or whose diameter is the length of those walls. Now by the mean original dimensions of the chamber the side walls are 412.25 inches long, and the ends 206.13, exactly half the amount. Taking, then, either of these as the basis of a diameter or radius of a circle, the wall height, if the sides are the circumference of such circle, will be 235.32 ±.10, and this only varies from the measured amount within the small range of the probable errors. This theory leaves nothing to be desired, therefore, on the score of accuracy, and its consonance with the theory of the Pyramid form strongly bears it out." [96]



The first two floor blocks of the king's chamber passage are limestone. The last three blocks are granite. The granite blocks in the passage and chambers are shaded. Petrie gives 126.2 inches for the limestone portion, and 204.1 inches for the granite portion. The total length of the floor of the king's chamber passage is 330.3 inches. The floor is divided in the φ proportion by the limestone and granite portions of the passage: $330.3/204.1 = 204.1/126.2 = \varphi$.

According to Petrie's measurements, the antechamber is 149.3 inches, or 7.236 cubits high. The chamber is 116.3 inches, or 5.64 cubits, or $2.5 + \pi$ cubits long. The mean of the measurements of the upper width of the chamber is 64.75 inches, or 3.14, or π cubits. Granite wainscots line the lower portion of the east and west walls of the chamber, making the floor and the lower portion of the antechamber two cubits wide.

The top of the wainscot on the east side of the antechamber is five cubits high, and the height above the wainscot is 2.236, or $\sqrt{5}$ cubits. The top of the wainscot on the west side is 5.427 cubits high, or 3/4 of the height of the chamber, and the height above the wainscot is 1.809, or $(\varphi^2 + 1)/2$ cubits. The total height of the chamber is equal to $5 + \sqrt{5}$, or $2\varphi^2 + 2$, or 7.236 cubits.

The second limestone floor block is the same length of 3.14 cubits as the upper width of the antechamber, and the second granite floor block is 4/3 of 3.14 cubits. The first granite floor block is the same length of 2.28 cubits as the course height of the king's chamber, and the third granite floor block is 3/2 of 2.28 cubits.

The length from the beginning of the granite floor to the south side of the antechamber is five cubits, the same as the height of the east wainscot, giving a square area of 25 cubits. A circle with a diameter of 5.64 cubits, equal to the length of the antechamber, also has an area of 25 cubits, and a circumference of 17.72 cubits, or $10 \times \sqrt{\pi}$. The rectangular area of the ceiling of the antechamber, with a length of $2.5 + \pi$ cubits, and a width of π cubits, is 17.72, or $10 \times \sqrt{\pi}$, square cubits.

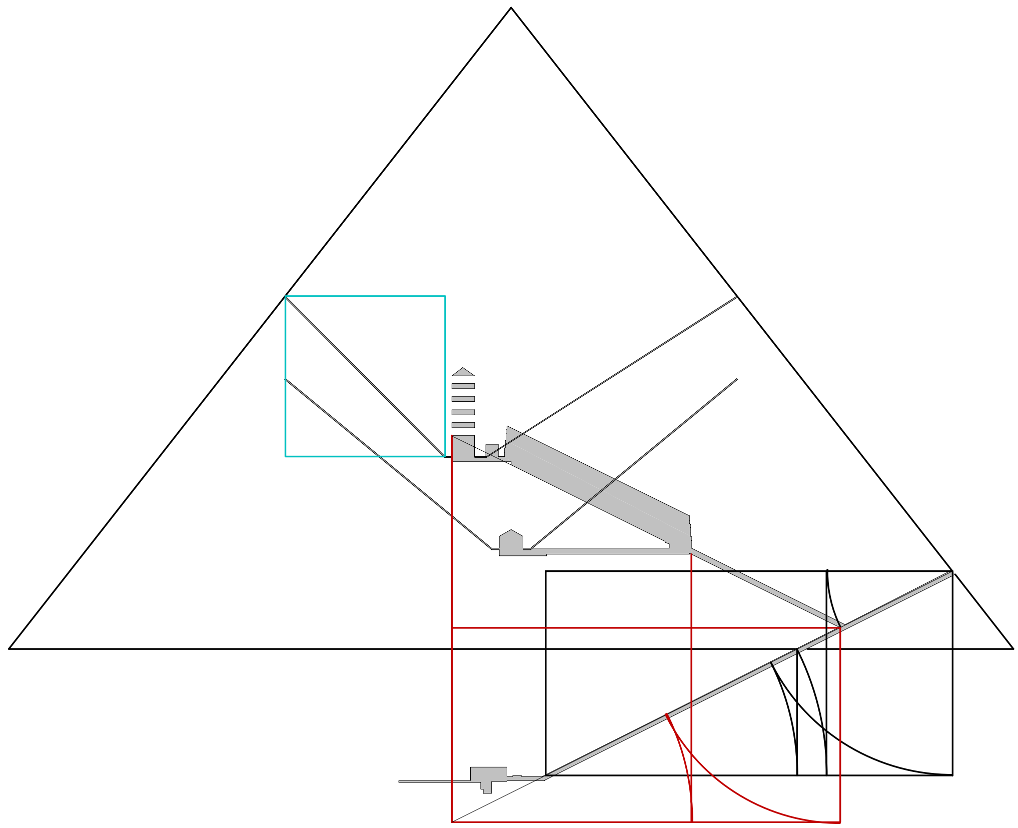


Diagram 8

Based on his robotic survey of the diagonal shafts in Khufu's pyramid, Rudolph Gantenbrink reported that the southern shaft of the king's chamber had an angle of 45º, that it began 2 cubits above the floor of the king's chamber, 84 cubits above ground level, and that it terminated at the outer casing of the pyramid, 154 cubits above ground level. The 45º angle means that the shaft is the diagonal of a square, and the 70 cubit height of the shaft means that the length of the shaft is 99 cubits ($70 \times \sqrt{2} = 99$). The square, with 70 cubit side lengths, as shown in diagram 8, is equal to 1/4 of the height of the pyramid. 16 of these squares are equal to the square area of the height of the pyramid. Taken as half-acres of 70 × 70 royal cubits, this gives eight acres for the square area of the height of the pyramid, and for the area of each face of the pyramid, as reported by Herodotus.

Based on his survey of Khufu's pyramid, Petrie reported that the height of the roof of the descending passage at the outer casing of the pyramid is 706 inches above ground level. The descending passage ends at the beginning of the horizontal subterranean passage, and Petrie reported that the roof at the beginning of the subterranean passage is 1143 inches below ground level. Petrie reported 4133 inches for the total length of the roof of the descending passage. Petrie stated: "The old theory of 1 rise on 2 base, or an angle of 26º 33' 54"; is far within the variations of the entrance passage angle, and is very close to the observed angle of the whole passage, which is 26º 31' 23"; so close to it, that 2 or 3 inches on the length of 350 feet is the whole difference; so this theory may at least claim to be far more accurate than any other theory."[97]

Given a slope of 1 rise on 2 base, the descending passage is the diagonal of a double square, making the length of the passage equal to the height of the double square, times the square root of five. 706 + 1143 = 1849 inches for the total height. $1849 \times \sqrt{5} = 4134$ inches. The height of the descending passage of 706 inches above ground level, times φ, or 1.618, equals 1142 inches, or the depth of the passage below ground level. The φ proportion divides the passage at ground level.

The double square defined by the diagonal of the descending passage is 34 royal cubits above ground level and 55 cubits below ground level, for a total height of 89 cubits. The length of the diagonal is $89 \times \sqrt{5}$, or 199 cubits. The above ground length of the passage is 76 cubits ($34 \times \sqrt{5}$) and the below ground length is 123 cubits ($55 \times \sqrt{5}$). $123/76 = \varphi$. The length of the descending passage from the outer casing to the intersection of the floor of the ascending passage is 55 cubits. $199 - 55 = 144$ cubits for the remaining length of the descending passage. $144/55 = \varphi^2$.

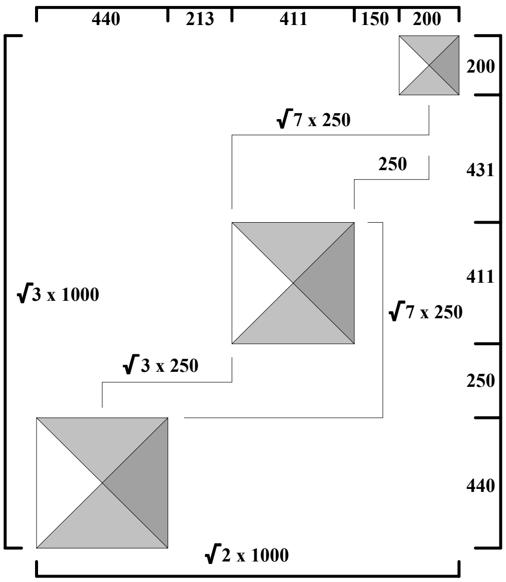
The 76 cubit length of the descending passage, from the outer casing to ground level, minus the length of 55 cubits from the outer casing to the ascending passage, leaves 21 cubits for the descending passage from the intersection with the ascending passage to ground level. The entire length of the descending passage is divided into the φ^2 proportion of 144/55 by the ascending passage; and the length of the descending passage from the outer casing to ground level is divided into the φ^2 proportion of 55/21 by the ascending passage.

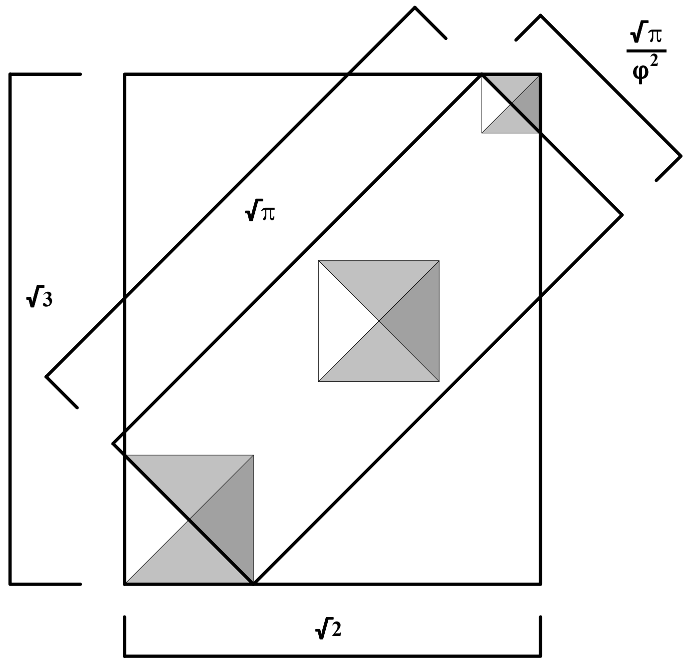
$55/\sqrt{5}$ is 24.6 cubits, for the difference in height from the beginning of the descending passage, 34 cubits above ground level, to the intersection with the ascending passage, 9.4 cubits above ground level ($34 - 24.6 = 9.4$). The height of the floor of the king's chamber is 82 cubits above ground level, plus 11.18 cubits for the height of the chamber, gives 93.18 cubits for the height of the ceiling, above ground level. The entire height from the beginning of the ascending passage to the ceiling of the king's chamber is 83.8 cubits ($93.2 - 9.4 = 83.8$).

The king's chamber complex of passages and chambers from the beginning of the ascending passage to the ceiling of the king's chamber is divided by the φ proportion at the intersection of the ascending passage and the grand gallery. $83.8/\varphi = 51.8$ cubits. $93.2 - 51.8 = 41.4$ cubits for the height of the end of the ascending passage and the beginning of the grand gallery. Petrie gave 852.6 inches above ground level for the height of the beginning of the horizontal queen's chamber passage at the end of the ascending passage. $852.6/41.4 = 20.6$ inches per cubit. The angle of the floor of the ascending passage and the grand gallery intersects the south wall of the king's chamber within 2 inches of the ceiling. As a result, the division of the king's chamber complex by the φ proportion may be demonstrated vertically, as explained above, or horizontally, as shown in diagram 8.

Given 55 cubits as unity: $21 = 1/\varphi^2$, $34 = 1/\varphi$, $55 = 1$, $70 = \sqrt{\varphi}$, $89 = \varphi$, and $144 = \varphi^2$.

The external dimensions of the pyramid; the division of the ascending passage and the descending passage in the φ proportion; the divisions of the descending passage in the φ^2 proportion by the intersection of the ascending passage; and the recognizable units of division, of 21, 34, 55, 70, 89 and 144 royal cubits, support a conclusion that the φ proportion, the Kepler triangle and the Fibonacci sequence of numbers were known and employed in the design of the great pyramid.





All three of the pyramids at Giza are precisely oriented to the cardinal directions. The first and second pyramids are oriented 5' east of due south, and the third pyramid is oriented 13' west of due south. Petrie's survey of all three of the Giza pyramids was based on the orientation of the first two pyramids, and his measures of the first two pyramids and the distances between them are as shown above. His NS measure of 1732, or $\sqrt{3} \times 1000$ cubits, suggests an EW measure of 1414, or $\sqrt{2} \times 1000$ cubits, although his survey gave a distance from the east side of the first pyramid to the west side of the third pyramid of 1417 cubits. If the third pyramid was originally sited based on a survey from the first two pyramids, but with the orientation of the third pyramid, the 18' difference would account for the difference of three cubits in Petrie's surveyed location of the third pyramid.

Petrie also gave 201.5 cubits for the base length of the third pyramid, although this measure is less certain than his measures of the first two pyramids, because the third pyramid was unfinished. The casing blocks were not dressed, paving stones around the pyramid were not installed, and Petrie had to calculate the base length from courses above the base because he was unable to clear stone debris that was up to 15 feet high around the base of the third pyramid. In 1997, Verner cited a 1965 survey by Maragioglio and Rinaldi, giving 104.6 m, or 200 cubits, for the base of the third pyramid. Given the measures shown above, the sides of the diagonal rectangle defined by the SE diagonals of the first and third pyramids are in the proportion of $\varphi^2 : 1$. The length of the long side of the rectangle is 1772 cubits or $\sqrt{\pi} \times 1000$, and the length of the short side is $\sqrt{\pi}/\varphi^2 \times 1000$.

This site plan for the pyramids at Giza solves the problem of drawing a circle and a square with equal areas. $\sqrt{\pi}$, or 1.772, plus $\sqrt{\pi}/\varphi^2$, or .677, equals 2.449, or $\sqrt{6}$, or the diagonal of a square with sides of $\sqrt{3}$. The length of $\sqrt{6}$, divided in the proportion of φ^2 to one, gives the length of 1.772. The diagonal of $\sqrt{2}$ gives the length of 2, and a circle with a diameter of 2 has a radius of one and the area of the circle is π. The area of a square with side lengths of 1.772, or $\sqrt{\pi}$, is also π.

$$\sqrt{6}/(\varphi^2 + 1) = .677... \quad :: \quad \sqrt{6} - .677... = 1.77246... \quad :: \quad \sqrt{\pi} = 1.77245...$$

Edme-Francois Jomard edited *Description de L 'Egypte.* In a chapter entitled *Geographical Knowledge and Maps of the Egyptians*, Jomard stated:

> "One should not be so surprised on seeing that a fact as important as the creation of maps, so honorable for the people who invented them, has remained obscure until the present day. But why is there today no authentic and impartial testimony to disperse these obscurities: Furthermore, is not the honor itself due to those who discovered maps, and is this not the cause of the silence of the Greeks concerning their true origin? Let us consider the Greeks in the era of Thales and Pythagoras, still plunged in an almost gross ignorance and suddenly proud to be in the possession of the sciences with which until then they had been unfamiliar. The Egyptians, on the contrary, a people isolated and ancient, worn by a long prosperity, communicating with reserve a small part of their knowledge to studious visitors, became indifferent to what use the visitors put their borrowings, and this knowledge was, moreover, lying unused on their ancient monuments. The petty thefts by the Greeks could not have been discovered in their own country; in Egypt these thefts were neither presumed nor prevented. How wonderfully Greek historians have concealed almost all the sources from which they had drawn!
>
> The Greek writers of early times, and the Latin writers who copied them in recounting the history of the exact sciences, usually pass over Egypt - who is their mother - in silence. In order to discover the heritage of the Egyptians, one must come to a more recent time, when the vanity of the Greeks had ceased along with their political existence. It is to the Church Fathers that we owe the most instructive facts.
>
> Saint Augustine said that the Egyptians were passionate about geometry. We cannot accuse Saint Clement of Alexandria of being too favorable to the Egyptians, so his account will not be suspect. Here is how he explains it in the sixth book of the *Stromates* (6.4), in an oft-cited passage where he described the functions of the priest of the colleges of Egypt: 'The sacred scribe is obliged to know the hieroglyphs, the cosmography and geography, the movements of the sun, the moon, and the five planets; the chorography of Egypt, the course of the Nile, the description of the temples and consecrated places, the measures and all things used in the temples.'
>
> I will compare this well-known passage with passages from the Bible where we see evidence of Egyptian methods. Moses and Joshua have indeed borrowed from Egypt what they understood of the exact knowledge. 'Choose three men from each tribe for me to send up and down the country so that they can make a survey with a view to its apportioning, and then come back to me...So the men left and went up and down the country, making a sevenfold list in the writing of all the towns.' (Joshua 18:4, 9). Joseph narrates also, but in more detail, the same fact: 'So he sent men to measure their country, and sent with them some geometricians, who could not easily fail of knowing the truth, on account of their skill in that art. He also gave them a charge to estimate the measure of that part of the land that was most fruitful, and what was not so good.' (*Complete works of Josephus*, 5.21, p. 108)

This measure of the country of Israel, ordered by Joshua in the same manner as the Hebrews had seen in Egypt, could pass for a true survey. It is the same work that had been done in the land of Egypt from a very distant age, and it is, in my opinion, the origin of topography and geography. To what exact and convenient use could one put these measures of each territory, the descriptions of the nomes, the knowledge of their limits and the subdivisions that Strabo described, if not by expressing all these proportions on the flat tables prepared for this work, such as those of which Apollonius of Rhodes speaks?

'It is said that a man left Egypt (Sesostris) to travel through Europe and all of Asia at the head of a strong and courageous army. He conquered a multitude of cities, some still inhabited today, and others abandoned because a great many years have elapsed since his time. The descendants of the men whom he established in Colchis still remain there and the colony is flourishing. They have preserved from their ancestors the engraved tables on which are drawn the borders of the earth and the sea, the routes and the roadways in a manner that serves as a guide for all travelers.' (*Argonautica*, 4.270-79)

There remain other monuments of the ancient topography of Egypt, and these monuments, although they are of a different type, are no less convincing or authentic: There are distances along routes that conform very closely to the latest observations, and the number of stades that the Egyptians reported to Herodotus, to Diodorus of Sicily, and to Strabo, when these travelers asked them about the distances to places, was very exact; there is a similar precision in the many measurements of Pliny taken in Egypt; and finally, those measurements of the ancient routes that the Romans adopted and translated without question and that we know today are accurate. I would ask how else these measurements, which are reported by Diodorus and Herodotus, could be as accurate as they are if the Egyptians had not possessed, as Clement of Alexandria reports, a detailed chorography, and if they did not have maps on which the distances were figured exactly. The distances one finds in these authors' works are not of the traveled routes; rather, they are straight-line measurements. They would have to have been measured from a bird's-eye view. How could the Egyptians have known these without the help of either maps or trigonometric observations? The existence of geographic maps among the Egyptians has been accepted by several scholars, including the celebrated author of the *Exposition of the World System*." [98]

Between 1636 and 1640, John Greaves traveled to Italy and Egypt to obtain the ancient measures of the Roman foot and the royal cubit. Greaves reported the Roman foot was divided into 16 digits and also divided into 12 uncaie (Roman inches).[99] Greaves stated: "My first enquiry was made after that monument of T. Statilius Vol. Aper in the Vatican gardens, from whence Philander took the dimensions of the Roman foot, as others have since borrowed it from him. In copying this upon an English foot in brass, divided into 2000 parts, it contains 1944 such parts as the English foot contains 2000." [100] 12 inches × 1944/2000 = 11.66 inches. Greaves also stated: "60 Roman feet is equal to 700 English inches." [101] 700/60 = 11.66 inches. 11.66/16 = Roman digit = .729 inches.

In an essay published posthumously in 1737, Newton stated: "From the pyramids of Egypt, measured by Mr. John Greaves, I collect the length of the ancient cubit of Memphis. In the middle of the first pyramid was a chamber most exquisitely formed of polished marble, containing the monument of the king. The length of this chamber was 34.38 English feet, and the breadth 17.19 English feet; that is, it was 20 cubits long, and 10 cubits broad, the cubit being 1.719 of the English foot."[102] $1.719 \times 12 = 20.628$ inches. $20.628/(20 \times \sqrt{2})$ = Egyptian digit = .729 inches.

Moody stated that 600 Attic feet = 607.9 feet. 607.9/600 = 1.013 ft. = 12.158 inches. In 1953, A. E. Berriman stated: "Stuart, assisted by Revett (c. 1750), was the first to measure the Parthenon professionally. A century later (1888) Penrose published independent measurements. Platform measurements in English feet; proportional Attic foot, in English inches:

	Platform		Attic foot	
	Width	Length	Width	Length
Stuart	101.141	227.587	12.137	12.138
Penrose	101.341	228.141	12.16	12.167

Mean of the four averages = 12.15 inches"[103]

Before Petrie went to Egypt, he published a survey of ancient linear measurements in 1877. He based his findings on reported measurements and from his own measurements of exhibits at the British Museum. This was before he recognized the connection between the digit and the remen, or the relationship of $20 \times \sqrt{2}$ between the digit and the royal cubit. Petrie stated:

> "The group of .728 inch digits are independent of the value of the cubit: if all these digits were exactly 28ths of the cubit, we should find that the group of digits and that of cubits were in exact connection; the mean of the cubits is 20.64 ± .02, and that of the digits is .73 ± .001, which × 28 = 20.47 ± .03; thus the probability against these two groups being really identical is about 650 to 1; and the probability of the principal group (.7276) being identical with the cubit is only 1 in about five million. Thus we may rest assured that the majority of the digits are independent of the 20.6 cubit; not a single example of that cubit appears as low as 28 × the mean value of the main group of digits; and the probability of the group of cubits and that of digits being identical is such as is not worth the least consideration.
>
> The Roman remains found in Britain, Africa, and other countries are included as being Italian work, and therefore probably constructed with Italian units. The standard Roman foot is found oftener than any other unit. Perhaps 11.64 ± .01 may be best adopted as the final result for the pure Roman foot. The colonial Roman seems to be longer, 11.68 ± .01.
>
> Greece: Under this head, the Sicilian remains are included, as the civilization under which they were executed was Greek in origin and in connections. The following is a compound group, part being due to the use of the Roman foot under Imperial rule, and part to the use of a unit nearly identical with it used in the

Kuklopeian remains. The mean of those marked as being free from Roman influence is 11.60 ± .02. This seems to be almost the sole Pelasgic or Kuklopeian unit; and it is the same as the 'ancient Greek foot' of 16 Egyptian digits, mentioned by some authors; 16 of those digits would make it 11.64 ± .02. The mean of the examples marked as very probably the Roman foot is 11.68 ± .02.

From the examples of the Olympic foot, the mean is 12.159 ± .004. This agrees well with the measure from the Parthenon step, 12.1375 ± .0003. The Olympic cubit is very closely equal to (and probably derived from) an Egyptian unit noticed by Boeckh, which is clearly a length of 25 digits, thus the foot would be ⅔ × 25 = 16 ⅔ digits - i.e., 6 feet, or 1 *orguia* of 100 digits. And further, 1/100th of the *orguia* is .72845 inches by the Parthenon foot (12.1375), or .7295 ± .02 by the value for the foot inductively found (12.16); so that the *orguia* is exactly 100 of the digits usually found in other countries.

It would therefore seem highly probable, as the 1/100th of the *orguia* corresponds with the usual, or early Greek, digit; and as the Olympic cubit and foot was probably derived from an Egyptian cubit of 25 digits, each = 1/100th of the *orguia*; that the Greek digit was originally similar to those of other countries, especially Egypt, from which the Olympic cubit and foot was derived. Also, that thus the digit completed a pure decimal system in 5 grades, culminated in the stadion; and that afterwards it was slightly altered, in order to accommodate it to a binary division of the Olympic foot; which instead of containing 100 digits to the 6 feet, or 25 to the cubit, as in Egypt (i.e. 16 ⅔ digits to the foot), contained only 16 of the new digits, a change which might seem practically convenient, but which overthrew the pure decimal system originally established.

This change would seem to have been caused by the decimal scale obtaining ascendency; and perhaps the date of this change may be pointed out by the Pelasgic remains being all in Ancient Greek feet of 16 original or Egypt digits, and later remains, probably from about the fifth century B.C., mainly dropping this old Pelasgic foot, and largely adopting other units, owing to the change in the digit which made the Pelasgic foot inconvenient to use.

The ratio of 24:25 between the Roman and Olympic feet, so often mentioned in metrology, is due to the Olympic foot being 1/6th of 100, or 16 ⅔, old digits; whereas the Roman and the Pelasgic foot had a binary relation, being 16 old digits; thus 16 : 16 ⅔ :: 24 : 25." [104]

According to Petrie, the Pelasgic foot was equal to the Roman foot of 16 Egyptian digits, or 4/5 of a remen. The Attic foot was based on 1/6 of 100 Egyptian digits, or 5/6 of a remen. The ratio between the Roman foot and the Greek foot is 24/25 (4/5 × 6/5 = 24/25), and the ratio between the 625 feet in the Roman stadium and the 600 feet in the Greek stadium is 25/24. Berriman stated that the Roman stadium and the Greek stadium both equal 500 remen, and 5000 remen = 10 Roman stadiums = 10 Greek stadiums = one minute of latitude.[105]

ACTS 27:28 "And sounded and found it twenty fathoms: and when they had gone a little further, they sounded again and found it fifteen fathoms." The Book of Acts was written in Koine Greek, based mainly on Attic and Ionic speech forms.[106] Fathom is translated from the Greek *orguia*. A fathom is presently defined as six English feet, although a previous British Admiralty definition was 1/100 of a cable, or 1/1000 of a nautical mile,[107] equal to 100 ancient Egyptian digits, or one *orguia*, or six Attic feet, or five remen, or 1/1000 of a minute of latitude. 10 stades, or 1000 fathoms, or 6,000 Attic feet, or 100,000 digits, is equal to one minute of latitude.

HERODOTUS 2.149: "One hundred fathoms equals a furlong of six hundred feet, the fathom being measured as six feet or four cubits, the feet being four palms each, and the cubits six." During the lifetime of Herodotus, Egypt was under Persian control. This may explain why Herodotus related the length of the Egyptian schoinos to the Persian parasang, and may explain the length of 12,000 royal cubits for the late period Egyptian schoinos. HERODOTUS 2.6 states that the Persian parasang was 30 furlongs, although Herodotus understood the relation between the Egyptian schoinos and the Persian parasang to be two to one, rather than the understanding of other ancient and modern sources, that the relation between the two is one to one.

The schoinos of 12,000 Greek cubits, or 400 Greek cubits times 30, is equal to one league, or three minutes of latitude. The length of the Persian parasang was 12,000 cubits, or 30 stadia of 400 royal cubits, although the Persian royal cubit was longer than the Greek cubit. According to Petrie, the length of the ancient Babylonian digit was .73 inches, the same as the Egyptian, Greek and Roman digit, and the length of the Babylonian royal cubit was 20.6 inches,[108] the same or very nearly the same as the early Egyptian royal cubit. Based on more recent studies of ancient cubit rods, Rolf Rottlander and Lelgemann gave 20.4 inches, equal to 28 digits, for the Nippur cubit. [109]

STRABO 17.1.24: "Artemidorus says that the navigation up the river from Alexandria to the vertex of the Delta is 28 schoeni, which amount to 840 stadia, reckoning the schoenus at 30 stadia. In sailing up from Pelusium to the same vertex of the Delta, is a distance, he says, of 25 schoeni, or 750 stadia." This is a correct measure of the distance from the apex of the Delta to Pelusium and it is the same distance given by Herodotus from Heliopolis to the sea, *in schoinos*. However, the measure given by Herodotus of 25 schoinos times 60, or 1500 stadia, from Heliopolis, extends nearly 100 miles into the Mediterranean Sea.

In 1882, Freidrich Hultsch stated: "Herodotus, through a misunderstanding, doubled the value of the schoinos to 60 stadiums." [110] In 1901, Kurt Sethe compared several of the schoinos measures of Herodotus to the actual distances between the given locations in Egypt, and found that the number of stades required to match the distances on the ground was between 30 and 40 stades per schoinos. According to Sethe, Herodotus erred in assigning 60 stadia to the Egyptian schoinos.[111]

The distances given by Herodotus are sometimes in schoinos, sometimes in furlongs (stades), and sometimes both. Where both measures are given, the number of schoinos, times 60, is the number of furlongs. In cases where the distance is given only in furlongs, the distances in furlongs, divided by 60, produce whole numbers of schoinos. This indicates that Herodotus started with what he described as the Egyptian schoinos, then multiplied by 60 for his measures in furlongs.

In 2003, Peter Thonemann described a Hellenistic distance marker from Ephesus and three Hellenistic distance markers known from Macedonia, "all of them with distances divisible by 10 stades." Thonemann proposed the designation of 'decastadion,' compared to previous designations of 'milestone' or 'stadion-stone.' Thonemann stated: "This dating (Hellenistic period) is certainly correct, on grounds of lettering, the absence of any mention of Roman authorities, and the use of stades rather than miles to measure distance." [112] The decastadion equals one minute of latitude, or one nautical mile, or 1852 m, or 6076 English feet. The Roman mile equals 1000 paces of five feet. The Roman stade of 625 feet × 8 = 5000 Roman feet = 4/5 of a nautical mile. The Greek stades are the same length as the Roman stade and also equal one eighth of a Roman mile.

In 1822, John Murray stated: "In the Itinerary of Antonius, the places, and their interjacent distances are stated as follows: Casium to Pentaschoenum - 20 M.P. (mille passus or Roman mile), Pentaschoenum to Pelusium - 20 M.P. STRABO 16.2.28, in placing Casium at three hundred stades from Pelusium, differs not much from the 40 M.P. of the Itinerary, or the ten schoenes indicated by the word Pentaschoenum, midway." [113] The distances from the Antonine Itinerary and from Strabo, indicate 40 Roman miles equals 10 schoinos; and 300 stades equals 40 Roman miles; thus 30 stades equals four Roman miles equals one schoinos.

PLINY 5.11 gives 30 stadia for the schoenus. PLINY 12.30: "the length of the schoenus, according to the estimate of Eratosthenes, is forty stadia; some persons, however, have estimated the schoenus at no more than thirty-two stadia." A schoinos of 32 Greek or Roman stadia produces a schoinos equal to four Roman miles (8 × 4). A schoinos of 30 stadia of 400 royal cubits, or Eratosthenes' schoinos of 40 stadia of 300 royal cubits, is slightly longer: 12,000 × 20.62 inches = 20,620 English feet (Egyptian schoinos). 6076 × 4/5 × 4 = 19,443 English feet (4 Roman miles).

Despite the evidence of the Egyptian schoinos of 12,000 cubits, a schoinos of 60 stades, or 24,000 cubits, sometimes rounded down to 20,000 royal cubits, and sometimes equated with the length of the *itr*, is attributed to Herodotus and asserted as evidence that ancient measures of Egypt, in *itr* and schoinos, are too long to be meridian measures. Also asserted as evidence of a long schoinos is the statement of Herodotus about Egypt's southern border area, the Dodekaschoinos.

HERODOTUS 2.29: "I went myself as an eyewitness as far as the city of Elephantine and from that point onwards I gathered knowledge by report." HERODOTUS 2.29 relates that the first cataract is bounded by the island of Elephantine at the lower end and by the island of Tachompso at the upper end; that the island of Tachompso is occupied one-half by Egyptians and one-half by Ethiopians; and that the distance from Elephantine to Tachompso is 12 schoinos.

STRABO 1.2.32: "Syene is entirely in Egypt, while Philae is inhabited by a mixed population of Ethiopians and Egyptians." Arthur Weigall stated: "The known history of Philae does not carry one back to a period earlier than the Ethiopian dynasty, the altar of Taharka (690-644 BC) being the oldest monument on the island." [114] Griffith stated: "On one face of the altar is an inscription of Tirhaqa: Beloved of Amun of Taqempso." [115] Dieter Arnold stated: Amasis II (570-526 BC) rededicated the temple on Philae to the cult of Isis. Nectanebo I (380-362 BC) expanded the temple of Isis on Philae and it was further expanded by Ptolemy II Philadelphus.[116]

Source of Image: Wikimedia Commons: Famine Stele - Photo Credit: Morburre

During the reign of Ptolemy V (204-180 BC), the Famine Stela was inscribed on the island of Sehel, in the vicinity of Elephantine. The inscription claims to be a copy of an Old Kingdom decree, donating the southern border area to Khnum. The claim of Old Kingdom authority for the donation to Khnum is disputed. Dating the description and boundaries given in the Famine Stela to the Old Kingdom is also disputed. The inscription contains mixed hieroglyphic forms, including *itr*, rather than the late period *ar*, or schoinos. As translated by Miriam Lichtheim:

"Its water rages on its south for an *itr*, a wall against the Nubians each day. There is a mountain massif in its eastern region, with precious stones and quarry stones of all kinds; likewise tall plants and flowers of all kinds that exist between Elephantine and Bigah, and are there on the east and the west; the stones that are there, lying in the borderland, those on the shores of Elephantine's canal, those in Elephantine, those in the east and west, and those in the river. A royal offering to Khnum, lord of the cataract region and chief of Nubia: In return for what you have done for me, I offer you Manu as western border, Bakhu as eastern border, from Elephantine to Tachompso, being twelve *itr* on the east and west." [117]

Maspero stated: "Their earliest horizon was limited. Their gaze might wander westward over the ravine-furrowed plains on the Libyan desert without reaching that fabled land of Manu where the sun set every evening, but looking eastward from the valley, they could see the peak of Bakhu, which marked the limit of accessible regions. Long after the Egyptians had broken through, the names of those places which had as it were marked out their frontiers, continued to be associated in their minds with the idea of the cardinal points. Bakhu and Manu were still the most frequent expressions for the extreme East and West." [118]

Laszlo Torok stated: "The decree of Ptolemy VI Philometor, inscribed in the Temple of Isis on Philae in 157 BC, gave twelve schoinos from Tachompso to Syene on the west bank and twelve schoinos on the east bank, making together twenty-four schoinos, to Isis, with all their fields, ponds, islands, stones, plants, trees, flocks, cattle, fish, birds, oils, and all things which exist there." [119]

According to Edwyn Bevan: "The temple at Pselcis (modern Dakkeh) is stated by its hieroglyphic inscriptions to have been built by (Meroitic king) Ergamenes, yet on the same temple we find reliefs added by Ptolemy IV Philopator (221-204 BC). The temple at Pselcis also contains the hieroglyphic statement of Ergamenes that Isis had given to him the Land of the Twelve *Ar*, from Syene to Tachompso. On Philae, Ergamenes had himself represented on the walls as Pharaoh, yet in close neighborhood to representations of Ptolemy IV in the same character." [120]

Griffith stated: "On the temple of Dakka is an inscription of a very late period by 'the agent of Isis in Philae and of the foreigners of Tacompso, chief of the region of thirty, scribe of the king of Cush,' etc. The 'region of thirty' may represent the triacontaschoenus, or district of thirty schoeni, of which there are a few records. In it a certain Boethus founded two cities named after Philometor and his wife Cleopatra, according to an inscription which probably came from Philae; and Cornelius Gallus, after his conquest of the Thebais, having passed the cataract and met the ambassadors of the Ethiopians on Philae, appointed an (agent?) for the triacontaschoenus. PTOLEMY 4.7.10 appears to place the district beyond the second cataract on the west side of the Nile; but he may be wrong, and it would not be surprising to find that it either included, or lay immediately to the south of, the Dodecaschoenus." [121]

Griffith stated: "So far as we know, no Ptolemaic or Roman ruler had his name inscribed on any building in Nubia south of the Dodecaschoenus. The earliest of the Ptolemies whose name is found south of Philae is Philopator at Dakkeh. South of the Dodecaschoenus we have only a few records of Cyrenaean Greeks in the temple built by Hatshepsut at Buhen opposite Wadi Halfa. From this place Professor Sayce published two graffiti of Cyrenaeans, attributing them to the second or third century BC; and in 1912 the present writer removed into the temple for safety an inscribed sandstone slab, perhaps a grave-stone. Mr. M. N. Tod assigns the lettering to the period from the fourth to the second century BC." [122]

Following the Roman conquest of Egypt in 30 BC, and an unsuccessful attack by the kingdom of Meroe on the first cataract region in 23 BC, the northern border of Meroe and the southern border of Roman Egypt was established at Maharraqa. Arnold stated: "The Roman presence was manifested at the southern border at Maharraqa (Hiera Sycaminos) by a temple dedicated to Isis and Serapis that cannot be securely dated because it was neither completed nor inscribed. However, since temple building in Nubia declined after the reign of Augustus, one might date the Maharraqa temple to this period (19 BC - 14 AD)." [123]

Griffith stated: "A few sculptures and hieroglyphic inscriptions from the south wall of the temple at Maharraqa were observed by Burckhardt and recorded by Lepsius and Erbkam, showing that Isis and Osiris of Philae 'in Kem-so' were here worshiped. The only date preserved is in a Greek graffito, of year 21 Thoth 12 of Trajan, i.e. 9 Sept. 117 AD, actually a month after the death of Trajan." [124]

PLINY 6.35 gives long lists of towns south of Syene (Elephantine) from accounts of Bion and Juba. The names and locations of virtually all of the towns are unknown, but Tacompsos is listed above the cataract on the east side of the river by Juba, and Tacompsos is listed above the cataract on both the east and west sides of the river by Bion. PLINY 6.35 states that persons sent by Emperor Nero determined that the distance from Syene to Hiera Sycaminos was 54 miles. PTOLEMY 4.7.10 lists five sites under the heading of the Dodecaschoenus: 1-After the lesser cataract; 2-Hiera Sycaminos; 3-Philae; 4-Metacompso; 5-in which region on the west bank of the river is Pselcis.

The opinion that Maharraqa is Tachompso is based on the Roman establishment of Maharraqa as the southern limit of the Egyptian border in 23 BC, and the inscription that mentions Kem-so in the subsequently constructed Roman temple at Maharraqa, and the subsequent listing in Ptolemy's *Geography* of Hiera Sycaminos in the Dodekaschoinos. This opinion is often accompanied by a statement that Maharraqa is 75 miles or 120 km away from Elephantine, and that 120 km/12 schoinos = 10 km per schoinos.

It has also been suggested that Derar is Tachompso. In 1907, Weigall stated: "It has been pointed out that Derar is probably to be identified with the ancient Tachompso. In the temple of Dakkeh, Ergamenes states that he ruled the land from Aswan to Takompso, and it seems that this island was, at various periods, the limit of the Egyptian or Lower Nubian dominions." [125] In 1976, Alan Lloyd stated: "The meaning of Tachompso is obscure. The exact location has given rise to some controversy." Nonetheless, Lloyd concluded that "Djerar was identical with Tachompso." Lloyd acknowledged that "Griffith objects to the identification of Djerar with Tachompso because Djerar was a shifting island without any antiquities." [126]

Due to the late establishment of Maharraqa as the southern border, and because Maharraqa is not an island, it has also been suggested that Tachompso was extended, approximately five miles south, from Derar to Maharraqa. It is generally acknowledged that the region of the first cataract, from Elephantine to Philae, is the traditional border of Egypt, but Philae as Tachompso has been rejected because the distance from Elephantine to Philae is much less than 12 schoinos.

In 1901, Sethe wrote an article in response to the assertion of Maharraqa, and/or Derar, as Tachompso. He was not yet aware of the inscription at Maharraqa that mentioned Kem-so, or the inscription at Philae on the altar of Taharqa, that had not yet been discovered. Sethe concluded that Tachompso was the island of Philae. He believed that this was the only conclusion that was consistent with the statements of Herodotus, the statements in the Famine Stela, and the statements by Bion and Juba, that Tachompso was at the head of the first cataract. Sethe pointed out that the first cataract area, from Elephantine to Bigah, was Egypt's traditional border, and that even though this was a border area, as opposed to a borderline, it was an east-west border, marking the southern limit of Egypt, and the northern limit of Ethiopia. Sethe believed that 12 schoinos for the length of the cataract was based on a very short schoinos. Sethe stated that the distance from Elephantine to Maharraqa was too far to be 12 schoinos in general, and too far to be 12 schoinos of Herodotus in particular, based on comparisons of the actual distances between known Egyptian localities, and the distances between these localities given by Herodotus in schoinos. Sethe added that it would be contradictory to conclude that the distance from Elephantine to Derar and the distance from Elephantine to Maharraqa were both 12 schoinos along the river.[127]

In 1903, Victor Loret objected to the idea of a very short schoinos. He believed the *itr* and the schoinos both had lengths of 10,000 and 20,000 royal cubits. Loret concluded that the distance of 12 schoinos from Elephantine to Maharraqa confirmed his opinion that the distance of 1113 km, or 106 *itr* of 20,000 cubits, was the average of his given distances of 831 km in a straight line, and 1415 km along the river, from Elephantine to the sea.[128]

After learning of the inscription at Maharraqa that mentions Kem-so, Sethe wrote another article about the Dodekaschoinos in 1904, that has been described as abandoning or renouncing his first article. Sethe acknowledged that the mention of Kem-so identified Maharraqa as the southern boundary of the Dodekaschoinos, at least as early as the time of the inscription, but he repeated that this would be difficult to reconcile with the older statements that identified Philae as Tachompso. In 1906, James Breasted stated: "According to an inscription in Maharraka, found by Sethe in one of Lepsius' notebooks, Takompso must be at least as far south as the former town, so that Sethe's ably defended thesis confining the dodekaschoinos to the cataract between Assuan and Philae is thus disproved for the Greco-Roman age at least, and probably also for the earlier time." [129] Sethe concluded his 1904 article by saying "The question of expansion of the Dodekaschoinos in Greco-Roman times is decided in favor, but must remain open for older times until further notice." [130]

In 1911, Petrie stated that the Harris papyrus recapitulated all of the offerings of Rameses III. One section dealt with Nile offerings over an eight year period. "A very frequent base is 848, of which there are 53 examples. Taking eight years, as stated, this yields 106 per year. What then can 106 be, as a basis of offerings to Hapi? The Nile was divided into a series of towing stages, called schoinoi, though longer than the land schoinos. The length of these stages is best fixed by those beyond Egypt in the dodekaschoinos from Aswan to Takompso. This district was about 81 miles along the river, giving 6.74 miles per schoinos or 5.92 miles if measured direct along the plain. This river schoinos is stated by Herodotus to be 60 stadia, or 6.9 miles. Probably it was 20,000 cubits, or 6.55 miles. On this scale of the schoinos the distance from Aswan to Memphis (by the river windings) would be 84 schoinoi, and from Memphis to the sea 21 on the plain or 27 by the river windings. His statement of the number of schoinoi is apparently corrupt, as he gives from the sea to Heliopolis as 25, and from Heliopolis to Thebes as 81, making 106 from the sea to Thebes. He then adds 30 from Thebes to elephantine. It is possible, however, that the schoinos varied, and that the 106 which we here deal with were the Nile stages only from Thebes down to the sea. The conclusion then is that the 106 offerings were made for, or at, each stage on the Nile." [131]

Irina Tupikova stated: "Hultsch also cites Herodotus, who equaled a schoinos to 60 stadia, an error which may be ascribed to the usage of the notation 'schoinoi' for stations for ship towing along the Nile, which were of different lengths." [132] Fixing the length of towing stages based on the distance along the river from Elephantine to Maharraqa assumes that Tachompso was Maharraqa; that 12 schoinos was a measure of towing stages along the river; that the length of towing stages were the same; and that the length of the *itr* and the schoinos were the same. HERODOTUS 2.9 "The distance inland from the sea to Thebes is 6120 furlongs (102 schoinos); and the distance from Thebes to Elephantine is 1800 furlongs (30 schoinos)." 132 schoinos, compared to 106 *itr*, is not corrupt: 106 *itr* × 15000/12000 = 132 schoinos. The length of 30 schoinos from Thebes to Elephantine also contradicts the proposed length of 12 schoinos from Elephantine to Maharraqa.

In 1930, after Griffith discovered the inscription 'Beloved of Amen of Taqempso' on Philae, Griffith stated: "The presence here of the remarkable name Taqempso is enough to revive Sethe's theory (in his *Dodekaschoinos*) that the island Tachompso in HERODOTUS II 29, is Philae. The evidence is clear in inscriptions of Ptolemaic and Roman date and in the geographical work of Ptolemy that Kem-so was in the neighborhood of Maharraqa; its temple lies 113 kilometres above Philae on the western mainland, dedicated to Isis and forming the southern boundary of the Dodecaschoenus. Its name, probably non-Egyptian, is essentially the same as that on the Tirhaqa monument, although the phonetic renderings of it are very different. Probably then in Tirhaqa's time Philae was Tachompso. Herodotus virtually puts Tachompso at the head both of the First Cataract and of the Twelve Schoeni, in two contradictory situations. Thus it would seem that the name had already moved southward by 450 BC, perhaps under Persian influence, but the tradition of its earlier position still survived to confuse the old historian and through him the lexicographers." [133]

David Klotz stated: "The installation of Persian garrisons at Elephantine and Syene (during the first Persian period 535-402 BC) reflects the continued engagement with Egypt's southern frontier. However, the pottery from the second cataract fort at Doginari, previously ascribed to the Saite-Persian period (Heidorn 1991, 1992), has more recently been dated to dynasties 25-26 (Heidorn 2013), and thus no longer confirms Achaemenid domination south of Elephantine." [134]

The opinion that Tachompso had already moved south when Herodotus made his statement in 450 BC is not supported by any remains concerning Tachompso or the Dodekaschoinos and is contradicted by the Famine Stela and by the later statements of Bion and Juba, placing Tachompso at the head of the first cataract. The opinion that Herodotus was wrong about the southern limit of the Dodekaschoinos at the head of the first cataract is based on the opinion that Herodotus was right about 12 schoinos being a north-south measure along the river.

The Famine Stela, the inscription of Ergamenes at Dakka, and Philometor's decree all give 12 schoinos on the east side of the river and 12 schoinos on the west side of the river. The Famine Stela and the decree of Philometor both give extensive descriptions of the east-west area of the Dodekaschoinos, including the river, fields, quarries, mines and the desert. The Famine Stela gives Manu and Bakhu as the east-west extent of the border area, but these are symbolic boundaries of uncertain distance from the river, and they are not mentioned in Philometor's decree. Instead, Ptolemy VI gives twelve schoinos from Tachompso to Syene on the west side of the river and twelve schoinos on the east side of the river, making together twenty-four schoinos. This is a complete and coherent definition of the north-south boundary, from Tachompso to Syene, and of the east-west boundary, of 12 schoinos on the east side of the river and 12 schoinos on the west side of the river, making together 24 schoinos. The statement of 12 + 12 = 24 schoinos as a north-south measure, or a river measure, makes Ptolemy's decree incomplete and incoherent.

The mentions of the triacontaschoinos give no indication of north-south or east-west borders. The Greek graffiti found at Buhen, consisting of unknown Greek names, makes no mention of the triacontaschoinos. The undefined area of the triacontaschoinos does not support the conclusion that the distance from Elephantine to Maharraqa is 12 schoinos. The inscriptions indicate that 12 schoinos was an east-west measure, which it would have to be, with the north-south limits of the ancient Egyptian border area between Elephantine and Philae. Not having traveled south of

Elephantine, Herodotus erred in giving the measure of 12 schoinos as the length of the first cataract. The error of Herodotus, applied to the distance from Elephantine to the southern border established by Rome at Maharraqa, has no bearing on the length of the ancient Egyptian *itr*, or *ar*, or schoinos.

In 1925, Herbert Thompson stated: From the metrological statements in the papyrus at Heidelberg (dem. no. 1289), "we obtain the following table:

100 cubits	=	1 *ẖ-nwh*				
4 "	=	1 stadion				
120 "	=	30 stadia	=	1 *shfe-t*		
240 "	=	60 "	=	2 "	=	1 *kmy* -

- It is unfortunate that the word for the highest measure is written with a single symbol only which gives no clue to the reading. The second part of the word is certainly 'of Egypt' and the whole must mean 'Egyptian schoenus'. Herodotus in a well-known passage (II, 6) states that thirty stadia are equal to a parasang and sixty stadia to a schoenus. The accuracy of this statement has been severely criticized by Prof. SETHE in his *Dodekaschoinos* (*Untersuch*., II, Heft 3, 63 seq.). He refers to Artemidorus (as quoted by Strabo), a traveler of circa B.C. 100 and therefore but little later than our document...and concludes in agreement with SCHWARZ (*Berl. Studien f. klass. Philologie*, XV, Heft 3, 1894) that the schoenus was in later times usually equal to 30 stadia, or less often to 40." [135]

This statement from the Heidelberg papyrus, written in the 2nd century BC, appears to be a demotic copy of the statement by Herodotus. The only metrological difference between the two is the stadia given by Herodotus, of 600 Greek feet, or 400 Greek cubits, or 185m, versus the stadia from the demotic papyrus of 400 royal cubits, or 210m. Named ancient Egyptian multiples of the royal cubit were the *Khet*, of 100 royal cubits, and the *itr*. There was no named ancient Egyptian equivalent for Greek or Roman stadia. In 2004, Gyula Priskin stated: "The relation that one stade is made up of 400 royal cubits seems, however, evident in Papyrus Heidelberg 1289." [136] Priskin concluded that the stade of Eratosthenes was 400 royal cubits, and based on 5300 Eratosthenian stades as a 50x multiple of 106 *itr* for the length of Egypt, concluded that the length of the *itr*, and the Egyptian schoinos, were both 20,000 royal cubits.

In 2018, after comparing straight line distances between known locations, with the distances given in stades by ancient authors, Dmitry Shcheglov stated: "This result might have been regarded as a brilliant confirmation of the short stade hypothesis, but strangely enough, in comparing ancient and modern distances, the proponents of the itinerary stade lose sight of a crucial factor, namely measurement error. Moreover, they proceed from a tacit assumption that distances recorded in ancient sources were almost as accurate as those measured on a modern map. However, this cannot be true for two main reasons. First, with rare exception, there is no indication that distances given by ancient sources derive from actual measurements on the ground, rather than from rough estimates deduced, for example, from the duration and the average speed of travel. Second, and most importantly, even when ancient distances do derive from actual and quite accurate measurements, as is the case with the late Roman itineraries, they were certainly measured not as the crow flies, but including all numerous twists and turnings of the route. Even when ancient surveyors were able to make accurate measurements of separate road sections, they simply had no need and most probably never tried to calculate the overall straight-line distance between start and end points. This is the major reason why distances recorded in ancient sources must inevitably be over-estimated in comparison with those measured in a straight line on the modern map." [137]

According to Pliny 5.9 "The Nile, dividing itself, forms on the right and left the boundaries of Egypt's lower part. By the Canopic mouth it is separated from Africa, and by the Pelusiac from Asia, there being a distance between the two of 170 (Roman) miles." $170 \times 4/5 = 136$ nautical miles, $\times\, 5000 = 680{,}000$ remen, $/\sqrt{2} = 480{,}832$ royal cubits, $/12{,}000 = 40$ Egyptian schoinos. The straight line distance from Alexandria to Pelusium is 136 nautical miles, or 40 schoinos.

HERODOTUS 2.15 "If we desire to follow the opinions of the Ionians as regards Egypt, who say that the Delta alone is Egypt, reckoning its sea-coast to be from the watch-tower called of Perseus to the fish-curing houses of Pelusion, a distance of forty schoinos." HERODOTUS 2.6 "As to Egypt itself, the extent of it along the sea is sixty schoines, according to our definition of Egypt as extending from the Gulf of Plinthine to the Serbonian lake, along which stretches Mount Casium; from this lake then the sixty schoines are reckoned." HERODOTUS 2.30 "In the reign of Psammetichos garrisons were set, one towards the Ethiopians at the city of Elephantine, another towards the Arabians and Assyrians at Daphnai of Pelusion, and another towards Libya at Marea."

The location of the watch-tower of Perseus is unknown and the precise location of Marea is uncertain. In 1854, William Smith stated: "Marea, the chief town of the Mareotic Nome, stood on a peninsula in the south of Lake Mareotis, nearly due south of Alexandria, and adjacent to the mouth of the canal which connected the lake with the Nile. Under the Pharaohs, Marea was one of the principal frontier garrisons of Egypt on the side of Libya." [138] Heinrich Kiepert's 1879 map of Roman Egypt gives locations for Marea, Pelusium and Mount Casium.[139]

Kiepert's map places Marea, and the peninsula above Marea, slightly farther west of Alexandria, and slightly farther west of the canal, than the description given by Smith. Alexandria's longitude is 29°55'E. Given a schoinos of 12,000 royal cubits, a longitude of 29°52'E places Marea 40 schoinos due west of Pelusium. Opposite Pelusium, the location of the watch tower of Perseus may be associated with Marea, on the fortified western frontier of ancient Egypt. According to Murray and Strabo, the distance from Pelusium to Casium is 10 schoinos. Although Lake Serbonis and the Gulf of Plinthine are diffuse locations, the given distance of 60 schoinos is also correct.

In 1882, Hultsch stated: "The fortieth of the schoinos is the Eratosthenian stadium of 300 royal cubits, or 157.5 meters." [140] In 1957, Ivor Thomas stated: "Heron of Alexandria (*Dioptra* 36), STRABO II.5.7 and Theon of Smyrna (ed. Hiller 124, 10-12), give Eratosthenes' measurement as 252000 stades, against the 250000 of Cleomedes. 'The reason of the discrepancy is not known; it is possible that Eratosthenes corrected 250000 to 252000 for some reason, perhaps in order to get a figure divisible by 60 and, incidentally, a round number of 700 stades for one degree. If PLINY XII.13.53 is right in saying that Eratosthenes made 40 stades equal to the Egyptian schoinos at 12000 royal cubits of .525 meters, we get 300 such cubits, or 157.5 meters, i.e., 516.73 feet, as the length of the stade. On this basis 252000 stades works out to 24662 miles, and the diameter of the earth to about 7850 miles, only about 50 miles shorter than the true polar diameter, a surprisingly close approximation, however much it owes to happy accidents in the calculation.' (Heath -1921, *H.G.M.* II.107)" [141]

Tupikova stated that given the circumference of 180,000 stadia for Ptolemy, and 252,000 stadia for Eratosthenes, and assuming both scholars used the same stadia, "The Ptolemaic earth is too small in comparison with the Eratosthenian earth (i.e. 28,305 km vs. 39,690 km, if one estimates the stadion as 157.5 m). The recalculation of spherical coordinates given on a sphere of one size to a sphere of another size is simple from the mathematical point of view, but requires some experience in the subject. The results of such a recalculation show that if Ptolemy had adopted Eratosthenes' figure, the majority of his positions would have had coordinates which match their modern counterparts remarkably well. As a consequence, one can confirm first the very high precision of Eratosthenes' result for the circumference of the earth and second, the near equivalence of the length of stadion used by both scholars." [142]

Based on a statistical analysis of the longitudes reported by Ptolemy, Lucio Russo stated: "In the Greek world several different 'stadia' had been in use and the value of the one used by Eratosthenes is a *vexata questio*. Hultsch, in 1882, had determined it as 157.5m and this measure was accepted by most of the scholars till the first half of the twentieth century. Among the many other values that have been proposed it seems that the most widely accepted nowadays is 185m, which is the length of the so-called 'Attic stadion'. This value is documented in many sources, but not explicitly referring to Eratosthenes, while Hultsch's argument was based essentially only on a single statement by Pliny, which nevertheless refers explicitly to Eratosthenes. If we accept Hultsch's value, the error of Eratosthenes' measure is less than 1%, while if we assume that his stadion was the Attic one the error is about 17%. Whereas there is no general agreement on the length of the 'stadion' used by Eratosthenes, all scholars agree that later geographers, like Hipparchus, Strabo, Marinus and Ptolemy, used his same stadion, as shown by the fact that many distances in stadia have the same value for all of them...We obtain for the stadion the value of 155.6m. Since $155.6 \times 252{,}000 = 39{,}211{,}200$m, this value would correspond to an error a little less than 2% on Eratosthenes' measurement of the great circle of the earth...lending strong support to Hultsch's determination and allowing us to exclude, in my opinion, that Eratosthenes had used the Attic stadion of 185m or the even larger stadia proposed by some scholars." [143]

Russo stated that while Cleomedes gave 250,000 stades for Eratosthenes' circumference, all other ancient sources gave 252,000 for Eratosthenes' figure, and that Eratosthenes may have given 5,250 stadia as 1/48 of the circumference, giving 252,000 for the circumference, while Cleomedes gave 5000 stadia as 1/50 of the circumference, to simply explain *The Method of Eratosthenes* in his short account. "In Eratosthenes' time the angles 1/12 of a turn (corresponding to one sign of the zodiac, or 30° in our notations), 1/24 of a turn (half sign or "step") and 1/48 of a turn ("part"), as well as sixtieths of a turn, were privileged as units of measurement, so that 1/48 of a turn was a very natural result of an angular measurement, while the angle reported by Cleomedes (1/50 of a turn) is hard to express in the units then used. 5,250 stadia is a plausible result of the measurement of Eratosthenes, because he used to express large distances as multiples of 250 stadia. An important piece of evidence is provided by Strabo, who reports that the distance between Syene and the Mediterranean was estimated by Eratosthenes as 5,300 stadia. Since Strabo always expresses large distances as multiples of 100 stadia, his figure has the best possible agreement with the value of 5,250 stadia." [144] The length of 5,250 stadia for 1/48 of the circumference gives 252,000 stadia for the circumference, or 700 stades per degree. The length of 5,300 stadia for 1/48 of the circumference gives 254,400 stadia for the circumference, or approximately 707 stades per degree.

1n 1851, Alexandre Vincent edited the posthumous publication of *Studies of Fragments of Hero of Alexandria - or - On the Egyptian Measurement System*, by Jean-Antoine Letronne. Letronne stated: "A vast number of distances were transmitted to us by geographers and historians in antiquity who did not themselves verify those distances. Sometimes they did not even bother to indicate how the measurements had been made...Throughout antiquity, historians and travelers alike have almost never taken different units of measurement into account when traveling across foreign lands. They simply wrote down distances they were told in each one of those countries, without bothering about the actual unit used for those measurements...That is how we quickly realize that the word stade was simply a Greek word here applied to an Asian measurement...Consequently, geographic distances defined in Asia by ancient scholars had actually rarely been measured in Greek stades, but rather in that particular nation's unit of measurement. If we focus on Greece, Italy, and a few parts of Asia Minor, we may recognize distances expressed in Greek stades, but as soon as we travel to other nations in Asia, or to Egypt, we face seemingly insurmountable obstacles. More specifically, it is very difficult to establish the basis on which geographers from the School of Alexandria created their units of measurement. Today, we realize that all of their measurements were incorrect, even in places they knew well. However, their errors are so significant, that we cannot blame them on the basis of ignorance. Rather, we may be the ignorant ones, because we do not know what units of measurement were actually used to express those distances." [145]

Vincent stated: "Throughout his thesis, Letronne was convinced that the Egyptian soil had been measured according to a triangular principle from very ancient days. These measurements had enabled Egyptians to know with extreme precision, the dimensions of all things. According to his conclusions, Letronne was convinced that the stadion used by Eratosthenes, and defined as contained 700 times in the degree of latitude, essentially belonged to and originated from Egypt." [146]

Letronne specialized in translations of ancient Greek and Roman inscriptions. From Letronne's translation of a Roman inscription from the first century AD: "As for those who became alarmed when they heard talk of a measurement of the lands in Alexandrian territory, insofar as the ancient evaluation was maintained and never the chain of a land surveyor was carried on these lands, they should not address such supplications to us. These would be utterly useless, as no one would dare, nor would they have permission, to renew a territorial measurement, as you must always benefit from the advantages of that which has been done since ancient times." [147]

Surveys of the cultivable fields of Egypt are well documented because they were required throughout Egyptian history, due to the annual changes in the Nile Valley caused by the inundation. An accurate survey to determine the length of Egypt, or the size of the earth, would only have to be done once, for so long as the results of the survey were preserved, by means such as the sacred cubit rods, and the inscriptions on the cubit rods and in the temples, giving 106 *itr* for the meridian length of Egypt, divided into 20 *itr* for lower Egypt and 86 *itr* for upper Egypt.

Herodotus gave 132 schoinos for the length of Egypt and stated that his measures were confirmed by the Egyptian Oracle of Ammon. Taken as measures of latitude and longitude, his measures are correct, based on the Egyptian schoinos of 12,000 royal cubits. His measures and his division of upper and lower Egypt are also the same as the ancient Egyptian *itr* measures from the Karnak cubit rods, the White Chapel, Edfu, and the Tanis papyrus, with the conversion of the ancient Egyptian *itr* of 15,000 royal cubits to the late period Egyptian schoinos of 12,000 royal cubits.

Eratosthenes' figure of 5000 stadia for the distance from Alexandria to Syene, as 1/50th of the meridian circumference, gives 250,000 stadia for the circumference, or 694.4 stadia per degree. His statement of 5000 stadia for the NS distance from Alexandria to Syene is correct. His statement of 5300 stadia for the length of Egypt from Syene to the sea is also correct, and it is also the same as the ancient Egyptian statement of 106 *itr* for the length of Egypt from Elephantine to the sea. According to Heron of Alexandria, Theon of Smyrna and Strabo, Eratosthenes and Hipparchus gave 252,000 stadia for the circumference, or 700 stadia per degree. As an artifact of the $\sqrt{2}$ relation between the royal cubit and the remen, 14.14 *itr* is equal to one degree of latitude, and each *itr* contains 50 stadia, giving 707 stadia per degree ($14.14 \times 50 = 707$). The difference between 700 and 707 stades per degree is the cause of the one percent error in the calculations of Eratosthenes.

Herodotus, Eratosthenes and Strabo may not have been aware of the geographic basis and relations between the digit, the remen, the royal cubit, the Greek foot and the Roman foot. However, the archaeological and textual evidence from throughout ancient Egyptian history, and the textual evidence from these Greek and Roman authors, support a conclusion that the correspondence between the length of the remen and the royal cubit, and the meridian length of Egypt and the earth, was known to their creators.

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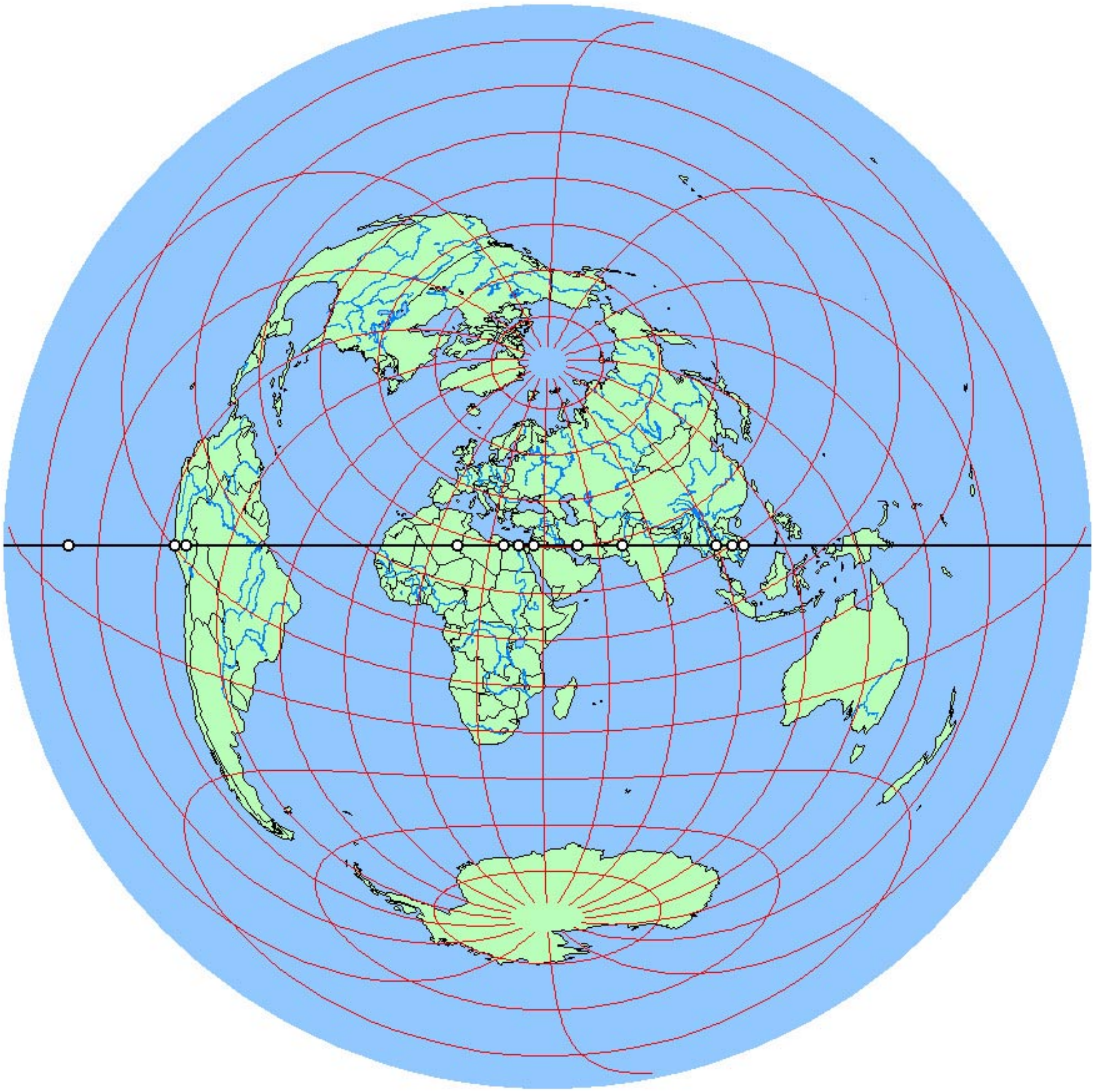
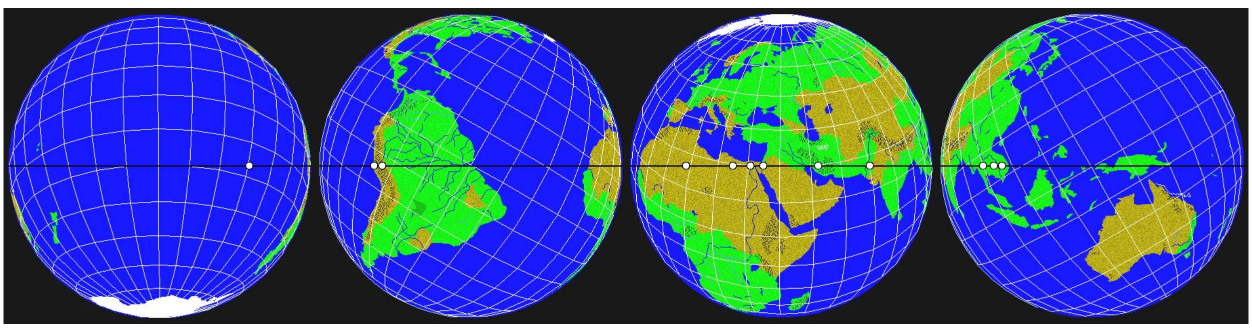


Azimuthal images from Great Circle Maps by Roger Hedin - qsl.net/sm3gsj/index.htm

Centered on: 30°22'N 41°24'E

Revisiting The Prehistoric Alignment of World Wonders

Jim Alison - June 2020
home.hiwaay.net/~jalison

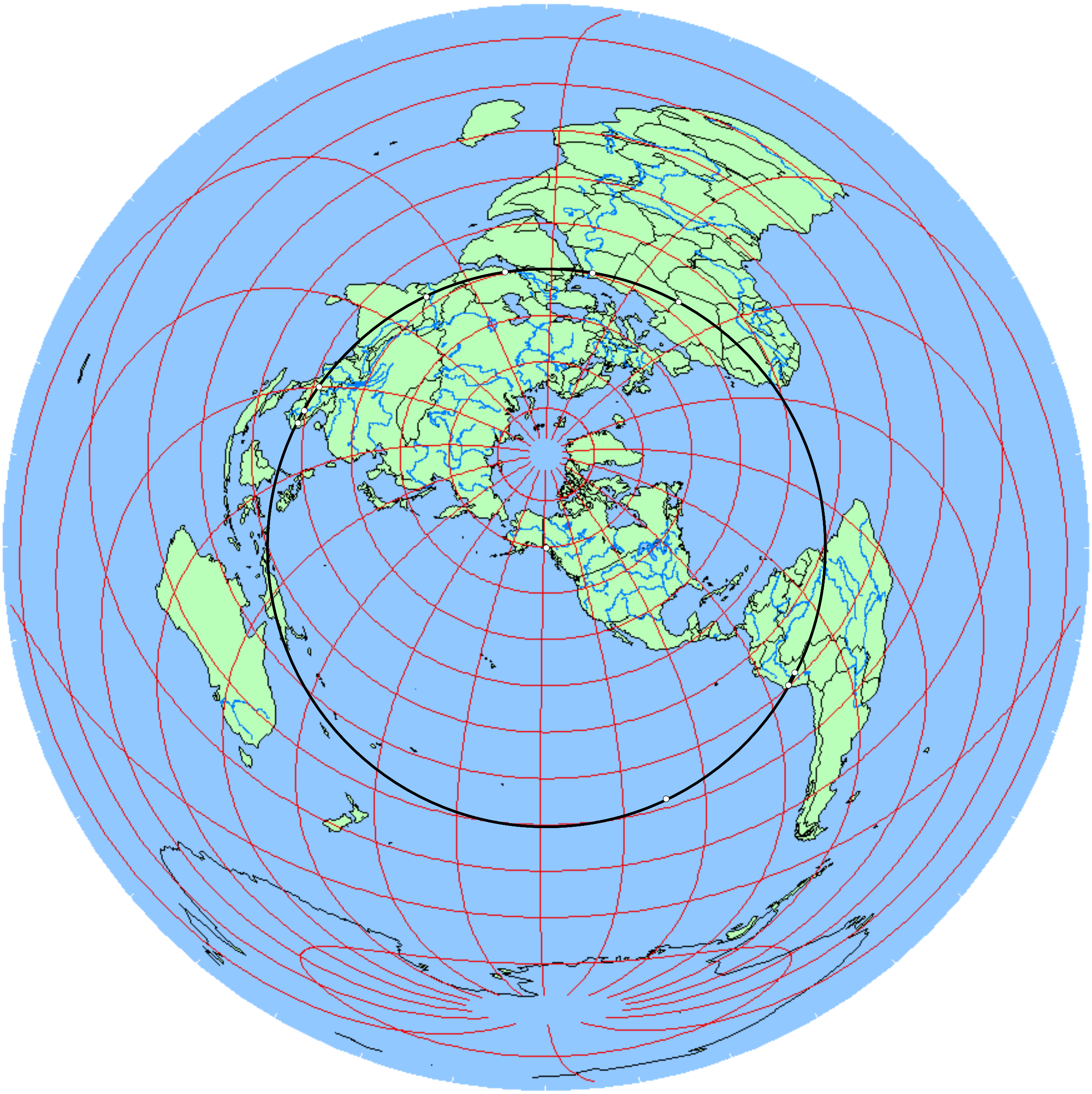


Map images from On Top of the World by Adrian Hylinka

Centered on:

30°22'S 138°36'W 0.00°N 48°36'W 30°22'N 41°24'E 0.00°N 131°24'E

Easter Island, Nazca, Ollantaytambo, Tassili n' Ajjer, Siwa, Giza, Petra, Persepolis, Jaisalmer, Sukothai, Prasat Ban Ben and Yang Prong are aligned on a single great circle around the earth.



Centered on: 59°53'N 138°36'W

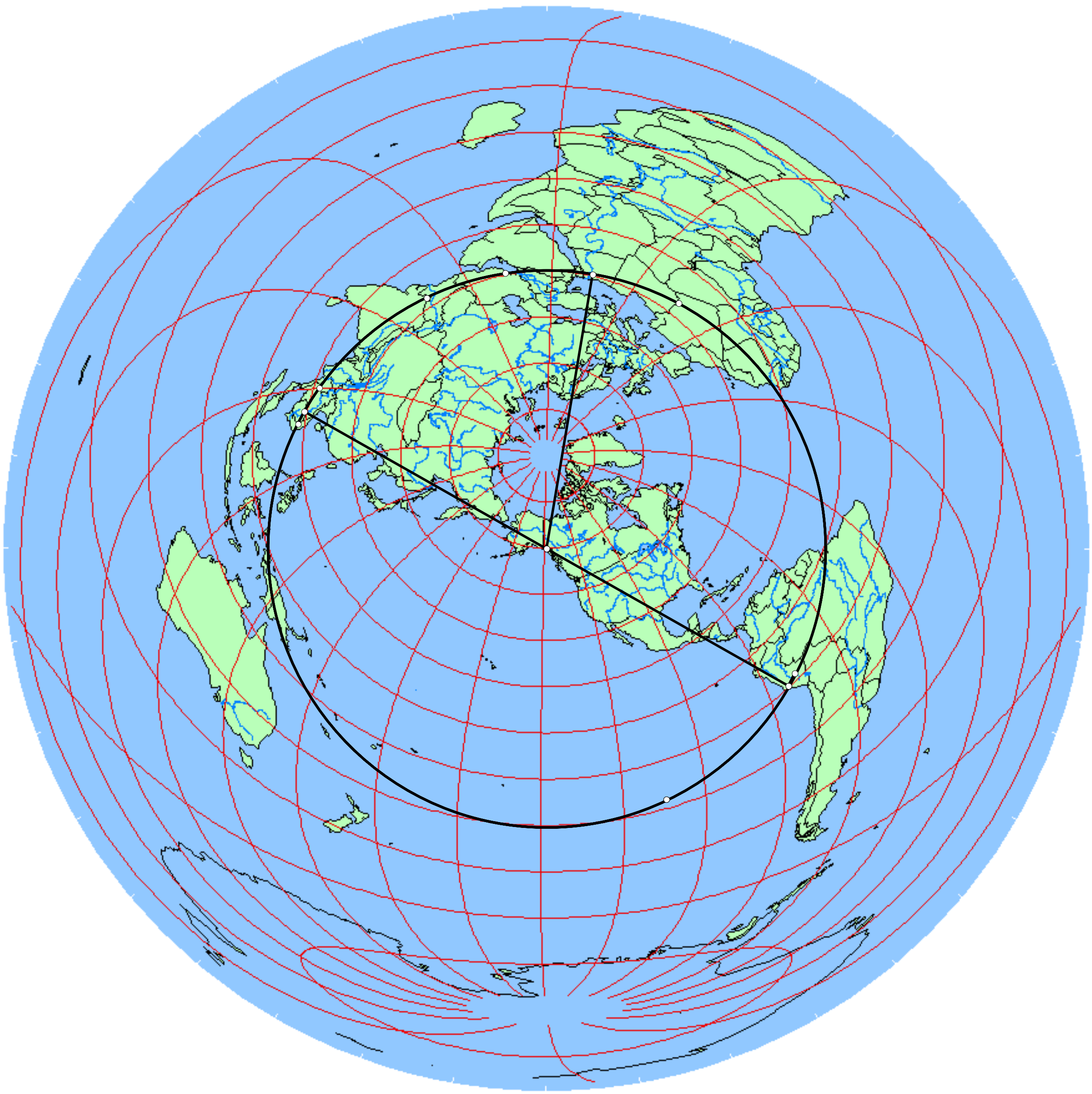
Easter Island, Nazca, Ollantaytambo, Tassili n'Ajjer, Giza and Yang Prong are all in exact alignment on a single great circle. Ancient sites located within one tenth of one degree of this alignment are Petra, Persepolis, Jaisalmer, Sukothai, Prasat Ban Prasat and Prasat Ban Ben. Ancient sites located within one quarter of a degree of this alignment are Cuzco, Machu Picchu, Siwa, Mohenjo Daro, Khajuraho and Preah Vihear.

The circle crosses over the source and the mouth of the Amazon River, the Nile, the mouth of the Tigris-Euphrates, the Indus River, the ancient Sarasvati River, the Bay of Bengal near the mouth of the Ganges, the Irrawaddy River and the Mekong River. The circle also crosses over areas that remain unexplored, including parts of the Sahara Desert, the Brazilian Rainforest, the highlands of New Guinea, underwater areas of the Atlantic and Pacific Oceans and the South China Sea.

The great circle crosses over the equator at 48°36'W and 131°24'E. The maximum latitude of the circle is 30°22'N at 41°24'E and 30°22'S at 138°36'W. The two axis points for the great circle are located at 59°53'N and 138°36'W; and 59°53'S and 41°24'E. The axis points are 90° of longitude east and west of the points where the great circle crosses the equator. Because degrees of latitude are slightly shorter near the equator and slightly longer near the poles, the latitude of the axis points is offset by 15' of latitude in relation to the maximum latitudes of the great circle. The north-south distance from the axis points to the maximum latitude of the great circle at 30°22' S and 138°36' W; and to the maximum latitude of the great circle at 30°22' N and 41°22' E, is 6,215 miles.

Because the distance from the axis point to the great circle is one quarter of the circumference of the earth, the great circle is halfway between the center and the outer edge of the equal azimuthal projection. Counterclockwise from Easter Island, the marked sites are Nazca, Ollantaytambo, Tassili n'Ajjer, Giza, Persepolis, Jaisalmer, Sukothai, Prasat Ban Ben and Yang Prong. The circumference of the earth along this great circle is 24,888 miles long.

	Latitude:	**Longitude:**	**To Alignment:**	**Azimuth from N. Axis point:**
Easter Island	27° 06' S	109° 20' W	0 miles	154.21°
Nazca	14° 42' S	75° 06' W	0 miles	119.90°
Ollantaytambo	13° 15' S	72° 16' W	0 miles	116.80°
Tassili n'Ajjer	26° 32' N	9° 50' E	0 miles	27.89°
Siwa	29° 14' N	25° 31' E	8 miles	13.80°
Giza	29° 59' N	31° 08' E	0 miles	8.85°
Petra	30° 19' N	35° 28' E	5 miles	5.11°
Persepolis	29° 56' N	52° 53' E	4 miles	350.08°
Jaisalmer	26° 55' N	70° 54' E	8 miles	333.99°
Sukhothai	17° 01' N	99° 42' E	3 miles	305.64°
Prasat Ban Prasat	15° 06' N	104° 01' E	1 mile	301.08°
Prasat Ban Ben	14° 47' N	104° 56' E	5 miles	300.14°
Yang Prong	13° 12' N	107° 50' E	0 miles	296.92°



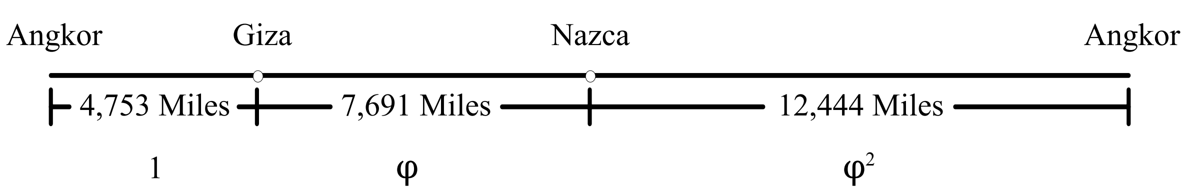
12,444/1.618 = 7691
7691/1.618 = 4753 and
7691 + 4753 = 12,444

NAZCA and ANGKOR

The great circle alignment crosses directly over the great pyramid at Giza, and directly over the main grouping of lines and figures at Nazca. The equal azimuthal projection is centered on the axis point. As a result, the great circle segments from the axis point to Giza, Nazca and Angkor are straight lines. The distance from Giza to Nazca along the great circle is 7691 miles, or 30.9% of the circumference (7691/24,888 = .309), or 111.245° along the great circle (.309 × 360° = 111.245°). Due to the obliquity of the great circle and the ellipsoidal shape of the earth, the difference in azimuths from the axis point vary slightly from the number of degrees along the great circle. The azimuth from the axis point to Giza is 8.85° and the azimuth from the axis point to Nazca is 119.9°, for a difference of 111.05°. The distance from the axis point to Giza and to Nazca is one quarter of the circumference of the earth, or 90°. The great circle segments from the axis point are perpendicular to the great circle itself, with angles at Giza and Nazca of 90°.

Spherical trigonometry converts the 90° - 90° - 111.245° spherical triangle to an equivalent flat surface triangle. The angle opposite the base length is divided by two (111.245°/2 = 55.6225°), and then divided by the angle adjacent to the base length (55.6225°/90° = .618 or 1/φ). Applying the inverted cosine function to .618 gives 51.83° for the flat surface equivalents of the 90° spherical angles at Nazca and Giza. In 1997, Jim Bowles observed that these flat surface angles between Nazca, Giza and the axis point are the same as the angular dimensions of the great pyramid. The two great circle segments from the axis point to Nazca and from the axis point to Giza are each 25% of the circumference of the earth while the great circle segment from Nazca to Giza is 30.9% of the circumference of the earth. The two great circle segments from the axis point to Nazca and to Giza add up to 50% of the circumference of the earth and 50/30.9 = 1.618, just as the pyramid slant height, of 356 + 356 = 712 and 712 cubits /440 cubits = 1.618, or φ.

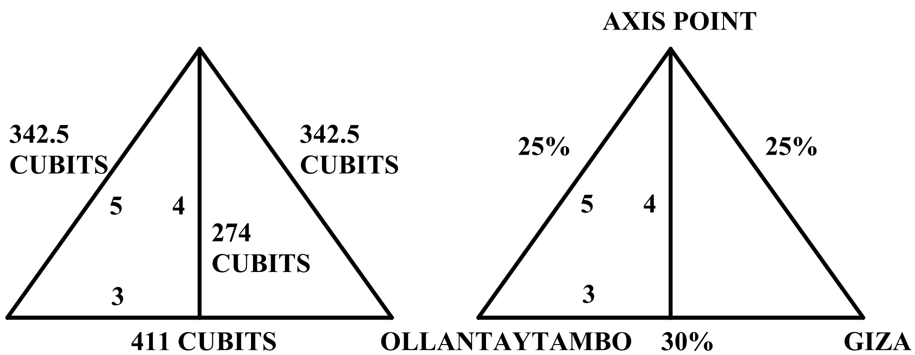
Prasat Ban Ben is an Angkor temple approximately 20 miles NNE of Prasat Preah Vihear and approximately 80 miles NNE of Angkor Wat. The alignment of these Angkor sites is approximately perpendicular to the great circle, and as a result, all three sites are very nearly the same distance from Giza, and from Nazca. The distance from Nazca to Prasat Ban Ben is 12,444 miles, or 50% of the circumference of the great circle. The distance from Giza to Prasat Ban Ben is 4753 miles, or 19.1% of the circumference of the great circle. The distance from the axis point to Giza and to Prasat Ban Ben is 25% of the circumference for each, or 50% combined. 50%/19.1% = 2.618. The ratio between both sides of this terrestrial triangle and the base length of the triangle is 2.618 to one or φ^2. Because Nazca and Prasat Ban Ben are antipodal sites, the two golden section relationships between these three sites are also shown along the circumference of the great circle:



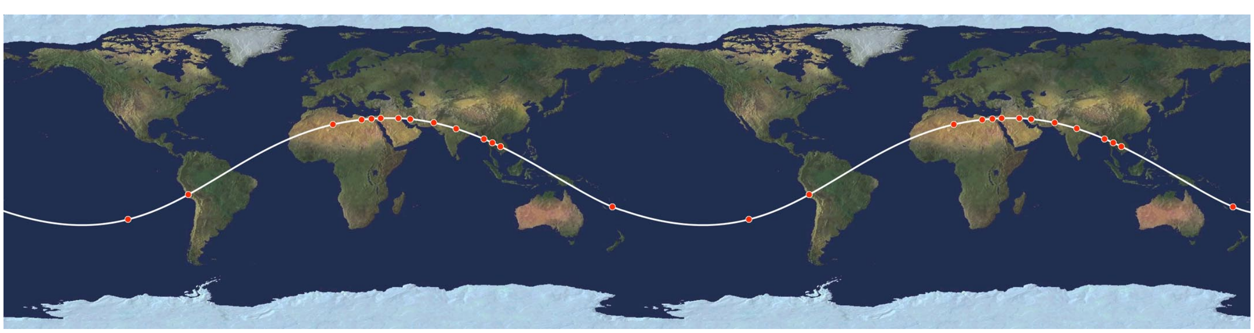
OLLANTAYTAMBO and YANG PRONG

Ollantaytambo and Yang Prong are antipodal sites that are both exactly aligned with the great circle. Ollantaytambo is approximately 20 miles southeast of Machu Picchu and approximately 30 miles northwest of Cuzco. Ollantaytambo is 7466 miles from Giza. This is 30% of the circumference of the great circle (24,888 × .3 = 7466), or 108° (360° × 30% = 108°) from Giza. The angles at Ollantaytambo and at Giza are 90° and the distance from the axis point to Ollantaytambo and to Giza is 90° or 25% of the circumference of the earth.

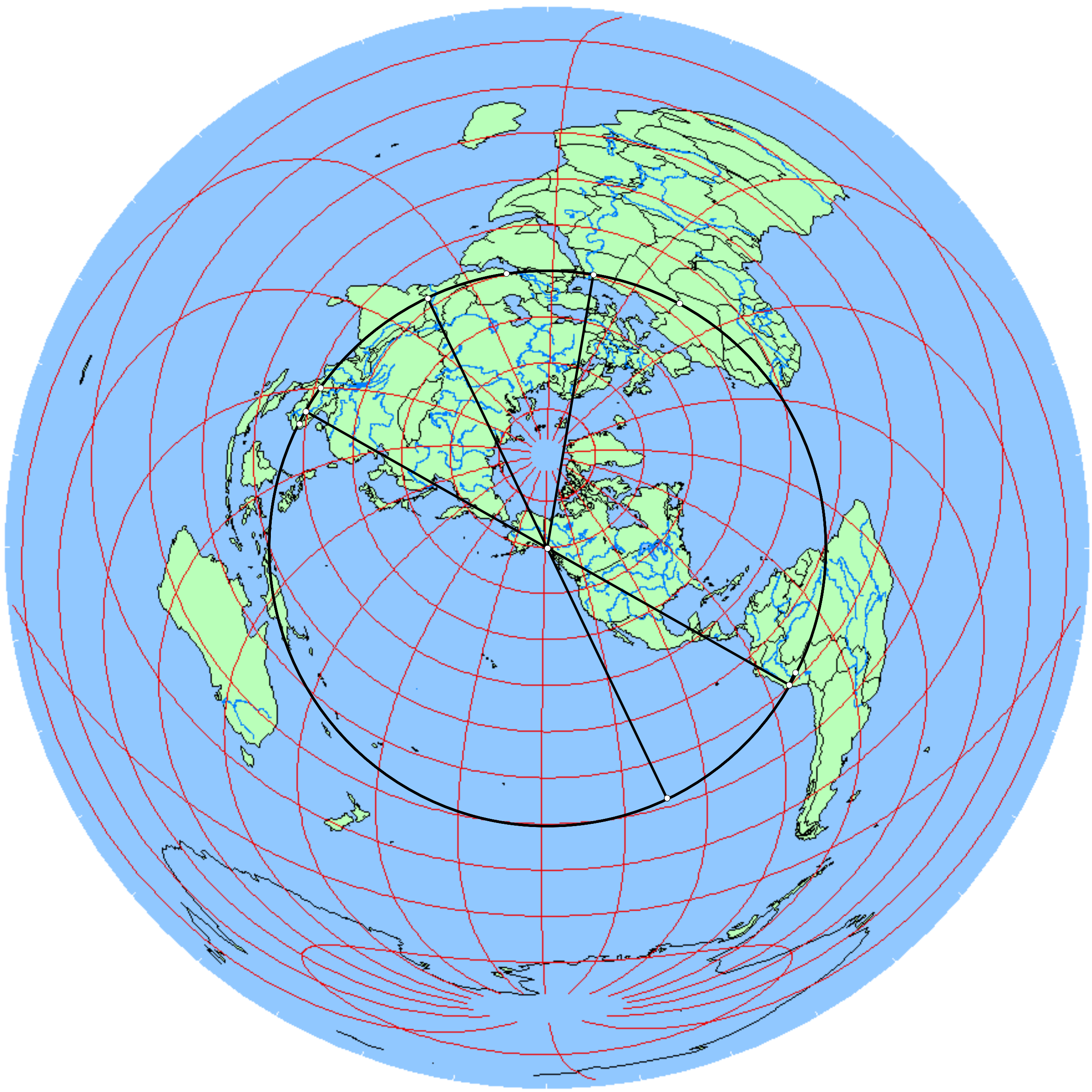
The height of the second pyramid at Giza is 274 cubits and the base length of the sides at ground level is 411 cubits. The ratio between the height and the base length is 2/3. The slant height of the pyramid is 342.5 cubits. The triangle formed by the half base, the height and the slant height is a 3-4-5 right triangle. The ratio between the base length and the slant height is 6/5. These are the same dimensions as the terrestrial triangle from the axis point to Ollantaytambo and Giza with a base length of 30% and side lengths of 25%.



Yang Prong is the only ancient tower in the highlands of Vietnam. Yang Prong is attributed to the Cham kingdom, near the ancient eastern boundary of the Khmer (Angkor) Empire. The Khmer temple at Sukothai is located near the ancient western border of the Khmer Empire, within five miles of the great circle. The Khmer temple at Prasat Ban Prasat is located within one mile of the Great circle. Yang Prong is precisely 50% of the circumference of the great circle from Ollantaytambo, and precisely 20% of the circumference from Giza.



Map © Cosmi 3-D World Atlas



EASTER ISLAND and THE INDUS VALLEY

The great circle crosses directly over Easter Island and within eight miles of Jaisalmer. The antipode for Jaisalmer is five miles east of Easter Island. Recent excavations at Rakhigarhi indicate that it was the largest site of the Harappan (Indus Valley) civilization. The ancient hill forts at Bikaner and Jaisalmer are approximately half way between Rakhigarhi and Mohenjo Daro. According to K. M. Pannikkar: "The centre of early civilization was the desert area in Bikaner and Jaisalmer, through which the ancient river Saraswati flowed into the gulf of Kutch." (Scarre, 2013)

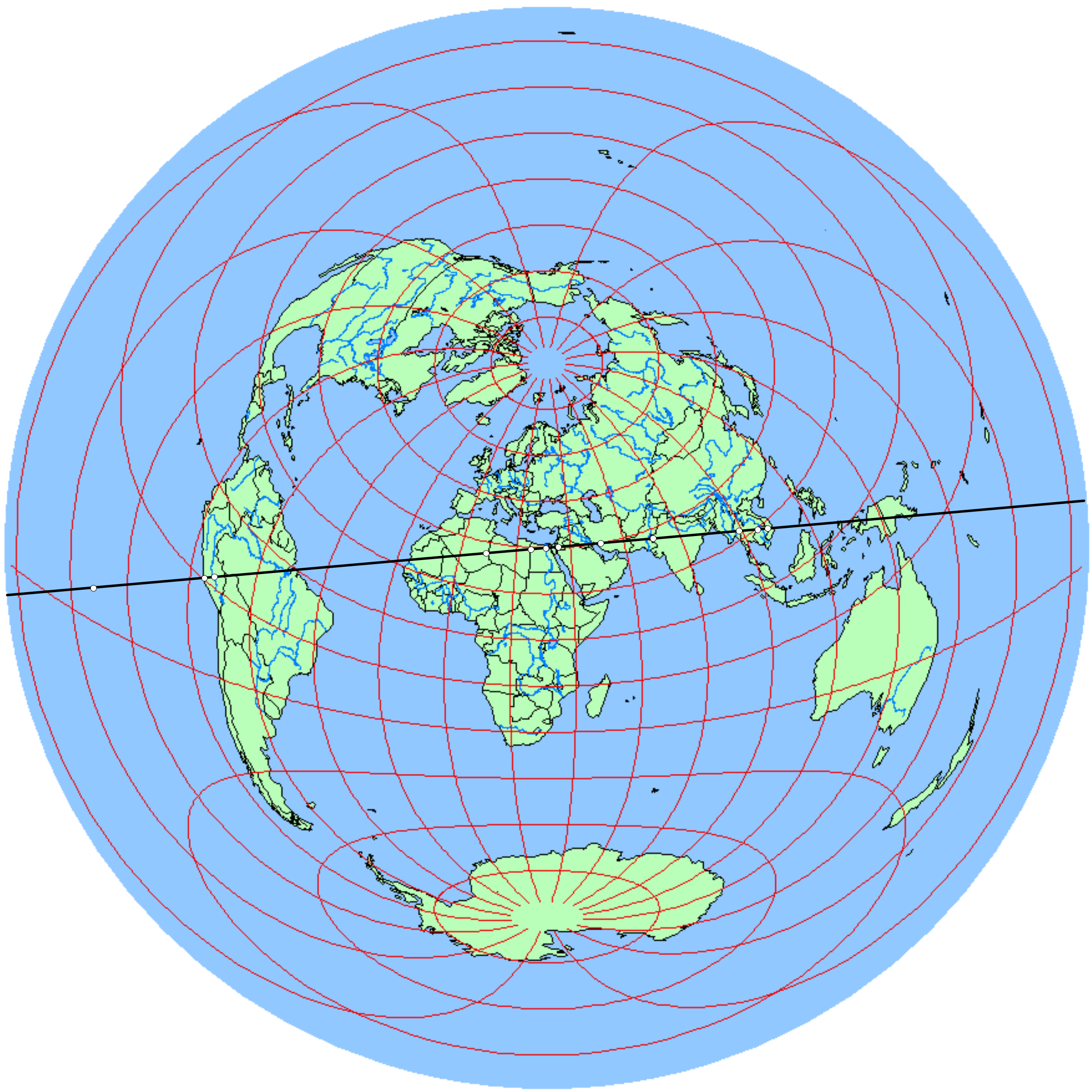
The distance from Giza to Angkor is 19.1% of the circumference. The mid point between Giza and Angkor, 9.55% each way, is approximately 30 miles west of Jaisalmer, and the antipode for the mid-point between Giza and Angkor is less than 10 miles west of Easter Island. The distance from Angkor to Jaisalmer, from Jaisalmer to Giza, and from Easter Island to Nazca, are all very nearly 9.55% of the circumference. The distance from the axis point to the great circle, of one quarter of the circumference, or 25%, divided by 9.55%, equals 2.618, or φ^2.

The distance from Giza to Nazca of 30.9% of the circumference, plus 9.55%, equals 40.45% of the circumference from Giza to Easter Island. 50% of the circumference from Angkor to Nazca, minus 9.55%, equals 40.45% of the circumference from Angkor to Easter Island. 40.45% divided by 25% equals 1.618, or φ. The distance of 40.45% of the circumference, divided by φ, is equal to the distance from the great circle to the axis point. Given 40,000 km for the circumference of the earth, 40,000 × .4045 = 16,180 km, divided by the distance of 10,000 km from the axis point to the great circle, equals φ.

40.45% × 360° = 145.6°. The azimuth from the axis point to Easter Island, of 154.2°, minus the azimuth from the axis point to Giza, of 8.85°, equals 145.4°. The azimuth from the axis point to Angkor, of 300.1°, minus the azimuth from the axis point to Easter Island, of 154.2°, equals 145.9°. The great circle segments from the axis point are perpendicular to the great circle, with 90° angles at Giza, Easter Island and Angkor. The 90° - 90° - 145.6° spherical triangles convert to flat surface triangles with angles of 36° - 36° - 108°, the same as the large triangles in a pentagram, with the same φ proportions.

PERSEPOLIS

The funerary and ceremonial city of Persepolis was built shortly before 500 BC, shortly after the Persian occupation of Egypt. The latitude of Persepolis is 30°N, the same as Giza, and the distance from Persepolis to Giza is almost exactly 4,000,000 royal cubits. Given that 40,000,000 meters is equal to the circumference of the earth, 4,000,000 meters is equal to 10% of the circumference, and given the .5236 ratio between the royal cubit and the meter, 4,000,000 royal cubits is equal to 5.236% of the circumference of the earth. 5.236 is equal to $2\varphi^2$. The distance from Giza to Persepolis in relation to the circumference of the great circle is $2\varphi^2/100$ or $\varphi^2/50$.



Centered on: 29°59'N 31°08'E

GIZA

The great circle is a straight line on the equal azimuthal image centered on Giza, and the azimuth of the great circle as it passes over Giza is 5° north of due east, and 5° south of due west.

The Wall of the Crow (photographs above © Jon Bodsworth) is believed to have been the original entrance to the Giza Plateau. The surviving section of the wall is over 600 feet long and over 30 feet high. The wall is visible in the upper left (southeast) corner of the 2002 © QuickBird satellite image below. The wall is aligned with the azimuth of the great circle as it passes over Giza, five degrees north of due east and five degrees south of due west.

Fractional relations between the Digit, Roman foot, Northern foot, Remen, Roman Cubit, Babylonian cubit, Egyptian Royal cubit, Ezekiel's cubit, Megalithic yard and Meter

	Digit	**R. foot**	**N. foot**	**Remen**	**R. cubit**	**B. cubit**	**E. R. C.**	**E. cubit**	**M. yard**	**Meter**
Digit	**1**	**1/16**	**1/18**	**1/20**	**1/24**	**1/27**	**7/198**	**1/30**	**1/45**	**1/54**
R. foot	**16**	**1**	**8/9**	**4/5**	**2/3**	**16/27**	**56/99**	**8/15**	**16/45**	**8/27**
N. foot	**18**	**9/8**	**1**	**9/10**	**3/4**	**2/3**	**7/11**	**3/5**	**2/5**	**1/3**
Remen	**20**	**5/4**	**10/9**	**1**	**5/6**	**20/27**	**70/99**	**2/3**	**4/9**	**10/27**
R. cubit	**24**	**3/2**	**4/3**	**6/5**	**1**	**8/9**	**28/33**	**4/5**	**8/15**	**4/9**
B. cubit	**27**	**27/16**	**3/2**	**27/20**	**9/8**	**1**	**21/22**	**9/10**	**3/5**	**1/2**
E. R. C.	**198/7**	**99/56**	**11/7**	**99/70**	**33/28**	**22/21**	**1**	**33/35**	**22/35**	**11/21**
E. cubit	**30**	**15/8**	**5/3**	**3/2**	**5/4**	**10/9**	**35/33**	**1**	**2/3**	**5/9**
M. yard	**45**	**45/16**	**5/2**	**9/4**	**15/8**	**5/3**	**35/22**	**3/2**	**1**	**5/6**
Meter	**54**	**27/8**	**3**	**27/10**	**9/4**	**2**	**21/11**	**9/5**	**6/5**	**1**

Fractional relations between the Shusi, Digit, Roman inch, Northern inch and Indus Valley inch

	Shusi	**Digit**	**Roman inch**	**Northern inch**	**I. V. inch**
Shusi	**1**	**9/10**	**27/40**	**3/5**	**1/2**
Digit	**10/9**	**1**	**3/4**	**2/3**	**5/9**
Roman inch	**40/27**	**4/3**	**1**	**8/9**	**20/27**
Northern inch	**5/3**	**3/2**	**9/8**	**1**	**5/6**
I. V. inch	**2**	**9/5**	**27/20**	**6/5**	**1**

Measurement Systems Related to the Digit

Jim Alison - April 2021

In *On the Heavens* 2:14, Aristotle stated: "Those mathematicians who try to calculate the size of the earth's circumference arrive at the figure of 400,000 stades." The length of the modern meter is based on 1/10,000,000th of the length of the quarter circumference, from the equator to the pole, or 1/40,000,000th of the length of the polar circumference. As a result, a stadia of 100 meters is contained 400,000 times in the polar circumference of the earth.

In *Historical Metrology* (1951), A.E. Berriman observed that the lengths of the Roman digit and the ancient Egyptian digit are the same, and that 54 of these digits equals one meter. 100 meters equals 5400 digits. The length of the polar circumference, in ancient Egyptian or Roman digits, is equal to 400,000 times 5400, or 2,160,000,000 digits. One quarter of the polar circumference is 540,000,000 digits. One degree of latitude is 6,000,000 digits. One nautical league is equal to three nautical miles, or three minutes of latitude, or 300,000 digits. One minute of latitude is equal to one nautical mile, or 100,000 digits. One-tenth of one minute of latitude is 10,000 digits.

Inscribed on ancient Egyptian cubit rods, the hieroglyphic sign for one digit is one finger. The hieroglyphic sign for the number10,000 is also one finger. In *The Sand Reckoner*, Archimedes equated 10,000 fingers with the length of one stadia. The polar circumference contains 216,000 of these stadia. The quarter circumference is 54,000 stadia. One degree of latitude is 600 stadia. One minute of latitude is 10 stadia. One-tenth of one minute of latitude is one of these stadia.

The ancient Egyptian remen contains 20 digits. The remen is contained 108,000,000 times in the polar circumference. One degree of latitude is 300,000 remen. One minute of latitude is 5000 remen. One-tenth of one minute of latitude is 500 remen, or 10,000 digits. Three minutes of latitude contains 30 units of 500 remen, or 15,000 remen, or 300,000 digits. The royal Egyptian cubit is the diagonal of a square remen, or 28.28 digits, or 20 digits times the square root of two. The diagonal of a square royal cubit is a double remen, or 40 digits. The short Egyptian cubit is 24 digits.

The Roman cubit is also 24 digits. The Roman cubit, or the short Egyptian cubit, is contained 90,000,000 times in the polar circumference. In addition to the digit, the Roman system also includes Roman inches that are 4/3 the length of the digit. The Roman cubit contains 24 digits or 18 Roman inches. The Roman foot is 2/3 the length of the cubit, or 16 digits, or 12 Roman inches. One degree of latitude is 250,000 Roman cubits, or 375,000 Roman feet, or 75 Roman miles of 5000 Roman feet. The Roman stadia of 625 feet is 10,000 digits, or one-tenth of one minute of latitude. One nautical league, or three minutes of latitude, contains 30 of these stadia of 625 Roman feet, or 18,750 Roman feet, or 300,000 digits, or one Roman schoenus. The Roman stadia, or furlong, of 625 feet is contained eight times in the Roman mile of 5000 feet. The Latin for the Roman mile is *mille passus*, or 1000 paces of five Roman feet. Each pace, or double step, contains 80 digits, and each step, of two and a half Roman feet, contains 40 digits.

The Greek *orguia*, or fathom, contains 100 digits. The Greek fathom is contained 21,600,000 times in the polar circumference. The Greek fathom is divided into six Greek feet, or four Greek cubits. The Greek cubit, of 25 digits, is contained 86,400,000 times in the polar circumference, or 21,600,000 times in the quarter circumference, or 240,000 times in one degree of latitude. The Greek stadia of 100 fathoms, or 600 feet, or 400 cubits, is 10,000 digits, or one-tenth of one minute of latitude. The Greek schoinos contains 30 of these Greek stades, or 12,000 Greek cubits, or 300,000 digits, and is equal to one nautical league, or three minutes of latitude.

Ezekiel's cubit, also known as the Talmudic cubit, or the Hebrew cubit, or the Jewish cubit, contains 30 digits. The original Greek version of EZEKIEL 40:5 gives this length as a cubit and a hand breadth. The Tosefta is a supplement to the Mishnah, which is a compilation of the laws of Judaism. TOSEFTA 6:12-13 gives the common cubit as five hand breadths and the cubit of the altar as six hand breadths. Ancient Egyptian cubit rods and texts indicate that a palm is four digits, and a hand, or hand breadth, is five digits. The cubit of five hand breadths referred to in the Book of Ezekiel and the Tosefta is a cubit of 25 digits, the same as the Greek cubit, and the additional hand breadth gives Ezekiel's cubit of 30 digits. In *Jewish Measures and Weights* (1686), Richard Cumberland stated: "The Jewish cubit is to the Roman cubit as five is to four." The Roman cubit of 24 digits, times 5/4, equals the Jewish cubit of 30 digits. The Jewish cubit is contained 72,000,000 times in the polar circumference, or 200,000 times in one degree of latitude.

According to Herodotus 1.78, the Babylonian cubit contains one cubit plus three finger breadths. The common Egyptian cubit or the Roman cubit of 24 digits, plus three additional digits, equals 27 digits. In *The Structure of Linear Units*, Livio Stecchini stated: "The cubit considered standard in Mesopotamia is the barley cubit, composed of 27 basic fingers." In *Ancient Mesopotamian's system of measurement* (2020), L A Kasprik and A C Barros translate cuneiform inscriptions that give 20 shusi for the foot and 30 shusi for the cubit. Kasprik and Barros also reference ancient Mesopotamian remains, including the statue of Gudea and the tower of Babylon, indicating that the length of the Babylonian cubit was 50 cm, and the length of the Babylonian foot was 33.3 cm. The shusi and the digit converge in the Babylonian foot of 18 digits = 20 shusi, and the cubit of 27 digits = 30 shusi, giving 9 digits = 10 shusi. In *The Excavations at Babylon* (1914), Robert Koldewey states that the length of the sides of the tower of Babylon measured 90 meters. In his text, Koldewey included the translation of the Smith cuneiform tablet that gives 180 cubits for the base lengths of the tower of Babylon. Ninety meters/180 = 50 cm, or 30 shusi, or 27 digits.

In *Babylonian, Assyrian and Persian Measures* (1944), Angelo Segre cited Babylonian texts translated by Francois Thureau-Dangin and C. F. Lehmann-Haupt, stating "the measure of volume of one qa is the cube of 6 shusi, or 216 cubic shusi," and "the capacity of the qa corresponded to the double mina." Segre states: "The length of the Babylonian cubit was certainly very near to 500 mm." Six shusi is 1/5 of the 30 shusi length of the Babylonian cubit or 1/5 of 50 cm or 10 cm. The volume of 6^3 shusi = 216 cubic shusi = 10^3 cm = 1000 cubic cm = one liter. The volume of the cubic cubit is the volume of the qa times 5^3 = 125 qa = 125 liters. Kasprik and Barros give 500 grams for the mina. The water weight of the qa is equal to the double mina of 1000 grams. The capacity of the cubic cubit corresponded to 250 mina. The correspondences of one qa = one liter; two mina = one kilogram; and two cubits = one meter, are due to the correspondence of 80,000,000 Babylonian cubits = 40,000,000 meters = the length of the polar circumference.

The Indus Valley inch is twice the length of the shusi, and the Indus Valley foot contains 10 Indus Valley inches, or 20 shusi, or 18 digits. In *The Indus Civilzation* (1968), Mortimer Wheeler gives the unit of measurement for the great bath at Mohenjo-daro of 13.1 English inches, and states "from the results of over 150 checks applied to the buildings of Harappa and Mohenjo-daro, the Harappan foot seems to vary between 13.0 and 13.2 English inches." One meter, or 39.37 English inches, divided by three, equals 13.12333... English inches, or 1/3 of a meter, or 1/3 of 54 digits, or 18 digits, or 20 shusi, or 10 Indus Valley inches.

In *Mohenjo Daro and the Indus Civilization* (1931), Sir John Marshall stated: "The great bath at Mohenjo-daro was cleared in 1925-1926, and in size and conception is the most elaborate structure found in any part of Mohenjo-daro. The sides of the bath are: west side, 39 ft 4 inches long; east side, 39 ft 3 inches; the south side is 22 ft 11 inches, and at the north 23 ft 4 ½ inches." The east and west sides are 471 inches and 472 inches. 472.44 inches equals 12 meters, or 36 feet of 10 Indus Valley inches, or 18 digits. The north and south sides are 280.5 inches and 275 inches. 275.59 inches equals 7 meters, or 21 feet of 10 Indus Valley inches, or 18 digits.

In *The Minoan Unit of Length and Minoan Palace Planning* (1960), J. Walter Graham stated that at Phaistos, a length of 3.34 m occurs four times in recessed panels and the adjoining lengths of the walls, and that at Mallia, a length of 3.31 m occurs twice on the same facade element. Graham proposed a Minoan foot of .303... m, giving 11 of these units for the measurements above, instead of 10 units for these measurements, in the range of .331 m - .334 m. Graham reported measures of 22, 33 and 66 of his Minoan feet, but these same measures give 20, 30 and 60 feet of .333 m. Graham gave a measure of 24.00 m as 80 of his Minoan feet, with a margin of error of 29 cm, although 24.00 m is exactly 72 feet of .333 m, or 18 digits, or 20 shusi, or 10 Indus Valley inches.

In *The Greek Stadion* (1890), Wilhelm Dorpfeld stated the Philaetarian, i.e. Pergamene, stadion was 600 feet of .333m, or 200 m. Philetaerus founded the Attalid dynasty of Pergamon in Anatolia. In *Carnac, the Alignments* (2012), Howard Crowhurst stated that the Northern foot was also .333m. In 12 B.C., Roman general Nero Claudius Drusus defined the Northern foot in the Germanic provinces as two digits longer than the Roman foot, or 18 digits, or .333m. 300 Northern feet, or 300 Babylonian feet, or 300 Indus Valley feet, are contained 400,000 times in the polar circumference. The Northern mile of 8 furlongs of 600 feet, or 4800 feet, is contained 25,000 times in the polar circumference. The Northern inch, contained 12 times in the northern foot, is equal to 5/3 of one shusi, or 3/2 of one digit, or 9/8 of one Roman inch, or 5/6 of one Indus Valley inch.

In *The Statute for Measuring Land* (1305), King Edward I defined the English foot and English yard in terms of 10/11 of the Northern foot and Northern yard as follows:

> "It is ordained that three grains of barley, dry and round, make an inch, twelve inches make a foot, three feet make an Ulna, five and a half Ulna make a rod, and forty rods in length and four in breadth make an acre. And it is to be remembered that the Ulna of our Lord the King, contains 3 feet and no more, and the foot must contain 12 inches measured by the correct measure of this kind of Ulna; that is to say the thirty-sixth part of the Ulna makes 1 inch neither more nor less; and five and a half Ulna make 1 rod, sixteen feet and a half, by the said Iron Ulna of our Lord the King."

The length of 16.5 English feet or 5.5 English yards for the rod, was equated with the length of 15 Northern feet, or 5 Northern yards, or 180 Northern inches. Fifteen Northern feet, or 180 Northern inches, times 11/10 equals 198 English inches, or 16.5 English feet.

In *The English Yard and Pound Weight* (1952), F. G. Skinner stated:

> "The current Imperial Yard and Pound Avoirdupois legalised in 1855, are directly descended from standards established by the English Statutes of the Realm some 600 years ago. These are the two basic standards from which all other British Weights and Measures are now derived. But the origins of these standards go back far beyond even Saxon times, having their roots in the remote ages of the most ancient civilizations of the Middle East...
>
> The Saxon foot was derived from a very ancient and widespread measure known as the Northern Cubit, a non-Semitic standard which can be traced in building work, and as actual Cubit Measures in Mesopotamia and Egypt from about B.C. 2000 and which was always associated with land measure. This cubit and its half or foot passed westwards into Europe with the early migrations from the east of the Teutonic tribes. In B.C. 12, this foot was recognized as the standard for land measurement among the tribes of Lower Germany and its length was recorded by the Roman general Drusus as 2 digits longer than the Roman foot, i.e. a length of 13.11 inches.
>
> After A.D. 410 with the departure of the Romans and the coming of the Saxons, the Northern Cubit and Foot became established in England in the Saxon kingdoms at a value of 26.4" for the Cubit or Ell as a cloth measure and a foot of 13.2" for building and for land measure for their 'Open Field' system of ploughlands, in which the various 'holdings' were rectangular strips side by side, known as 'Roods' (1/4 acres). The sides of the Rood were always in the relation of 1 × 40, i.e. one Land Rod in breadth x 40 Land Rods in length, i.e. the 'Furrow length' or 'Furlong', Four Roods side by side made up the 'Acre', the sides of which were thus 4 × 40 Rods. The Land Rod was 15 Saxon feet of 13.2", i.e. 16 ft. 6 ins. in our current Imperial Measure. The Furrow-length of 40 Land Rods or 600 Saxon feet was thus 660 feet or 220 yards by our current measure and is still the recognized length of the furlong, eight of which make our mile of 5,280 feet or 1,760 yards."

The English mile of eight furlongs of 660 English feet, or 5280 English feet, is equal to the Northern mile of eight furlongs of 600 Northern feet, or 4800 Northern feet. Based on 10/11 of the old Northern foot of 18 digits, rather than the lengthened Northern foot from after the fall of Rome, an old English mile of 5280 old English feet would also be contained 25,000 times in the polar circumference. The length of this English foot is .9944 times the length of the modern English foot: 24,860 (polar circumference in modern English miles) divided by 25,000 = .9944. Given 18 digits for the Northern foot, 54 digits for the meter, and 10/11 of 18 digits for the old English foot, an old English inch would be contained 39.6 times in the meter. One tenth of one mile, of 480 old Northern feet, or 528 old English feet, would be contained 250,000 times in the polar circumference.

The megalithic yard is two and a half Northern feet, or 45 digits. In *Stonehenge* (1956), Richard Atkinson gave 1168 inches for the inner diameter of the sarsen circle and 42 inches for the width of the lintels. 1168 + 42 + 42 = 1252 inches, or 1252/39.37 = 31.8m. The outer diameter of 31.8m × π = 99.9m. The outer circumference of the sarsen circle is 100 meters, or 120 megalithic yards, or 216 Greek cubits, or 225 Roman cubits, or 270 remen, or 300 Northern feet, or 1/400,000 of the earth's polar circumference. The curved outer length of each of the 30 lintels in the sarsen circle is 10 Northern feet, or 4 megalithic yards, or 1/12,000,000 of the earth's polar circumference.

In *Stonehenge* (1880), Petrie gave several other measures that suggest the use of a Northern foot of 18 digits at Stonehenge. Petrie gives 472.7 inches, plus or minus .5 inches, for the diameter of the inner bluestone circle. 472.7/.39.37 = 12.00m or 36.00 Northern feet for the diameter of the inner bluestone circle. Petrie gives 198 inches for the length of the alter stone, or 15.08 Northern feet. Petrie gives the length of the trilithon capstones as 184 inches, or 14.01 Northern feet, and he gives 39.7 inches for the average width of the trilithon capstones, or 3.02 Northern feet.

In *A New Look at the Astronomy and Geometry of Stonehenge* (2012), Euan MacKie states "Stonehenge was examined by Ranieri using the most accurate available plans in the English Heritage report. Analysis of the various measurements showed a unit of length .665m - or two of these making 1.33m - could have been used. These lengths are multiples of the Drusian or 'Northern' foot of .333m which survived in England into Saxon times when it was used in the dimensions of religious buildings such as the 7th century minster at Winchester." MacKie compared the distances at Stonehenge using Thom's megalithic yard of .829m and the Drusian foot of .333m and "In almost every case the Drusian foot is closer to the actual measurements on the ground."

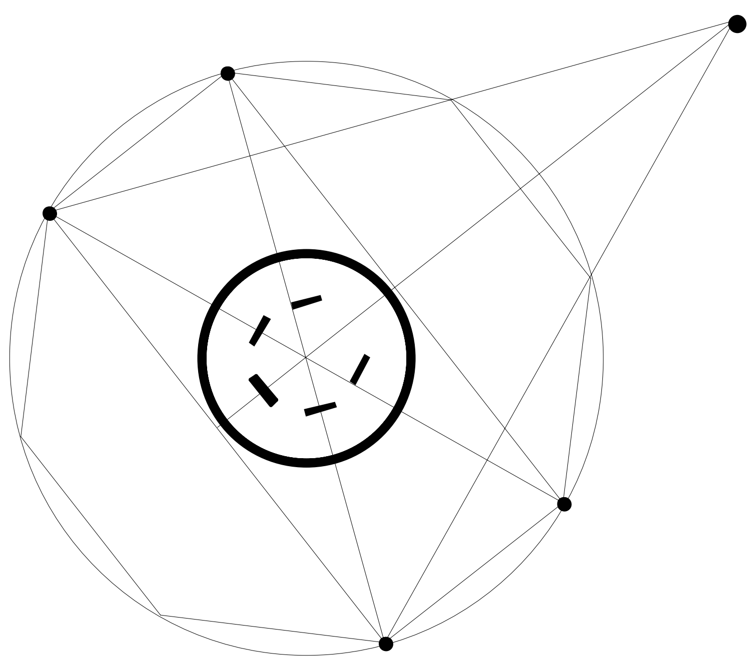
The axis of Stonehenge is marked by an avenue aligned with sunrise on the summer solstice. The distance along the avenue from the center of Stonehenge to the heel stone is five times the distance from the center of Stonehenge to the outer circumference of the sarsen circle. The 100m circumference of the sarsen circle has a radius of 15.9m, and 15.9 × 5 = 79.577m. In *Geometry at Stonehenge* (2003), Marcello Ranieri gave 79.355m from the center of Stonehenge to the heel stone.

The long sides of the station rectangle are perpendicular to the axis of Stonehenge and the intersection of the diagonals of the station rectangle marks the center of Stonehenge. The corners of the station rectangle consist of two diagonally opposed standing stones, and two diagonally opposed tumuli. Holes below the centers of the tumuli are believed to have held standing stones. The Aubrey circle consists of 56 covered holes that form a circle with a radius of 43m from the center of Stonehenge. The station stones and tumuli are on the Aubrey circle. The diagonals of the station rectangle give the diameter of the Aubrey circle. The long sides of the station rectangle are the same length as the distance from the center of Stonehenge to the heel stone, or 79.577m. Ranieri gave a measurement of 79.75m for the long sides of the station rectangle.

In *The Lost Science of Measuring the Earth* (2006), John Michell stated that "The Station rectangle is either the central rectangle of an octagon or it is made up of two Pythagorean triangles with sides of 5-12-13. The difference is too slight to be measured." The ratio between the long side and the short side of the Pythagorean triangles is 12/5, or 2.4. The ratio between the long side and the short side of an octagon rectangle is one plus the square root of 2, or 2.4142...

Given an octagon rectangle, long sides of 79.577m, divided by 2.4142 = 32.962m for the short sides of the rectangle. Atkinson gave 32.70m for the length of one of the short sides of the station rectangle and Ranieri gave 33.23m for the same side of the rectangle. The average of these two measures is 32.96m. Given long sides of the station rectangle equal to five times the radius of the sarsen circle, or 79.577m, and short sides of 32.962m, $79.577^2 + 32.962^2 = 86.133^2$, or 86.133m, for the diagonals of the station rectangle and for the diameter of the Aubrey circle. In *Stonehenge* (1880), Flinders Petrie gave a length of 3391 inches from the centers of the diagonally opposed station tumuli, or 3391/39.37 = 86.132m.

The short sides of the heel stone triangles are one half of the long side of the station rectangle, and the ratio between the short sides and the long side of the heel stone triangles is the same as the ratio between the short sides and the long sides of the station rectangle. One half of the long side of the station rectangle is 79.577/2 = 39.788m and 39.788m × 2.4142 = 96.058m for the long side of the heel stone triangles. The short side of the station rectangle is 32.962m, divided by 2 = 16.481m. The 96.058m length of long side of the heel stone triangles, minus 16.481 = 79.577m for the distance from the center of Stonehenge to the heel stone, exactly the same length as the long sides of the station rectangle. The octagon rectangle proportions for the heel stone triangles result in the hypotenuse of the heel stone triangles exactly intersecting the corners of the octagon that is defined by the station rectangle and circumscribed by the Aubrey circle.

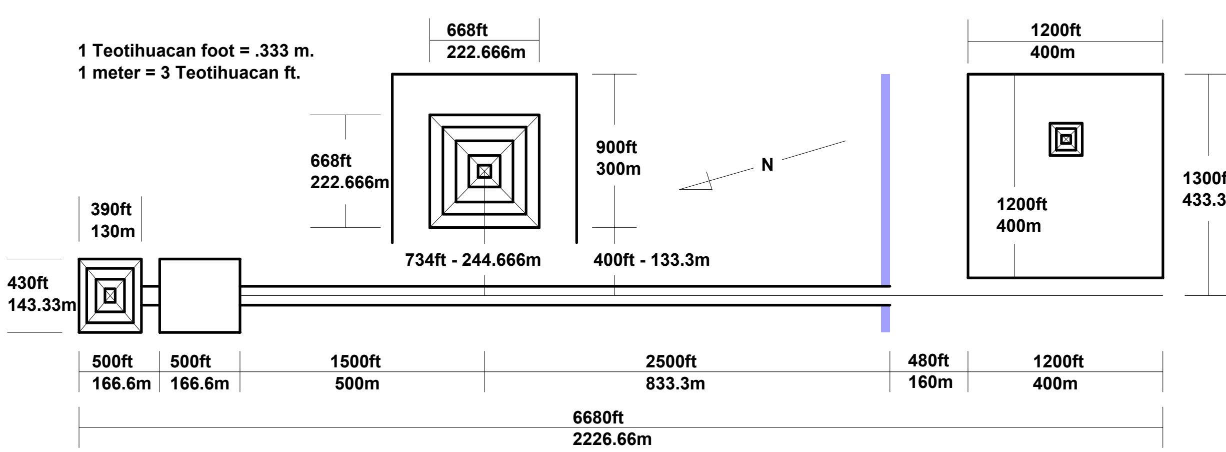


Given a 5-12-13 station rectangle with long sides of 80m, or 240 Northern feet, or 96 megalithic yards, the short sides are 240 × 5/12 = 100 Northern feet, or 40 megalithic yards. The diagonals are $100^2 + 240^2 = 260^2$, or 260 Northern feet, or 104 megalithic yards. The short sides of the heel stone triangles are 120 Northern feet, giving 120 x 12/5 = 288 Northern feet for the long sides, and $120^2 + 288^2 = 312^2$, or 312 Northern feet for the hypotenuse of the heel stone triangles.

In *A Mayan Milestone* (2005), Hugh Harleston gives a measure of 1.0594m for his Standard Teotihuacan Unit that is very close to Michell's measure of 3.476 feet for the width of the lintels on the sarsen circle at Stonehenge, and like Michell, Harleston identifies this as 1/6,000,000 of the polar radius of the earth. He gives the side lengths of the square enclosure of the Citadel as 378 STU and 378 × 1.0594 = 400.4m. *Mexico Archaeology* (2021) gives 1312 feet, or 400m, or 1200 Northern feet, for the side lengths of the Citadel. Harleston gives 231 STU for the distance fom the apex of the Sun pyramid to the NS axis of Teotihuacan and 244.666m/1.0594 = 230.94 STU. The base lengths of the Sun pyramid are 222.66m, or 668 Northern feet, and 244.66m − 111.333m = 133.33m, or 400 Northern feet, from the west side of the Sun pyramid to the NS axis of Teotihuacan.

In *Worldview Materialized in Teotihuacan, Mexico* (1993), Saburo Sugiyama gives a unit of measure of .83m, that he calls the Teotihuacan Measurement unit, that is very close to 2.72 feet, given by Thom for the megalithic yard, and also close to the megalithic yard of 45 digits, or .8333m. Sugiyama gives 833m, or 2500 Northern feet, from the northern edge of the Moon pyramid to the EW axis of the Sun pyramid, and the same length from the EW axis of the Sun pyramid to the canal. In *Archaeology of Measurement* (2010) Sugiyama gives 1194.9m from the EW axis of the Sun pyramid to the EW axis of the Feathered Serpent pyramid, that is also the EW axis of the Citadel. Harleston gives 1125 STU from the EW axis of the Sun pyramid to the EW axis of the Citadel, and 1125 × 1.0594 = 1191.8m. The average of Sugiyama and Harleston's measures is 1193.3m, or 3580 Northern feet. 1193.3m + 833.3m + 200m = 2226.66m, or 6680 Northern feet, from the northern edge of the Moon pyramid to the southern edge of the Citadel. Sugiyama gives 222.7m for the square base length of the Sun pyramid, or 1/10 of 2226.66m. He gives 433.3m, or 1300 Northern feet, from the NS site axis to the eastern edge of the Citadel, and to the eastern edge of the Sun pyramid enclosure. Sugiyama also gives 60m, or 180 Northern feet, as the standard length for the sides of the square compounds that surround the monumental zone at Teotihuacan.

Except for the base lengths of the Feathered Serpent pyramid in the Citadel, of 65m, or 195 Northern feet, and the base lengths of the Sun pyramid, equal to one tenth of the distance from the northern edge of the Moon pyramid to the southern edge of the Citadel, the ground plan of the main features of Teotihuacan is based on 10x and 100x multiples of the Babylonian/Indus Valley/ Northern/Teotihuacan foot, of 18 digits, or 20 shusi, or 12 Northern inches, or .333m.



Sources and Accounts of Eratosthenes

Jim Alison - February 2022

In *Thesaurus* (1884), Heinrich Brugsch published drawings of the inscriptions on the walls of the Ptolemaic temple at Edfu, giving 106 *itr* for the length of Egypt. In *Two Hieroglyphic Papyri from Tanis* (1889), Petrie published drawings of the charred papyrus from Tanis, giving 106 *itr* for the length of Egypt, and giving an additional length of 20 *itr*. Written in Ptolemaic times, this papyrus stated it was a copy of an Old Kingdom inscription. In *Altagyptische Zeitmessung* (1920), Ludwig Borchardt published photographs of inscribed Middle Kingdom cubit rods from Karnak, giving 106 *itr* for the length of Egypt, 86 itr from Elephantine to Pi-Ha‘py and 20 itr from Pi-Ha‘py to the northern limit of Egypt. In *Une Chappelle De Sestostris* (1956), Pierre Lacau published Middle Kingdom inscriptions from the White Chapel at Karnak and from the Temple of Montu at Madamud, giving 106 *itr* for the length of Egypt, 86 *itr* from Elephantine to Pi-Ha‘py, and 20 *itr* from Pi-Ha‘py to the northern limit of Egypt. In *Horus of the Bedhet* (1944), Alan Gardiner stated that after being allowed to view Lacau’s findings prior to publication, he was convinced they were copies of Old Kingdom inscriptions. Gardiner also stated: “Obviously the compiler of these figures set before himself the task of stating the lengths of the upper and lower Niles respectively.” These inscriptions indicate that these measures were known throughout dynastic Egypt, including the time of the Persian occupation when Herodotus was in Egypt, and including Ptolemaic times when Eratosthenes was the head librarian at Alexandria.

During the lifetimes of Herodotus and Eratosthenes, the length of the standard Greek schoinos was 30 stadia of 400 Greek cubits, or 12,000 Greek cubits. The length of the Greek cubit was 25 digits, or 10,000 digits for the Greek stadia, or 300,000 digits for the Greek schoinos. The length of the Egyptian remen is 20 digits, or 10,000 digits for a stadia of 500 remen, or 300,000 digits for 30 of these stadia. A stadia of 500 remen is contained 600 times in one degree of latitude. The length of a stadia of 300 remen is 6,000 digits, or 300,000 digits for 50 of these stadia. A stadia of 300 remen is contained 1,000 times in one degree of latitude, and this stadia of 300 remen is contained 360,000 times in the meridian circumference. However, the standard unit of measure throughout dynastic times in ancient Egypt was the royal cubit, equal to the diagonal of a square remen, or 20 digits times the square root of two, or 1.414... remen. A stadia of 300 royal cubits is contained 707.1 times in one degree of latitude (1000/1.414... = 707.10...). The ancient Egyptian *itr* contained 50 stadia of 300 royal cubits, or 15,000 royal cubits, and 14.14... *itr* are contained in one degree of latitude.

The latitude of Elephantine/Syene is 24°05'N and the northern limit of Egypt is 31°35'N, or a difference of 7°30', or 1/48th of the meridian circumference. The length of 14.14... *itr* per degree, times 7.5°, equals 106.06 *itr,* as specified in the ancient Egyptian inscriptions. The most significant division from the single stream of the upper Nile to the multiple streams of the lower Nile is the division of Canopic and the Sebennytic channels at the latitude of 30°10'N, in the immediate vicinity of Heliopolis. The difference between 31°35'N and 30°10'N is 1°25', or 1.416°. The length of 14.14 *itr* per degree times 1.416° = 20.03 *itr*, as specified in the ancient Egyptian inscriptions. If the distance from the apex of the delta to the northern limit of Egypt was regarded as 1.414°, or one degree times the square root of two, then 1.414° times 14.14 *itr* per degree, equals 20 *itr*.

HERODOTUS II.6 names the Egyptian schoinos as his unit of measure for the lengths of Egypt. Like the Greek schoinos, the Egyptian schoinos contained 12,000 cubits, but like the Egyptian *itr*, the Egyptian schoinos contained royal Egyptian cubits, rather than Greek cubits. In *A Season in Egypt* (1887), Petrie published his survey of the inscribed Ptolemaic schoinos markers on the Fayum road, showing the distance between the markers was 12,000 royal cubits. HERODOTUS II.7 states that the distance from Heliopolis to the sea is 25 Egyptian schoinos, and this corresponds exactly with the statement of 20 *itr* from Pi-Haʻpy to the northern limit of Egypt (20 × 15000/12000 = 25). HERODOTUS II.6 states that the Egyptian schoinos contained 60 furlongs and the long distance measures of Egypt given by Herodotus in furlongs are equal to whole numbers of Egyptian schoinos. HERODOTUS II.9 gives 7920 furlongs from Elephantine to the sea, and 7920/60 =132 Egyptian schoinos. The measure of 106 *itr* × 15000/12000 = 132.5.

In *The Well of Eratosthenes* (1914), J.L.E. Dreyer stated: "When it is stated that we do not know the length of the stadia adopted by Eratosthenes, it is only a repetition of what used to be stated in books written long ago. It is now generally conceded that the fact recorded by Pliny, that Eratosthenes put a schoinos = 40 stadia settles the matter. An Egyptian schoinos was 12,000 royal cubits. Therefore the stadia of Eratosthenes was 300 such cubits." In *The Egyptian Measurement System* (1816), Jean-Antoine Letronne states that the schoinos of both Herodotus and Eratosthenes was the Egyptian schoinos of 12,000 royal cubits, that was divided into 60 furlongs of 200 royal cubits by Herodotus, and divided into 40 stadia of 300 royal cubits by Eratosthenes. The stadia of 300 royal cubits is contained 254,588 times in the polar circumference (707.1 × 360° = 254,558). The polar circumference of 40,008km/254558 = 157.148m per stadia and 157.148m/300 = .5238m, or 20.62 inches per cubit.

STRABO 2.5.8 states that according to Eratosthenes: "The whole circle contains 252,000 stadia, that is, the space from the equator to the pole contains fifteen of the sixty divisions. There are four divisions between the equator and the summer tropic or parallel passing through Syene. From Syene to the equator there are 16,800 stadia (for such is the amount of the four-sixtieths, each sixtieth being equivalent to 4200 stadia), and from Alexandria to the equator 21,800." The measure of 252,000 stadia for the meridian circumference, divided by 360°, equals 700 stadia per degree.

Eratosthenes believed that Syene and the Tropic of Cancer were both on the 24th parallel. The 16,800 stadia from Syene to the equator is 700 stadia, times 24 degrees. The 21,800 stadia from Alexandria to the equator is the 16,800 stadia from Syene to the equator, plus the 5000 stadia from Alexandria to Syene. The center of Ptolemaic Alexandria is located at 31°12'N. The older Egyptian port city of Rhakotis, now in the southern quarter of Alexandria, is located at 31°09'N.

The difference of 5000 stadia between Alexandria and Syene, divided by 700 stadia per degree, gives 7.142°, or 7°09', for the difference in latitude. Eratosthenes believed that Syene and the Tropic of Cancer were both located on the 24th parallel, but the actual latitude of Syene is 24°05'N, five minutes of latitude closer to Alexandria. Subtracting five minutes of latitude from 7°09' gives 7°04', or 7.07°, and 5000 stadia divided by 7.07° equals 707 stadia per degree. The calculation of 700 stadia per degree, instead of the correct calculation of 707 stadia per degree, may have been due to the known length 5000 stadia, divided by an incorrect difference in latitude, based on the incorrect latitude of 24°00'N for Syene.

STRABO 17.1.2 states that Eratosthenes gives the distance from Syene to the sea of 5300 stadia. Alexandria is on the sea. The additional length of 300 stadia is the north-south distance from Alexandria to the sea at the northern limit of Egypt. The 5300 stades of Eratosthenes, from Syene to the sea, divided by 40, equals 132.5 Egyptian schoinos, compared to 132 schoinos given by Herodotus, and 5300 stades divided by 50 equals 106 *itr*, given in the ancient Egyptian inscriptions. In *The Dawn of Civilization* (1894), Gordon Maspero stated that "Not only did the Delta long precede the coming of Menes, but its plan was entirely completed before the first arrival of the Egyptians." and that "From that time Egypt made no further increase towards the north, and her coast remains practically such as it was thousands of years ago." The latitude of the northern limit of the Delta is 31°35'N, or 7.5° north of Syene. 707 stadia per degree, times 7.5°, equals 5303 stadia.

STRABO 2.5.24 states "The passage between Rhodes and Alexandria from north to south is about 4000 stadia; sailing round the coasts it is double this distance. Eratosthenes informs us that, although the above is the distance according to some mariners, he himself, from observations of the shadows indicated by the gnomon, calculates it at 3750 stadia." According to the UNESCO World Heritage Centre, the latitude of the ancient city of Rhodes is 36°26'50" N, or 36.4472°. According to CLEOMEDES 1.7, Eratosthenes determined the latitude of Alexandria with a sundial, and according to Strabo, Eratosthenes also determined the latitude of Rhodes with a sundial (gnomon). The center of Ptolemaic Alexandria is located at 31°12'N, or 31.2°. The older Egyptian port city of Rhakotis, now in the southern quarter of Alexandria, is located at 31°09'N, or 31.15°. The difference between Rhodes and Ptolemaic Alexandria is 36.45° − 31.2° = 5.25°, and 5.25° × 700 = 3675 stadia. The difference between Rhodes and Rhakotis is 36.45° − 31.15° = 5.3°, and 5.3°× 700 = 3710 stadia. The difference of 5.3° × 707 stadia per degree = 3747 stadia.

STRABO 2.1.20 states "Philo, in his account of a voyage by sea to Ethiopia, has given us the clima of Meroe. He says that at that place, the sun is vertical 45 days before the summer solstice and he also informs us of the proportion of shadow thrown by the gnomon at the equinoxes and solstices. Eratosthenes agrees almost exactly with Philo." Hamilton and Falconer's annotation of Strabo states: "Philo's observation indicates a latitude of 16°48'34" for Meroe." STRABO 2.5.7 states that Eratosthenes gives the latitude of Syene as midway between Alexandria and Meroe, and gives the same distance of 5000 stadia from Meroe to Syene, and from Alexandria to Syene.

In *A Historical and Topographical Guide to the Geography of Strabo* (2018), Duane Roller wrote: "The latitude of Meroe was well known through the report of Philon, who was in the service of Ptolemy II and spent time in Meroe, publishing an Aithiopika, about which little is known. The observation that the sun was in the zenith 45 days before the solstice indicates a latitude of 16°46', an error of only a few minutes. Philon may have been commissioned to make astronomical calculations at Meroe - since it was generally considered the southernmost inhabited place - building on the techniques of determining latitude that had been developed by Pytheas half a century earlier."

In *The Marriage of Philology and Mercury* 6.598 (c. 410 A.D.), Martianus Capella states that Eratosthenes gave 252,000 stadia for the great circle of the earth, and that Eratosthenes determined the distance from Meroe to Syene based on Philo's measurement of the number of portions of the great circle of the earth between Meroe and the Tropic of Cancer, by multiplying according to the ratio of this part of the circle, i.e.: 700 stadia × 7.1428° = 5000 stadia.

The actual latitude of Meroe is 16°56'. The meridian distance from Meroe to the 24th parallel is 7°04', or 7.07°, and 7.07°× 707 stadia per degree = 5000 stadia. The meridian distance from Meroe to Syene is 24°05'N − 16°56'N = 7°09', or 7.15°, and 7.15°× 700 stadia per degree = 5005 stadia. However, Eratosthenes believed Syene was on the 24th parallel and that each degree contained 700 stadia, giving 16°51' for the latitude of Meroe, compared with his calculation of 31°09' for the latitude of Alexandria, in relation to his understanding of 24°00' for the latitude of Syene. Based on his calculations, the difference between Meroe and Alexandria was 31°09' − 16°51' = 14°18', or 14.30°, and 14.30 × 700 = 10,010 stades.

Pytheas of Massalia traveled by sea (c. 325 BC) from Massalia (modern Marseille, France), through the Strait of Gilbralter, around England, and then further north, to Thule. Only fragments of the report of Pytheas have survived, but Pytheas' description of the location of Thule has been commented on by numerous ancient and modern sources. What has been reported from the accounts of Pytheas is that the sun shone at midnight on the solstice at Thule. Citing Pytheas, PLINY 4.30 equates Thule with the Arctic Circle, stating "the most remote of all that we find mentioned is Thule, in which, as we have previously stated, there is no night at the summer solstice, when the sun is passing through the Tropic of Cancer, while on the other hand at the winter solstice there is no day." STRABO 1.4.2 states "After this he (Eratosthenes) proceeds to determine the breadth of the habitable earth: he tells us, that measuring from the meridian of Meroe to Alexandria, there are 10,000 stadia. From thence to the Hellespont about 8100. Again; from thence to the Dnieper, 5000; and thence to the parallel of Thule, which Pytheas says is six days' sail north from Britain, and near the Frozen Sea, 11,500; To which if we add 3400 stadia above Meroe in order to include the Island of the Egyptians, the Cinnamon country and Taprobane, there will be in all 38,000 stadia."

This definition of the breadth (range of latitude) of the habitable earth is north to the Arctic Circle, and south to 3400 stadia south of Meroe. Given Eratosthenes calculation of 16800 stadia from the 24th parallel to the equator, and 5000 stadia from the 24th parallel to Meroe, Eratosthenes is giving the southern limit of habitable latitudes as 16,800 − 5000 − 3400 = 8400 stadia north of the equator, and 8400 stadia south of the Tropic of Cancer, or halfway between the tropic and the equator. Eratosthenes gives 21,800 stadia from the equator to Alexandria, plus about 8100 stadia to the Hellespont, plus 5000 stadia to the Dnieper, plus 11,500 stades to Thule, equals 46,400 stadia from the equator to the Arctic Circle, and 46400/700 = 66.28°N for the Arctic Circle, and this is very nearly correct. In Eratosthenes' time, the sun was partially above and partially below the horizon at midnight on the summer solstice at 66.28°N.

The relation of all of these measures to the equator, the tropic and the Arctic Circle indicates that these are all due north-south meridian measures, as Strabo also stated. The astronomical observations for differences in latitudes are accounted for, but the determination of 700 stadia per degree is not. CLEOMEDES 1.7 states that Eratosthenes arrived at the number of stadia by comparing his calculated difference in latitude between Alexandria and Syene to the distance of 5000 stadia, but how Eratosthenes arrived at the distance of 5000 stadia is unaccounted for. Determining this distance would require a long distance meridian survey. Eratosthenes' distance of 5300 stadia from Syene to the northern limit of Egypt, the same as the measure given earlier by Herodotus, and also the same as the measure given throughout ancient Egyptian history, indicates that Eratosthenes and Herodotus knowingly or unknowingly relied on a very old and very accurate meridian survey.

Sizes and Orbits of the Earth and Moon

Jim Alison - February, 2022

In *Ancient Metrology* (1981), John Michell diagramed the radius of the moon as .2727... times the radius of the earth. Given the earth radius as unity, or one, the radius of the earth plus the radius of the moon is 1.2727..., or 14/11. The NASA moon fact sheet (2021 update), gives the volumetric mean radius of the moon as .2727, compared to one for the volumetric mean radius of the earth, just as Michell stated. NASA gives the polar radius of the moon as .2731, compared to one for the polar radius of the earth. Michell concluded that the perimeter of a square with sides equal to the diameter of the earth equals the circumference of a circle with a radius equal to the radius of the earth plus the radius of the moon.

Given .273 for the moon/earth ratio, the reciprocal of .273 is 3.663, for the earth/moon ratio. In 2021, Manu Seyfzadeh stated on the Graham Hancock message board that .273 x 100 = 27.3 and 3.663 x 100 = 366.3, and that 27.3 and 366.3 are the number of days in the sidereal orbit of the moon around the earth, and the number of the earth's axial rotations in the sidereal orbit of the earth around the sun. Since .273 and 3.663 are reciprocals, .273 x 3.663 = one. Since these numbers are multiplied by 100 for the sidereal orbits of the moon and the earth, 27.3 x 366.3 = 10,000.

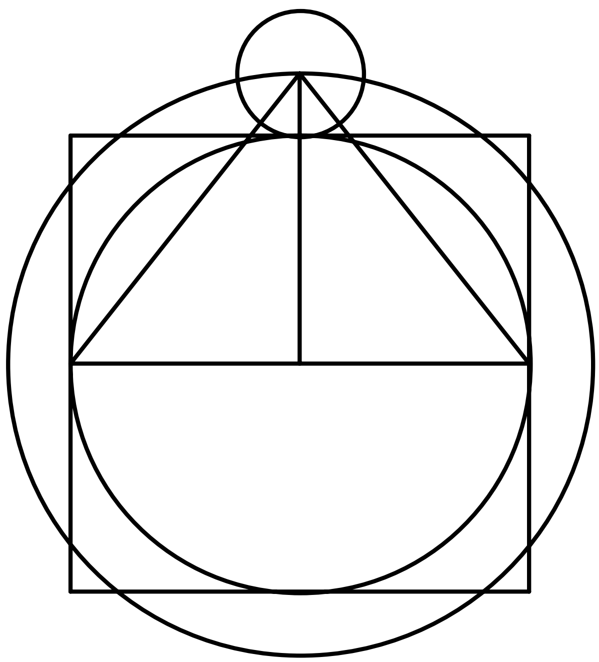
The range of the distance from the earth to the moon is 357,000km to 407,000km, for a mean distance of 382,000km. The speed of light is 299,792km/s. The speed of light per second, times $4/\pi$, or 1.2732, equals 381,707km, for a distance from the earth to the moon of 1.2732 light seconds. Taking this distance as the radius of the orbit of the moon, $1.2732 \times 2 \times \pi = 8$ light seconds for the length of the orbit of the moon around the earth. Eight seconds × 299,792km = 2,398,336km. The orbital distance of 2,398,336km/27.3 days/24 hours/60 minutes/60 seconds = 1.016km per second. NASA gives 1.1km/s to .996km/s for the range of the orbital speed of the moon. The perimeter of a square with side lengths of two light seconds, giving a distance of one light second from the center of the square to the midpoint of each side of the square, equals the length of the orbit of the moon.

The ancient Egyptian *itr* contains 15,000 royal cubits. As Livio Stecchini observed in *Secrets of the Great Pyramid* (1971), the polar diameter of the earth contains 1618 *itr*, or 1000φ *itr*, or 15,000,000φ cubits, or 24,270,000 cubits. The polar diameter of 12,713,500m/24,270,000 = .5238m per cubit. The diameter of $15{,}000{,}000\varphi \times 4/\pi = 19{,}098{,}593\varphi$ royal cubits for the polar diameters of the earth plus the moon. The perimeter of a square with sides of 15,000,000φ cubits is 60,000,000φ cubits. The diameter of $19{,}098{,}593\varphi \times \pi = 60{,}000{,}000\varphi$ cubits for the circumference of a circle with a diameter equal to the diameter of the earth plus the diameter of the moon.

In *The Temple of Man* (1956), Schwaller de Lubicz equated a diameter of one meter with a circumference of six royal cubits, giving a diameter of 10,000,000m × φ, or 16,180,339m, for a circumference of 60,000,000φ cubits. The diameters of 12,713,500m + 3,472,000m = 16,185,500m. The meter was intended to measure 1/40,000,000 of the polar circumference but the actual measure is 1/40,00800. Adjusting for this discrepancy gives 16,185,500m x 40,000/40,008 = 16,182,200m, or an error of 1861 meters in the combined polar diameters of the earth and the moon.

The earth's maximum orbital velocity is 30.29km/s, the mean velocity is 29.78km/s, and the minimum velocity is 29.29km/s. The speed of light is 299,792km/s. One ten-thousandth of the speed of light is within the narrow range of the orbital velocity and very nearly the same as the mean orbital velocity of the earth. 299,792km/10,000 × 365.25 × 24 × 60 × 60 = 946,071,601.9km in the orbit of the earth and 946,071,601.9km/π/2 = 150,571,971.9km for the distance from the earth to the sun. The minimum distance from the earth to the sun is 147,095,000km and the maximum distance is 152,100,000km. The velocity of 29.979km/s, divided by the synodic orbit of the moon, of 29.53 days, equals 1.015km/s for the orbital velocity of the moon. The length of the synodic orbit of the moon is equal to the length of one day in the orbit of the earth around the sun.

In *Historical Metrology* (1953) A.E Berriman equated the ancient Egyptian digit with 1/54 of one meter. 299,792,000m/s × 54 = 16,188,768000 digits/s, or 10 billion digits × φ for the speed of light per second, with an error of approximately 1/2000. Correcting the 40,000/40,008 discrepancy in the meter reduces the margin of error to approximately 1/3000. This gives the orbital velocity of the earth as 1,000,000φ digits per second; the orbit of the moon as 80,000,000,000φ digits; given 500 light seconds from the earth to the sun, 5,000,000,000,000φ digits; and the orbit of the earth around the sun, 10,000,000,000,000φ × π digits. This gives 125π for the ratio between the length of the orbit of the moon and the orbit of the earth, which is also the ratio between the distance from the earth to the moon and the distance from the earth to the sun. 500 light seconds from the earth to the sun, divided by 125π equals 1.2732 light seconds from the earth to the moon.



With the sides of the square equal to the base length of the great pyramid, the outer circle has a radius equal to the height of the pyramid and the circumference equals the perimeter of the square. With the sides of the square equal to the polar diameter of the earth, the outer circle has a radius of the earth radius plus the moon radius, and the circumference equals the perimeter of the square. With the sides of the square equal to two light seconds, the outer circle has a radius equal to the distance from the earth to the moon, and the circumference equals the perimeter of the square.

Ancient Egyptian Volumes, Weights and Exchange Rates

Jim Alison - May 2022

Named ancient Egyptian volumes are defined by the volume of the cubic royal cubit. The *khar*, or sack, is 2/3 of a cubic cubit. The *hekat* is 1/30 of a cubic cubit. The *henu* is 1/300 of a cubic cubit. The *ro* is 1/9600 of a cubic cubit. The length of the royal cubit is known, giving the volume of the cubic cubit, and also giving the other named volumes as fractions of the cubic cubit.

In *Canonical Grain Weights* (2014), Jon Bosak observed that the edge length of the *ro* equals one Roman inch. The Roman digit is the same length as the Egyptian digit and the length of the Roman inch is 4/3 the length of the digit. The length of the Egyptian royal cubit is 20 digits times the square root of two, or 28.28... digits, times 3/4 = 21.21... Roman inches, giving 9545.9 cubic Roman inches for the volume of the cubic royal cubit. 9545.9/9600 = .9944 cubic Roman inches for the volume of the *ro*, and the cube root of .9944 is .998 Roman inches for the edge length of the *ro*.

In *Ancient Weights and Measures* (1926), Petrie stated: "The *qedet* is by far the most numerous standard in Egypt, and has generally been regarded as especially Egyptian. It is the basis of nearly all statements of weight from the xviii dynasty onward. The multiple of 10 *qedets* was termed the *deben*, and 10 *debens* were termed the *sep*." Petrie also stated: "The *qedet* greatly preponderate in every period." In *Notes on Egyptian Weights and Measures* (1892), Griffith stated: "In early times there were probably several units of weight for various metals. Later, probably in the XVIIIth Dynasty, the *deben* of 1400-1500 grains (90.7-97.2 grams), with the qedet of 140-150 grains (9.07-9.72 grams), became the only unit recognized in documents. The unmarked weights in Mr. Petrie's lists under the *qedet*, show every shade of graduation between these two varieties of the *qedet*, with a large preponderance in favor of the higher value."

In 1892 Griffith also stated: "The Ptolemaic texts of Edfu (Dumichen, Geographical Inscription, II, PI. LXXXIII-IV) equate the *henu* of wine or water with 5 *deben* weight. The *henu* at 29.2 cubic inches gives a *deben* of 1474 grains, the *qedet* of which, 147.4 grains, lies between the Heliopolite *qedet* of 140 grains, and the royal *qedet* weight of Aahmes II, 150 grains. This is a fixed point of great value."

In *The Origin of Metallic Currency and Weight Standards* (1892), William Ridgeway stated that according to Michel Soutzo: "All the weight systems both monetary and commercial of Asia, Egypt, Greece, come from one primordial weight, the Egyptian *deben* of 96 grams, or from its tenth, the *qedet* of 9.6 grams. He ascribes the origin of these weights to an extremely remote epoch not far perhaps from the time of the discovery of bronze in Asia, and the invention of the first instruments for weighing." In *The Legacy of Egypt* (1942), R. W. Sloley stated: "The principal capacity measure (the *henu*) held water weighing 5 *debens*. The *deben* was the weight (1,470 grains) of the anklet of the same name, of which the tenth part was the *qedet,* the weight of the finger ring."

The length of the royal Egyptian cubit is 52.38 centimeters, giving a cubic cubit of 143,713 cubic centimeters, divided by 300 equals 479.0 cubic centimeters for the *henu*, or 479 grams for the water weight contained in the volume of the *henu,* or 479/5 = 95.8 grams for the weight of the *deben*.

In 2014, Bosak also observed that the weight of one *qedet* of barley grain is contained in the volume of one *ro*. The volume of the *ro* is 1/32 the volume of the *henu*. The 479 gram water weight of the *henu*, divided by 32, equals 14.97 grams, or 231 grains, for the water weight of the *ro*. The modern test weight for grade one barley is 48 pounds per bushel. A bushel is 2150.43 cubic inches, or 35,239 cubic centimeters, or 35,239 grams, or 77.69 pounds of water weight.

The barley weight of 48/77.69 = .618 for barley weight in relation to water weight, but the average weight of barley grain is higher than the minimum test weight, and the test weight for brewing barley is 50 pounds per bushel. The barley weight of 50/77.69 = .643 for barley weight in relation to water weight. The 231 grain water weight of the *ro*, times .64, equals 147.84 grains, or 9.58 grams for the barley weight of the *ro*, compared to 95.8 grams for the weight of the *deben*, or 9.58 grams for the weight of the *qedet*.

The weight of wheat in relation to the weight of barley is generally given as 5/4. This is reflected in the minimum test weight of 60 pounds per bushel of wheat compared to the test weight of 48 pounds per bushel of barley: 60/48 = 5/4. However, like barley, the average weight of a bushel of grade one wheat is higher than the minimum test weight, averaging 62 pounds per bushel. The 77.69 pound water weight of a bushel, times 4/5 = 62.15 pounds of wheat per bushel.

Given the weight ratios of water × 4/5 = wheat, and wheat × 4/5 = barley, the ratio between barley weight and water weight is 4/5 × 4/5 = 16/25, or 32/50. The *henu* contains 32 *ro*, and five *deben* contains 50 *qedet*. Given the 32/50 relation between barley weight and water weight, the barley weight of one *qedet* per *ro* gives a water weight of five *deben* per *henu*.

1 *ro* of barley = 1 *qedet*
1 *ro* of wheat = 5/4 *qedet*
1 *ro* of water = 50/32 *qedet*

1 *henu* of barley = *32 qedet*
1 *henu* of wheat = *40 qedet*
1 *henu* of water = 50 *qedet*

1 *hekat* of barley = 32 *deben*
1 *hekat* of wheat = 40 *deben*
1 *hekat* of water = 50 *deben*

1 *Khar* of barley = 64 *sep*
1 *Khar* of wheat = 80 *sep*
1 *Khar* of water = 100 *sep*

1 cubic cubit of barley = 96 *sep*
1 cubic cubit of wheat = 120 *sep*
1 cubic cubit of water = 150 *sep*

In *Ancient Weights and Measures*, Petrie lists the *peyum* as another Egyptian standard: "This standard is guaranteed and named P-Y-M by three weights found in Palestine." Petrie also states: "This standard is known from documents of the xviiith dynasty. In papyri (Z.A.S., 1906, 45) values are reckoned in rings of gold weighing 12 to the *deben*. The ring appears to be called *shoti* in a papyrus, so that may be the Egyptian name of the *peyum*." Petrie describes an inscribed but deteriorated limestone ball from Tell Amarna, that, allowing for loss, may have been 14360 grains. The inscription is 12 units of 10, but Petrie points out that "this could not have been 12 *deben* Egyptian, and probably indicates 12 units of 10 *peyum* each," or 120 *peyum* for the 10 *deben* weight of the limestone ball, or 12 *peyum* per *deben*. Petrie also comments that most of the weights in the *peyum* standard are in even numbered multiples of the weight of the *peyum*, also indicating the weight of a double *peyum*, of 1/6 of a *deben*.

I SAMUEL 13 19-21: "There was no blacksmith to be found throughout all the land of Israel, for the Philistines said, 'Lest the Hebrews make swords or spears.' But all the Israelites would go down to the Philistines to sharpen each man's plowshare, his mattock, his ax, and his sickle; and the charge for a sharpening was a *pym* for the plowshares, the mattocks, the forks, and the axes, and to set the points of the goads."

In *Ancient Egyptian Science* (1989), Marshall Clagett translates the Rhind Mathematical Papyrus, problem 62: "A bag containing equal weights of gold, silver, and lead is bought for 84 *shaty*, what is the amount of each precious metal. As for what is given for a *deben* of gold, it is 12 *shaty*, for silver it is 6 *shaty*, and for lead it is 3 *shaty*." One *deben* of gold is 12 *shati*, one *deben* of silver is 6 *shati*, and one *deben* of lead is 3 *shati*, for a total of 21 *shati*. The price of 84 *shati*, divided by 21 equals 4, giving the answer of four *deben* of each metal in the bag. In a footnote to this paragraph, Clagett states: "*Shaty* is a unit of value and *deben* is a unit of weight."

In *The Rhind Mathematical Papyrus* (1923), Eric Peet says that Griffith translates the hieroglyph as *shati*, but that Peet prefers 'ring' because of the presence of a ring like determinative that sometimes accompanies the hieroglyph, as well as remains of metal rings, mostly silver or gold, that are considered to be fractions or multiples of a *shati* of silver or gold.

Jaroslav Černý studied a number of commercial inscriptions that recorded exchange rates from the workmans village of Deir el Medina, near the Valley of the Kings, in Upper Egypt. In his 2017 masters thesis, Daniel Johnson gives the following chart from Černý's findings:

One *deben* of Gold

Time Period	**Silver equivalent**	**Copper equivalent**	**Barley equivalent**
18th and 19th dynasties	**1.6 *deben***	**166 *deben***	**166 *khar***
Early 20th	**2 *deben***	**120 *deben***	**60 *khar***
Mid 20th	**2 *deben***	**120 *deben***	**30 *khar***
20th in famine periods	**2 *deben***	**120 *deben***	**15 *khar***
Late 20th	**2 *deben***	**120 *deben***	**60 *khar***

Although the chart shows fluctuations in the relative values, the most frequent exchange rate between gold and silver is one *deben* of gold equals two *deben* of silver, which is the same as the relative value given in RMP #62, of 1/12 *deben* of gold = 1/6 *deben* of silver = one *shati*. The most frequent exchange rate of gold for copper is 1:120 and of silver for copper is 1:60. Given 1/12 *deben* of gold or 1/6 *deben* of silver for a *shati*, 10 *deben* of copper = one *shati*. The exchange rate for grain is the most variable on the chart, but the approximate average and the most common exchange rate is 60 sacks = one *deben* of gold, or five sacks = one *shati*.

Exhibit number EA23067 in the British Museum is a Middle Kingdom sandstone weight from Gebelein in Upper Egypt, weighing 943 grams, but noting that when received, the weight was labeled 948 grams. The British Museum photograph shows deterioration of the upper surface. The inscription is a ring and 60 units. This is a 10 *deben* weight, divided into units of 1/6 of a *deben*, or one double *peyum*, or one silver *shati*. This weight balances 120 *shati* of gold, or 60 *shati* of silver, or one *shati* of copper.

In *A Lawsuit Arising from the Purchase of Slaves* (1935), Alan Gardiner stated: "Let us now turn to the second point of special interest upon which the papyrus throws light, the relative values of copper and silver. Hitherto the only sure testimony from Pharaonic times has been a passage adduced by Peet from a late Ramesside papyrus in Turin, where the ratio works out at 60 to 1. In the Cairo papyrus we now obtain certain evidence that in the sixteenth year of Ramesses II silver was worth 100 times as much as copper, where a weight of 10 *deben* (or 100 *qedet*) of beaten copper is valued at one *qedet* of silver." This indicates the change mentioned by Griffith, from the double *peyum* or silver *shati* standard of 1/6 *deben*, to the *qedet* standard of 1/10 *deben*. While the copper standard of 10 *deben* is the same, the relative value of silver increases from 1/6 *deben* to 1/10 *deben*.

In Černý's chart for the 18th and 19th dynasties, 1 *deben* gold = 1.66 *deben* silver = 166 *deben* copper = 166 sacks of grain. The ratio between silver and copper is the same as one *qedet* silver for 10 *deben* copper. All of the entries in Černý's chart have a base line of one *deben* of gold. If the base line was 10 *deben* of copper, then the entry for the 18th and 19th dynasties would be .6 *qedet* gold = 1 *qedet* silver = 10 *deben* copper = 10 sacks of grain. The weight of .6 *qedet* × 9.58 grams is 5.75 grams, or 88.7 grains. In *Ancient Weights and Measures*, Petrie states: "During recent years many weights have been found in Palestine bearing a sign, of which one example occurs in Egypt. This sign appears to be a monogram of *kh* and *o*, presumably beginning with the name *kho*... It may be said that the stated range of the *kho* series is from 173.6 to 179.4 grains, with a mean value of about 177.5 grains... There is an interesting group of five cowry shells carved in grey syenite, evidently all from one source, though bought singly. The largest weighs 4 of the next one, and that double of the next, and these are respectively 2, 1/2 and 1/4 *kho*." One-half of Petrie's 177.5 grain weight for the *kho* is 88.75 grains, in agreement with Černý's exchange rate of .6 *qedet* of gold for 1 *qedet* of silver or 10 *deben* of copper or 10 sacks of grain in the 18th and 19th dynasties.

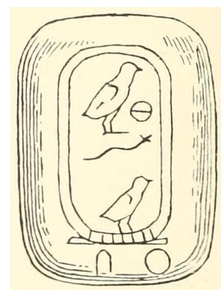
In *Ancient Weights and Measures*, Petrie lists the *beqa* as another named weight: "This standard has been recognized in Egypt during the last 20 years, and commonly called the gold standard, as the weights often have the hieroglyph of gold upon them. The name of this standard is given by three marked weights found in Palestine, each with the word *beth-qof-ayn*, spelling *beqa*. This standard was used in the earliest Hebrew literature, as it is named as the weight of the gold ring given to Rebekah, and the poll tax stated in Exodus xxxviii, 26."

Petrie includes variations in units of weight for the *beqa* from 190 grains to 215 grains but he indicates that the weights in this range are concentrated in two groups: "In the Old Kingdom, the lower standard is the more compact, 196-202. The higher standard spreads out from 206 to 213.5; and the extreme amounts are important examples, one the gold bar with the name of Aha (213.5), the other the fine weight with the cartouche of Khufu (206)." In *Architecture and Mathematics in Ancient Egypt* (2003), Corinna Rossi gives 13.6 grams, or 210 grains, for the weight of this standard during the the Old Kingdom and Middle Kingdom.

The Hilton-Price catalogue provides a drawing and a description of the Khufu weight:

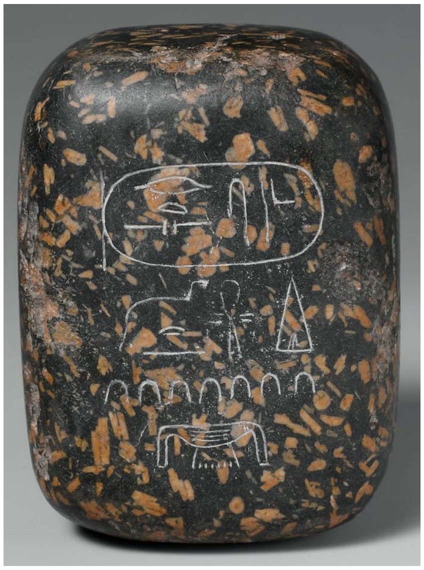
3007. Weight, oblong, rectangular, the top rounded, bearing the cartouche of Khufu; 10 ring units, 2060 grains. Unit: 206 grains or 13.348 grams. Basalt. IV dynasty. Bought at Cairo.

[See W. M. F. Petrie, *Academy*, January, 1891, No. 977, p. 95. F. L. Griffiths, "Notes on Egyptian Weights and Measures," *Proc. Soc. Bibl. Arch*., June 1892, Vol XIV., p. 442.



In the Metropolitan Museum of New York, Accession Number 35.9.5, is a weight bearing the cartouche of Userkaf; 5 ring units, 68.22 grams. Unit: 13.64 grams or 210 grains. Yellow jasper or opal. Fifth dynasty. Donated to the museum by Edward Harkness in 1935.

In the Metropolitan Museum of New York, Accession number 15.3.233, is a rhyolite weight from the 12th dynasty. Seventy units. Weight: 2.1 pounds. Unit weight: .03 pounds, or .48 ounces, or 13.6 grams, or 210 grains.

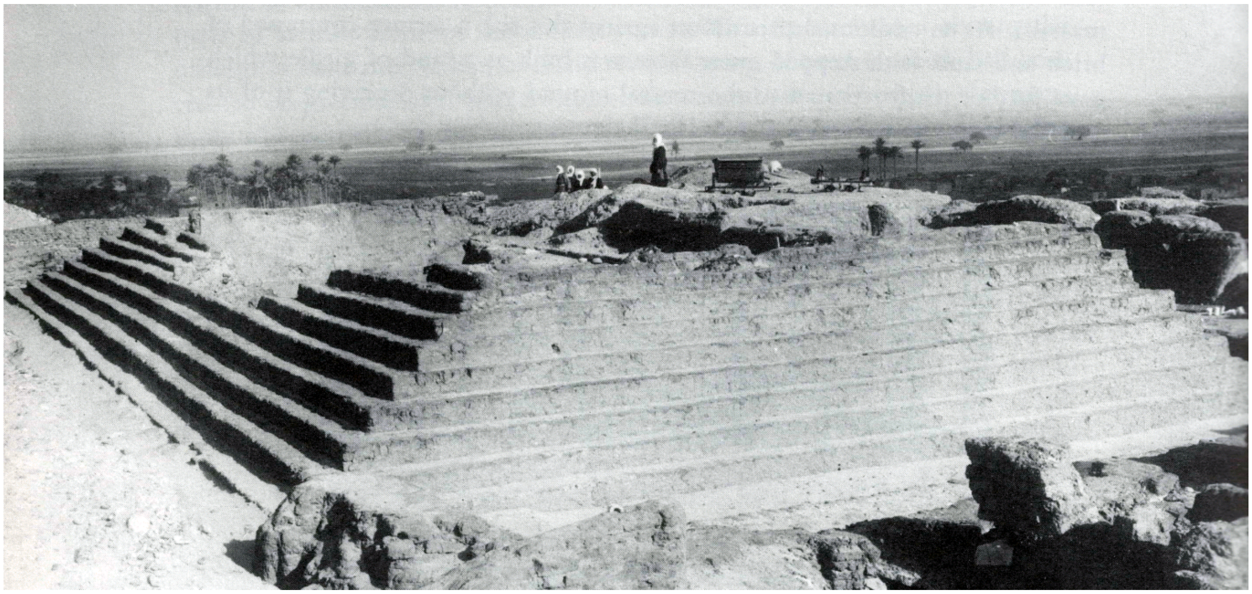


The weight of 2.1 pounds = 33.6 ounces = 952.5 grams = 14700 grains = 10 *deben* = 1 *sep*. The division of this 10 *deben* weight into 70 units indicates the *beqa* standard was 1/7 of one *deben*, just as RMP 62, and the research by Černý, indicate that the *peyum*, or gold *shati*, was 1/12 of one *deben*; the double *peyum*, or silver *shati* was 1/6 of one *deben*; and 1/2 *kho* = .6 *qedet*, or .06 *deben*.

In the Royal Ontario Museum is an uninscribed basalt weight excavated at Naukratis, dated to the late 18th or early 19th dynasty. Weight: 478.3 grams, or five *deben*, or 95.6 grams per *deben*. The volume of five *deben* of water is one *henu*. The volume of 478.3g of water at maximum density (4°C or 39.2°F) is 478.3cc, and 478.3cc × 300 = 143490cc for one cubic cubit, with a cube root of 52.35c, or 20.61 inches, for the length of the royal cubit. At room temperature (21°C or 70°F), water is 99.8% of maximum density, and 478.3g/.998 = 479.3cc, giving 479.3cc × 300 = 143777cc for one cubic cubit, or 52.387c, or 20.625 inches, for the length of the royal cubit.

Early Egyptian Pyramids

Jim Alison - July 2022

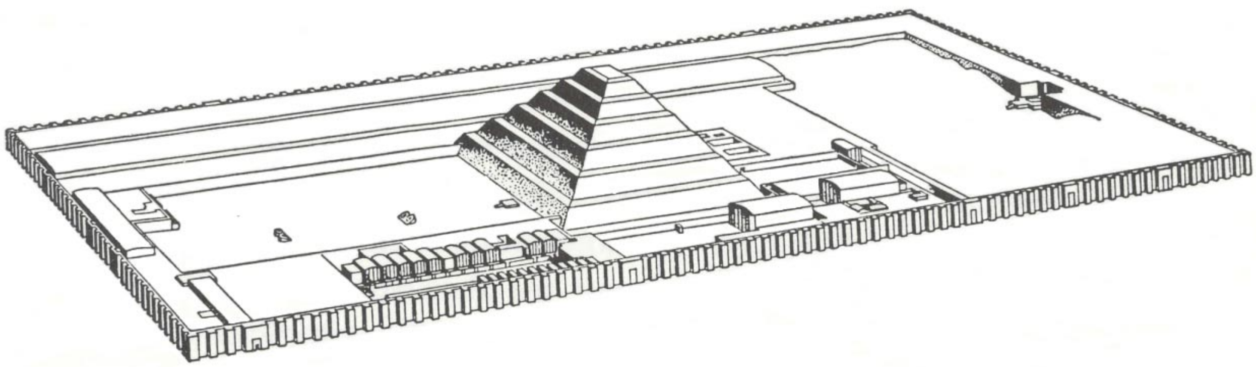


The first dynasty tomb of Adjib is near the northern limit of the Saqqara necropolis. This image, from the Egyptian Antiquities Service, is in *The Pyramids of Egypt* (1947) by I.E.S. Edwards.

In *Archaic Egypt* (1961), Walter Emery stated: “Sakkara tomb 3038 presents some very interesting and significant architectural features which so far have not been preserved in any other monument of the period. The building is dated to Enezib (Adjib), and it would appear probable that it is the burial place of the king. When first excavated, the superstructure of the tomb appeared to follow the familiar design of a rectangular platform, with its exterior decorated with recessed paneling. But further digging revealed a stepped pyramid structure hidden within it. Only the lower part of the stepped structure was preserved and it is possible that it continued upwards in a pure pyramid form. The tomb of Queen Her-nit at Sakkara was found to have a similar feature, although of a more primitive character, which takes the form of a rectangular earthen tumulus faced with brickwork, an obvious prototype of Enezib’s interior superstructure.”

Emery also stated: “The form of superstructure above the tombs of the kings of upper Egypt originally consisted of the rectangular tumulus faced with brick which developed into the superstructure of elongated pyramid form. In lower Egypt, the superstructure above the royal tombs took the form of the mastaba with its paneled facade. At Sakkara, notably in the case of the tombs of queen Her-nit and Enezib, the two forms of superstructure were welded together in one building; the tumulus pyramid directly over the burial and surrounded and covered by walls of the palace facade mastaba. Comparison of the plans of the tombs of Her-nit and Enezib with that of the step pyramid enclosure of King Zoser of the Third Dynasty reveals such similarities in design and proportion that we may well consider the design of the latter to be a development of the composite royal tomb of the First Dynasty.”

In *Mathematical Bases of Ancient Egyptian Architecture* (1985), Robins and Shute cite Perring's survey of the step pyramid, giving a slope of seven over two for the steps, and one half the height of each step, or 3.5 for the distance between the outer edge of each tread and the foot of the step above. This gives a rise of seven over a run of 2 + 3.5 = 5.5, for an overall slope of 7/5.5, or 14/11, or $\sqrt{\varphi}$, or $4/\pi$, or the same slope as the great pyramid. In *The Complete Pyramids* (1997), Mark Lehner gives 1645 meters for the perimeter of the enclosure, or $1000 \times \pi = 3141.6$ royal cubits ($3141.6 \times .5238 = 1645.5$ meters).



Step pyramid enclosure, from Edwards (1947), after J. P. Lauer (1936).

In the appendix to *Operations Carried on at the Pyramids of Gizeh* (1842), Perring states that the step pyramid "is the only pyramid in Egypt, the sides of which do not face the cardinal points, the northern front being 4°35' east of true north. In *A Season in Egypt* (1987), Petrie states "The azimuth of the step pyramid of Sakkara was observed by eye on its rough core masonry, as pointing to parts of the southern (bent) pyramid of Dashur. This resulted in showing it to be + 4°14' for E. and 4°28' for W. side, or mean 4°21' E. of true N."

The step pyramid enclosure has the same azimuth as the pyramid, and in addition to pointing south to the bent pyramid, the azimuths of the bent pyramid and the enclosure also point north to Adjib's tomb. A line from Adjib's tomb to the bent pyramid crosses over the step pyramid, in alignment with the orientation of the step pyramid and enclosure. The distance from Adjib's tomb to the bent pyramid is 10,475 meters. The length of 20,000 royal cubits × .5238 = 10,476 meters.

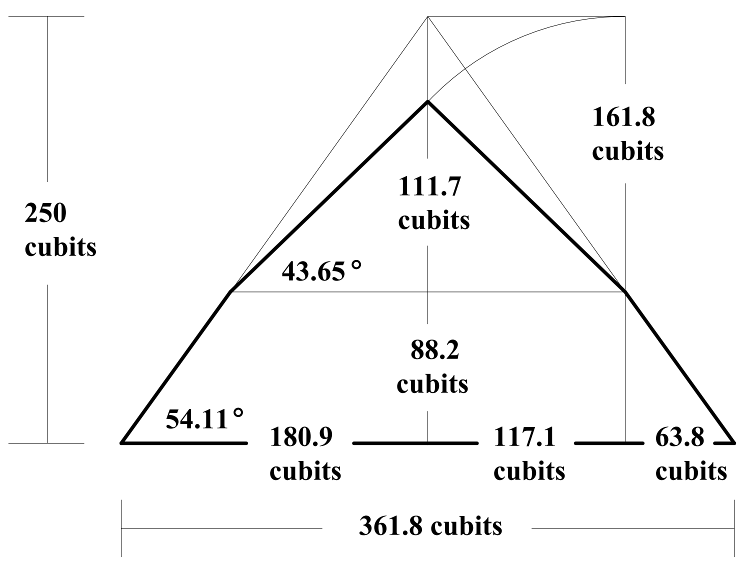
The northern (red) pyramid of Dashur has a base length of 420 cubits and a height of 200 cubits. The sides of the right triangle formed by the half base and the height are 210 and 200 cubits. The slant height (hypotenuse) of the pyramid is 290 cubits. This is a 10x expression of the Pythagorean 20-21-29 right triangle ($200^2 + 210^2 = 290^2$). The southern (bent) pyramid has the same or very nearly the same height of 200 cubits, and the upper part of the bent pyramid has the same or very nearly the same angular dimensions as the red pyramid. The slant height over the half base of 29/21 gives a base angle of 43.6° and the slant height over the half base of $(\varphi^2 + 1)/\varphi^2$ gives a base angle of 43.65°. In *The Temple of Man* (1957), R. A. Schwaller proposed that the ratio between the slant height and the half base of the upper part of the bent pyramid is $(\varphi^2 + 1)/\varphi^2$ and the ratio between the height and the half base of the lower part is $(\varphi^2 + 1)/\varphi^2$, giving a base angle of 54.11° for the lower part of the bent pyramid.

Schwaller relates the slant height over the half base of the red pyramid, of $(\varphi^2 + 1)/\varphi^2$, with the Fibonacci series (1, 2, 3, 5, 8, 13, 21...), Giving φ^2 for 21, φ for 13 and one for 8. This gives a slant height of $21 + 8 = 29$, and a half base of 21, as expressed by the red pyramid's slant height of 290 cubits over the half base of 210 cubits.

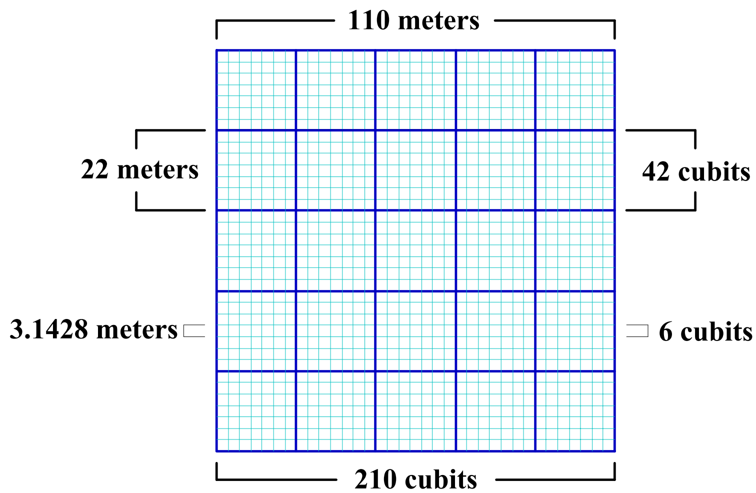
Petrie (1887), gives 7459", or 361.7 royal cubits, for the mean base length of the bent pyramid (7459/20.62 = 361.7). A base length of $(\varphi^2 + 1) \times 100$, or 361.8 cubits × 20.62 = 7460.3". Petrie also gives the completed height of the pyramid as 200 cubits. If the pyramid had been completed with the $(\varphi^2 + 1)/\varphi^2$ slope of the lower portion for the whole pyramid, the height would have been 250 cubits. The half base is 361.8/2 = 180.9 cubits, and $180.9 \times (\varphi^2 + 1)/\varphi^2 = 250$ cubits.

From the height of 250 cubits, the height of $\varphi \times 100$, or 161.8 cubits, is arced from the outer casing to the midline of the pyramid. This gives 161.8 cubits for the slant height of the upper portion and an 88.2 cubit bend height (250 − 161.8 = 88.2 cubits). The half base of the upper portion is $161.8 \times \varphi^2/(\varphi^2 + 1) = 117.1$ cubits, and the height for the upper portion is $161.8^2 - 117.1^2 = 111.7^2$, or 111.7 cubits.

The height of 88.2 + 111.7 = 199.9 cubits for the height of the bent pyramid. The base length, the slant height of the upper portion, and the height of the pyramid may also be expressed in terms of the Fibonacci series with φ^2 as 21, φ as 13 and one as 8, giving 21 + 8 = 29 for the base length, 13 for the slant height of the upper portion and 16 for the height. The red pyramid expresses the Fibonacci series directly in terms of 29 × 10 cubits for the slant height and 21 × 10 cubits for the half base, and the bent pyramid expresses φ and $\varphi^2 + 1$ directly with 161.8 cubits for the slant height of the upper portion and 361.8 cubits for the base length of the pyramid.

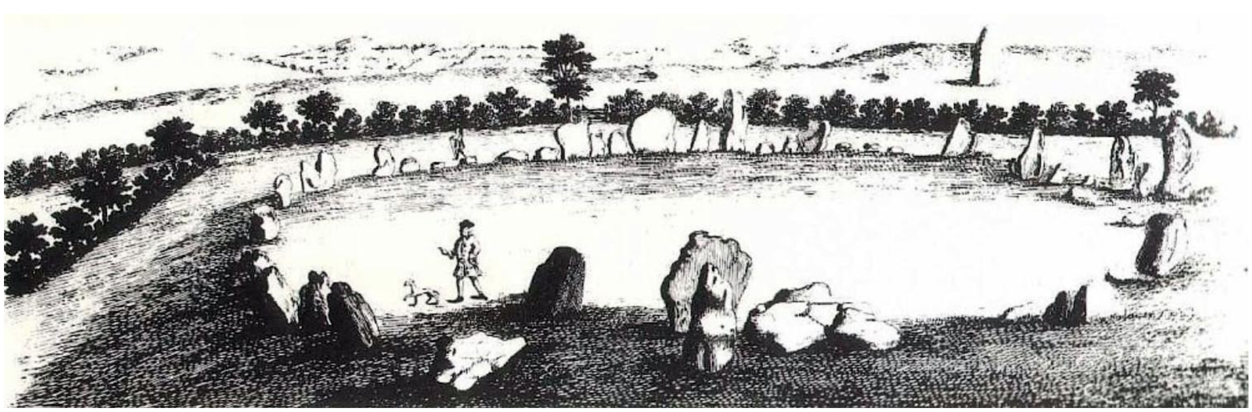


The 420 cubit base length of the red pyramid is given as 220 meters, or 110 meters for the half base of 210 cubits, illustrating the 11/21 = .5238 relation between the cubit and the meter. This relation is also illustrated by the Grande Arche in Paris. With a height of 110 meters, a width of 108 meters and a depth of 112 meters, the Grande Arche is also called the cube. The out sides of the Grande Arche are divided into 5 x 5 large panels and within each large panel are 7 x 7 smaller panels. A geometric cubic construction, with edge lengths of 110 meters, and without allowance for the spaces between the windows and the edges, gives 22 meters for the sides of the large panels and 22/7 meters, or 3.1428 meters, for the sides of the smaller panels. Edge lengths of 110 meters are equal to 210 cubits, giving 42 cubits for the large panels and 6 cubits for the smaller panels.



Source of image: Insecula.com

In *Abury* (1743), William Stukeley gives 35 yards for the diameter of the Rollright circle and states that in "the comparative scale of English feet and cubits, we discern 60 cubits of the Druids is the measure sought for." In *The Rollright Stones* (1983), George Lambrick gives 103-106 feet for the diameter. In stonepages.com, Diego Miozzi gives 103 feet, or 31.4 meters. Stukeley's discerned measure of 60 cubits × 20.62 inches/12 = 103.1 feet, and 60 cubits × .5238 = 31.428 meters.

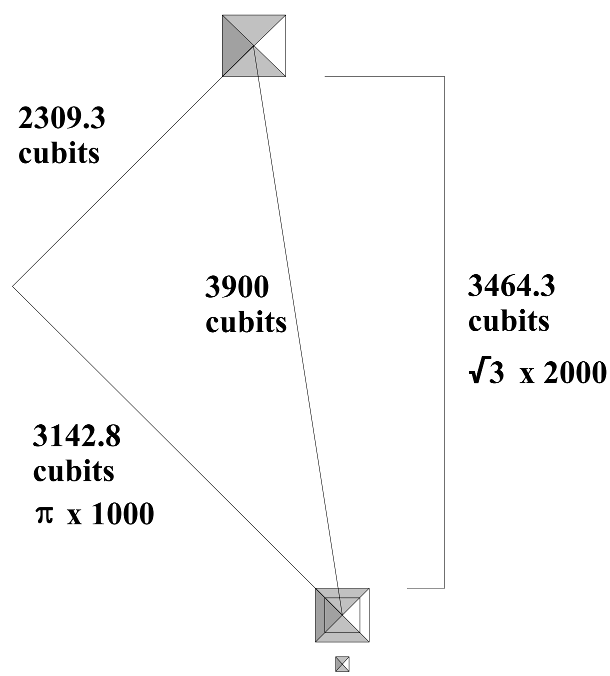


Rollright Circle, from Stukeley (1743)

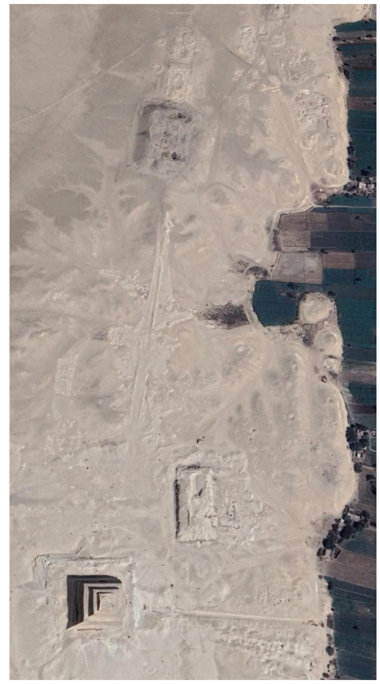
Petrie (1887) gives 6702 feet, or $6702 \times 12/20.62 = 3900$ cubits, for the distance between the summits of the red and bent pyramids, and 171.216° for the azimuth from the summit of the red to the summit of the bent, or 8.784° east of due south. This gives the azimuth from the summit of the bent to the summit of the red of −8.784°, or 8.784° west of due north. Petrie gives the mean orientation of the sides of the bent pyramid as −9' 12", or −.15°. Josef Dorner (1986) gives the orientation of the west side of the red pyramid as −8.7', or −.145°. These measurements show an error of less than 10' of orientation with the cardinal directions and an error of less than 30" in the orientation of the pyramids with each other. Adjusting the −8.784° azimuth by .145° gives −8.639° for the azimuth of the summits in relation to the orientation of the pyramids.

In relation to the cardinal orientation of the pyramids, diagonal 45˚ lines from the summits intersect in a 90° angle, 3142.8 cubits from the summit of the bent pyramid, and 2309.3 cubits from the summit of the red pyramid. This gives $3142.8^2 + 2309.3^2 = 3900^2$, or 3900 cubits for the distance between the summits. A right triangle with sides of 3142.8 cubits and 2309.3 cubits gives 36.31° for the angle at the summit of the bent pyramid, and 36.31° − 45° gives −8.69° for the azimuth of the line between the summits in relation to the orientation of the pyramids.

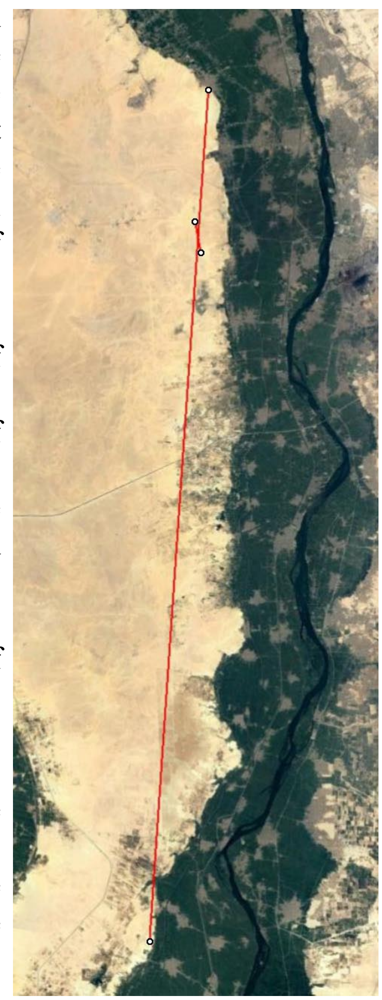
From the summit to the SE corner of the bent pyramid is $180.9 \times \sqrt{2} = 255.8$ cubits and from the summit to the NE corner of the red pyramid is $210 \times \sqrt{2} = 297$ cubits. The 45° distance from the south side of the bent to the north side of the red is 3142.8+ 255.8 + 2309.3 + 297 = 6004.9 cubits. The NS distance from the N side of the red to the S side of the bent is $6004.9/\sqrt{2} = 4246.1$ cubits. Subtracting the 420 cubit side length of the red and the 361.8 cubit side length of the bent gives 3464.3 cubits for the NS distance from the south side of the red to the north side of the bent. This equals $\sqrt{3} \times 2000$ ($1.73205 \times 2000 = 3464.1$), compared to $\sqrt{3} \times 1000$ for the 1732 cubit distance from the north side of the great pyramid to the south side of the third pyramid at Giza.



The Meidum pyramid is oriented to the cardinal directions with less than 30' of error. The large mastaba near the pyramid is also oriented to the cardinal directions. However, the largest mastaba at Meidum, attributed to Nefermat, and the path from the pyramid to the mastaba (Google Earth image), both have an azimuth of just over six degrees east of due north, pointing south to the Meidum pyramid, and pointing north to the step pyramid and Adjib's tomb.



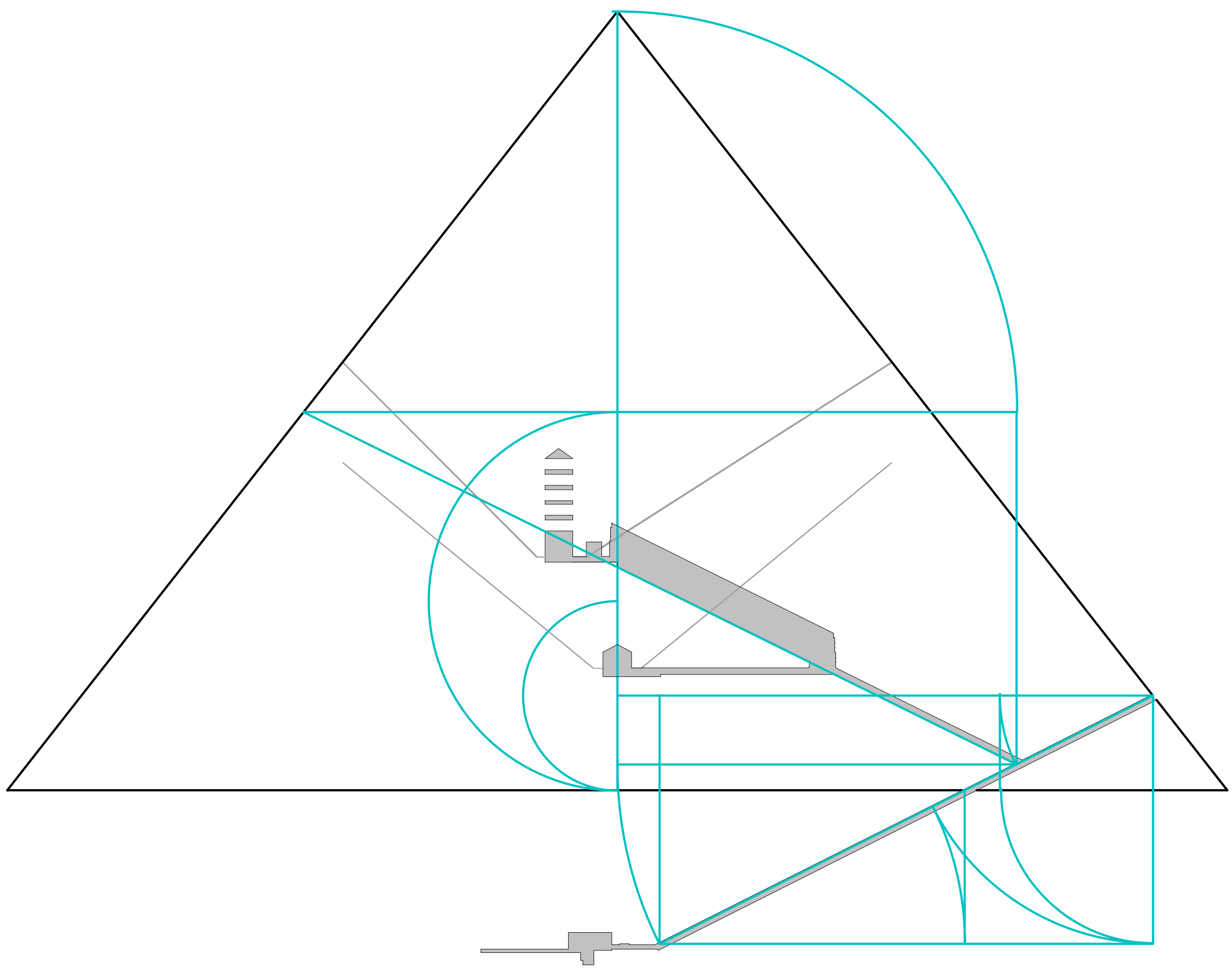
With an azimuth of +6° 11', the line from the Meidum pyramid to Adjib's tomb crosses directly over the step pyramid and crosses in between the summits of the red and bent pyramids (Google Earth image). Like the Giza pyramids and the Meidum pyramid, the Red and the Bent were cased in Tura limestone blocks that were quarried on the east side of the Nile. In *Egyptian Pyramid Geometry* (1998), Hadyn Butler gives the haulage distance of 1600 meters from the Nile for the red and the bent, as opposed to 400 meters from the Nile for Giza and 230 meters from the Nile for Meidum.



This is necessary for the alignment of the red and the bent with the Meidum pyramid, the step pyramid and Adjib's tomb, but the location of the red and the bent, so far from the Nile, is contrary to the conventional explanation that the pyramids were simply built at convenient locations on the west side of the Nile, unrelated to the locations of other pyramids. Also contrary to this conventional explanation is the location of the Meidum pyramid, so far south of Memphis and the other early Egyptian pyramids.

The latitude of Adjib's tomb is 29° 53' 07" and the latitude of the Meidum pyramid is 29° 23' 17", or 10" less than 30' of latitude. The latitude of the bent pyramid is 29° 47' 26" or 5' 41" south of Adjib's tomb. The distance from the Meidum pyramid to Adjib's tomb is 55,420 meters, or 55,420/.5238 = 105,800 royal cubits. The distance from Adjib's tomb to the bent pyramid is 20,000 cubits and the distance from the bent to the Meidum pyramid is 85,800 cubits.

106 *itr* is given in the ancient inscriptions for the length of Egypt from Elephantine to the northern limit of the delta and 20 *itr* is given for the length from the northern limit to the apex of the delta. Each *itr* contains 15,000 cubits. On a scale of 15 to one, the distance from Adjib's tomb to the bent pyramid is 20,000 cubits, and the distance from Adjib's tomb to the Meidum pyramid is within 200 cubits of 106,000 cubits. 106 itr is 7.5° from Elephantine to the northern limit, compared to 106,000 cubits for 30' of latitude, and the length of 20 itr is 1° 25', or 85', and 85/15 = 5.66', or 5' 41" for 20,000 cubits from Adjib's tomb to the bent pyramid.



Planning The Great Pyramid

Jim Alison - Updated - August 2022

The plan begins with a double square, divided into golden sections. The horizontal section of the height marks ground level. The scale is 55 cubits for the below ground section. The above ground section is $55/\varphi = 33.99$ cubits. The horizontal sides are $88.99 \times 2 = 177.98$ cubits. The diagonal is $88.99 \times \sqrt{5} = 198.99$ cubits. The below ground section is $198.99/\varphi = 122.98$ cubits. The above ground section is $122.98/\varphi = 76$ cubits. The upper end of the diagonal marks the outer casing on the north side of the pyramid, 33.99 cubits above ground level.

The diagonal is also divided into the proportion of φ^2 to one. The upper section is 55 cubits and the lower section is $55 \times \varphi^2 = 143.99$, giving $143.99 + 55 = 198.99$ cubits. This division of the diagonal, 55 cubits from the upper end, and 143.99 cubits from the lower end, marks the axis point. The length of the lower section of the diagonal is equal to the horizontal length of 143.99 cubits from the axis point to the vertical midline of the pyramid.

On the midline of the pyramid, the height of 33.99 cubits $\times$ 4 = 135.97 cubits above ground level. The length of 143.99 cubits, added to the height of 135.97 cubits, marks the apex of the pyramid, 279.96 cubits above ground level. *In The Pyramids and Temples of Gizeh* (1883), Petrie gives a height of 5776 inches ± 7.0 inches. 279.96 cubits $\times$ 20.62 inches per cubit = 5773 inches.

The rise over the run of the pyramid is the height on the midline, from 33.99 cubits above ground level to the apex, or $279.96 - 33.99 = 245.96$ cubits, over the horizontal length from the midline to the outer casing of the pyramid, 33.99 cubits above ground level. The horizontal length from the upper end of the diagonal, to a point directly above the axis point, is the diagonal length of 55 cubits, divided by $\sqrt{5}/2$, or $55/1.118 = 49.19$ cubits. The horizontal distance from the outer casing to the midline, 33.99 cubits above ground level, is $49.19 + 143.99 = 193.18$ cubits. The height of $245.96/193.18 = 1.2732$ for the rise over the run of the pyramid. This ratio is exactly four divided by the π coefficient of $\varphi^2 \times 6/5 = 3.14164$, or $4/3.14164 = 1.2732$.

Extending the line of the outer casing from the apex, through the upper end of the diagonal, to ground level, gives the length of $279.96/1.2732 = 219.88$ cubits for the half base, or 439.76 cubits for the base length. Petrie gives 9068.8 inches for the mean base length of the sides of the pyramid. The height of 439.76 cubits $\times$ 20.62 inches per cubit = 9068 inches.

A line from the axis point to the outer casing on the south side of the pyramid, 135.97 cubits above ground level, marks the floor of the ascending passage and the floor of the grand gallery. The height of the double square is 33.99 cubits above ground level and the length from the upper end of the diagonal to the axis point is 55 cubits. The height of the axis point below the height of the double square is $55/\sqrt{5} = 24.59$ cubits. The height of $33.99 - 24.59 = 9.4$ cubits for the height of the axis point above ground level. The height from the axis point to the outer casing, 135.97 cubits above ground level, is $135.97 - 9.4 = 126.57$ cubits.

From 135.97 cubits above ground level to the apex of the pyramid is 143.99 cubits. The run from the midline to the outer casing of the pyramid, 135.97 cubits above ground level, is 143.99/1.2732 = 113.09 cubits. The run from the axis point to the midline is 143.99 cubits. The horizontal run of the line from the axis point to the outer casing on the south side of the pyramid, 135.97 cubits above ground level, is 143.99 + 113.09 = 257.08 cubits.

The run over the rise of the line from the axis point to the outer casing on the south side of the pyramid, 135.97 cubits above ground level, is 257.08/126.57 = 2.031, and a run of 2.031 over a rise of one gives 26° 12' 47" for the angle of the floor of the ascending passage and the floor of the grand gallery. Petrie calculated 26° 12' 50" for "the mean angle of both passage and gallery."

The horizontal run of 143.99 cubits, from the axis point to the midline, divided by the horizontal run of 113.09 cubits from the midline to the outer casing, 135.97 cubits above ground level, is 143.99/113.09 = 1.2732.

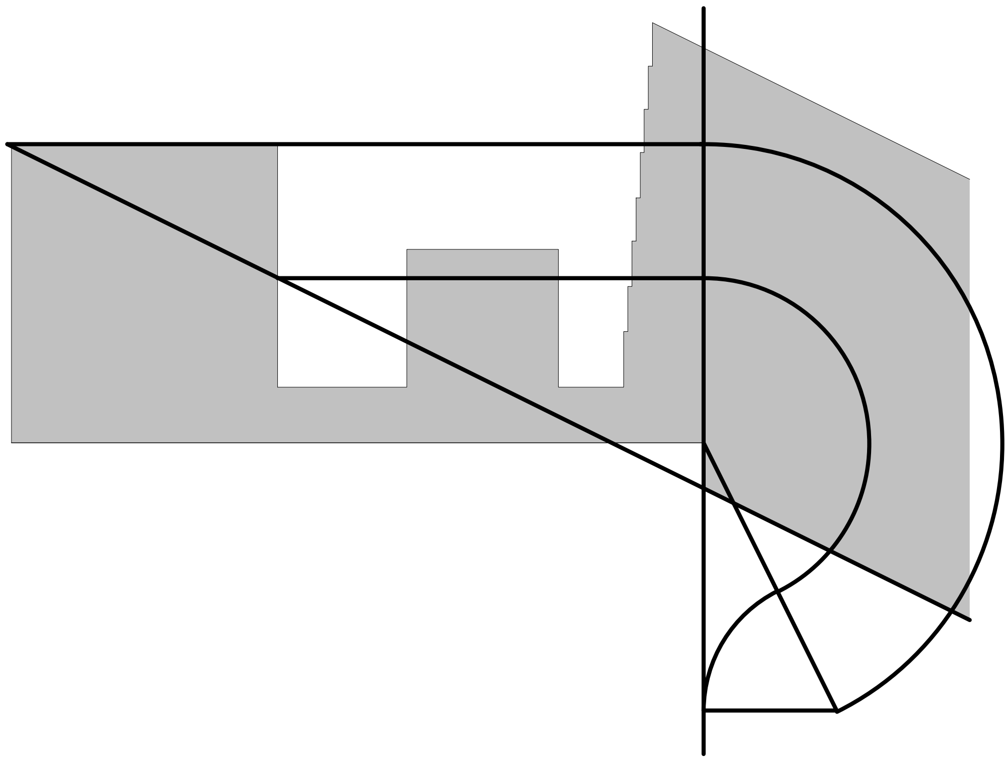
The height of 126.57 cubits from the axis point to the outer casing is also divided by the midline of the pyramid in the proportion of 1.2732 to one, or 126.57 × 1.2732/2.2732 = 70.89 cubits, giving 70.89 + 9.4 = 80.29 cubits above ground level for the floor of the grand gallery at the midline of the pyramid.

The height from the floor of the king's chamber to the apex of the pyramid equals the full height of the pyramid divided by the square root of two. The height of 279.96/$\sqrt{2}$ = 197.96 cubits and 279.96 minus 197.96 equals 82 cubits above ground level for the floor height. Petrie gives 1691.4 to 1693.7 ± .6 inches above ground level for the height of the floor of the king's chamber. The height of 82 cubits × 20.62 inches per cubit = 1691 inches.

Petrie states that "the face of the great step at the head of the gallery is .4 ± .8 inches south of the pyramid center. It may therefore be taken as intended that the face of this step, and the transition from sloping to horizontal surfaces, signalizes the transit from the northern to the southern half of the pyramid." Petrie also stated that "the top of the step itself, though straight, is far from level, the W. side being about 1.0 higher than the E. side. And the sloping floor seems to be also out of level by an equal amount in the opposite direction; since on the half width of the step (i.e., between the ramps) the height of the step face is 34.92 or 35.0 on E. And 35.80 or 35.85 on W."

Thirty five inches divided by 20.62 inches per cubit equals 1.7 cubits. Given the height of 82 cubits above ground level for the floor of the king's chamber passage, 82 − 1.7 = 80.3 cubits for the height of the floor of the grand gallery at the midline of the pyramid.

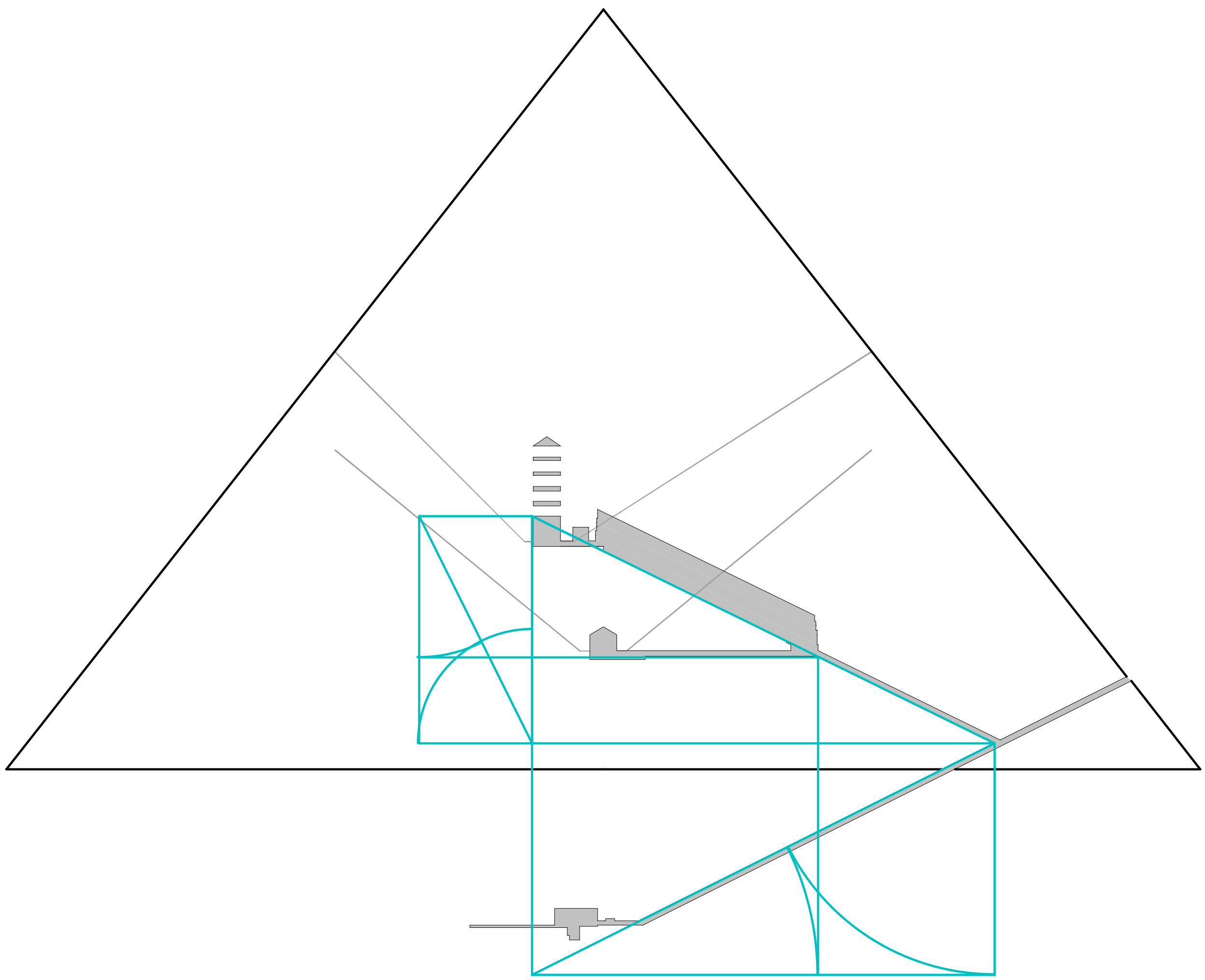
The floor plan of the king's chamber is 10 cubits by 20 cubits, giving 22.36 cubits for the diagonal of the floor. One half of the diagonal length, or 11.18 cubits, is the height from the floor to the ceiling of the king's chamber. The ceiling is 82 + 11.18 = 93.18 cubits above ground level. Petrie gives 1921.6 to 1923.7 ± .6 inches above ground level for the ceiling of the king's chamber. The height of 93.18 cubits × 20.62 inches per cubit = 1921 inches.



The height of 72 cubits above ground level is 10 cubits below the floor of the king's chamber and one-half of the length from the axis point to the midline of the pyramid. The length of the horizontal line at the bottom of the diagram is five cubits. The diagonal line at the bottom of the diagram is 11.18 cubits, the same as the height from the floor to the ceiling of the king's chamber.

Subtracting five cubits from the 11.18 cubit diagonal gives 6.18 cubits. The sloping floor of the gallery at the midline is 1.71 cubits below the top of the great step. The length of 6.18 cubits, plus 1.71 cubits, equals 7.89 cubits. The run over the rise of the sloping floor of the gallery is 2.031 and the height of 7.89 × 2.031 = 16.02. The Petrie gives 330.3 inches from the midline to the north wall of the king's chamber and 330.3 divided by 20.62 inches per cubit equals 16.02 cubits.

The south wall of the king's chamber is 10 cubits south of the north wall, or 26.02 cubits south of the midline. The rise from the sloping floor of the gallery at the midline, to the height of the ceiling of the king's chamber, is 11.18 + 1.71 = 12.89 cubits. The height of 12.89 cubits times 2.031 gives 26.18 cubits south of the midline for the intersection of the height of the ceiling of the king's chamber with the extension of the ascending floor of the gallery.



The axis point is 9.4 cubits above ground level. The height from the axis point to the ceiling of the king's chamber is $93.18 - 9.4 = 83.78$ cubits. Division of the height of 83.78 cubits into golden sections gives $83.78/\varphi = 51.78$ cubits, and $51.78/\varphi$, or $83.78/\varphi^2 = 32.00$ cubits for the height of the lower section. The height of 32 cubits, plus 9.4 cubits for the height of the axis point above ground level, equals 41.4 cubits above ground level.

The height of 41.4 cubits above ground level marks the junction of the floor of the ascending passage at the north wall of the grand gallery, the beginning of the undressed and unfinished floor of the horizontal passage to the queen's chamber, and the beginning of the sloping floor of the gallery. Petrie calculated 852.6 inches from ground level to the floor of the ascending passage at the north wall of the gallery and $852.6/41.4 = 20.6$ inches per cubit.

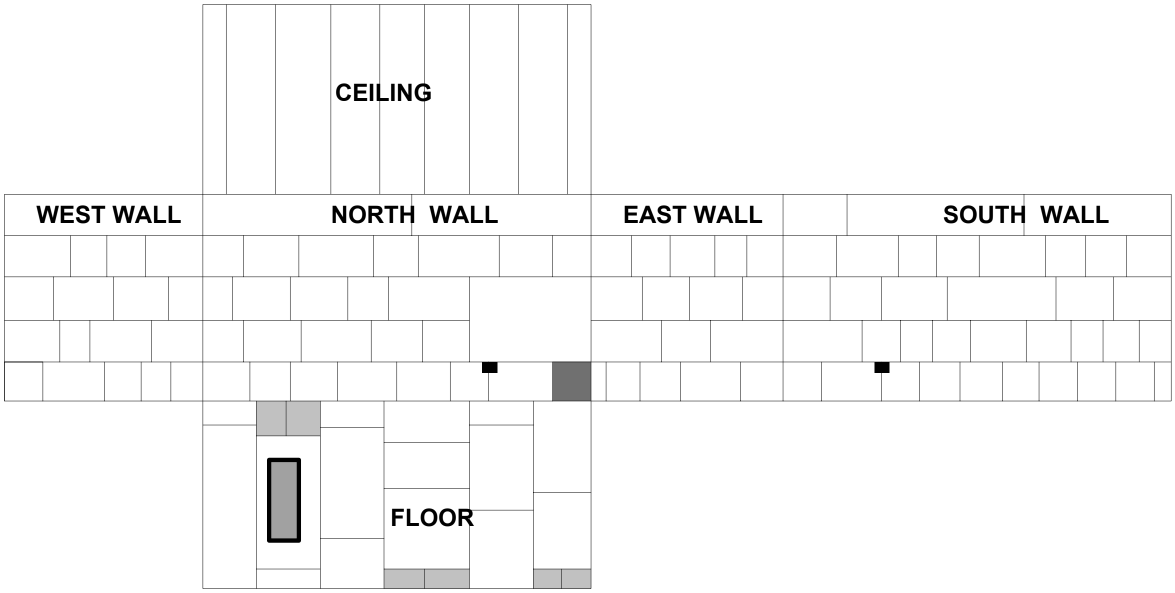
The diagonal from the outer casing to the axis point intersects the height of the king's chamber ceiling 26.18 cubits south of the midline. The horizontal length of the diagonal is 143.99 cubits from the axis point to the midline, plus 26.18 cubits from the midline to the point the diagonal intersects the king's chamber ceiling, or $143.99 + 26.18 = 170.17$ cubits.

Dividing 170.17 cubits into golden sections gives $170.17/\varphi = 105.17$ cubits for the horizontal length from 26.18 cubits south of the midline to the beginning of the sloping floor of the gallery, and $105.17/\varphi = 65.00$ cubits for the horizontal length from the axis point to the end of the floor of the ascending passage. The horizontal length from the axis point to the midline is 143.99 cubits, minus 65.00 cubits, equals 78.99 cubits. Petrie calculated 1627 inches from the junction of the floors of the ascending passage and the gallery to the midline, and $1627/78.99 = 20.6$ inches per cubit.

The run over the rise of the ascending passage from the axis point to the intersection of the floor of the ascending passage with the north wall of the gallery gives the run over the rise of the ascending passage and the sloping gallery in even numbers of cubits. The run of 65 cubits divided by the rise of 32 cubits equals 2.031.

The height from the north end of the gallery floor to the south end of the floor at the midline of the pyramid is $80.29 - 41.4 = 38.89$ cubits. The horizontal length from the north end of the floor to the midline is 78.99 cubits, and $38.89^2 + 78.99^2 = 88.0^2$, or 88 cubits for the sloping length of the floor of the gallery. Petrie gives 1815.5 inches for the distance on slope of the floor of the gallery, and $1815.5/88 = 20.62$ inches per cubit.

The floor blocks in the queen's chamber and the last 15 cubits of the queen's chamber passage are missing, and the floor blocks in the rest of the passage are rough and unfinished. The height above ground level of the unfinished floor blocks range from approximately two to six inches above the 41.4 cubit height of the end of the floor of the ascending passage. Petrie gives the height of the apex of the queen's chamber as 1078.7 inches, or 52.4 cubits above ground level and gives the top of the north and south walls of the chamber as three cubits below the apex. The floor plan of the queen's chamber is 10 cubits wide and 11 cubits deep. If the intended height of the floor of the chamber and the passage was also 41.4 cubits then the north and south walls would be 8 cubits high and the height from the floor to the apex of the chamber would be 11 cubits.



Every block in the ceiling of the king's chamber is cracked. There are damaged blocks in the south wall near the east side, and in the west wall near the south side. Several of the flooring blocks were removed and three of those have been replaced with six blocks (lightly shaded blocks in the diagram). In *Life and Work at the Great Pyramid* (1867), Piazzi Smyth reported that the south wall is "remarkable for fissures near east end, passing through several courses as they stand." Smyth also reported that the west wall "in two courses near its southern end, low down, are seriously chipped." In his diagram of the blocks in the king's chamber, Smyth shows an irregular line on the west wall, running from the top of the second course to below the top of the first course, approximately 15 inches from the south wall. Smyth counts this as a joint, giving 19 blocks in the west wall and a total of 100 blocks in all four walls. In *L'architettura Dele Piramidi Menfite* (1965), Maragioglio and Rinaldi regard this as a damaged single block, giving a total of 18 blocks in the west wall.

Originally there were 18 blocks in the floor, 9 blocks in the ceiling and 99 blocks in all four walls. The number of blocks in all six surfaces are multiples of nine: The east wall, west wall and floor each contain 18 blocks; the north wall contains 27 blocks; the south wall contains 36 blocks; and the ceiling contains 9 blocks. This is a total of 126 blocks in the chamber and 99 blocks in all four walls. The ratio between the total number of blocks in the chamber and the number of blocks in the walls is 126/99, or 14/11 or $4/\pi$, the same ratio found in the external slope of the pyramid.

The north wall of the king's chamber has 27 blocks and the first course of all four walls of the king's chamber also has 27 blocks. The entrance passage to the king's chamber is on the first course of the north wall. The south wall of the king's chamber has 36 blocks giving a total for the north and south walls of 63 blocks. The eye of Horus fractions are 1/2, 1/4, 1/8, 1/16, 1/32 and 1/64, adding up to 63/64, compared to 63/64 for the north and south walls of the king's chamber, counting the entrance passage as the missing 1/64.

Petrie gives the following measures in inches for the dimensions of the coffer:

	Outside	Inside
Length	89.62	78.06
Width	38.5	26.81
Height	41.31	34.42

These measures convert to ancient Egyptian Cubits as follows:

	Outside	Inside
Length	4.345	3.786
Width	1.867	1.301
Height	2	1.67

The diagonal of the inside base of the coffer is 4 cubits (3.7862 + 1.3012 = 42). This is twice the height of the coffer, just as the diagonal of the base of the King’s Chamber is equal to twice the 11.18 cubit height of the King’s Chamber ($20^2 + 10^2 = 22.36^2$).

A sphere with an area equal to the area of the inside base of the coffer has a volume equal to the volume of the inside of the coffer. The area of the inside base of the coffer is 4.925 square cubits (3.786 × 1.301). A circle with an area of 4.925 square cubits has a radius of 1.252 cubits. A sphere with a radius of 1.252 cubits has a volume of 8.224 cubic cubits. The inside volume of the coffer is 8.224 cubits (3.786 × 1.301 × 1.67 = 8.224).

A circle with a radius of φ (1.618 cubits) has a circumference of 10.17 cubits, equal to the perimeter of the inside base of the coffer (3.786 × 2) + (1.301 × 2) = 10.17. A circle with a radius of φ has an area of 8.224 square cubits, equal to the cubic measure of the inside volume of the coffer and a volume of 17.74 cubic cubits, approximating $\sqrt{\pi} \times 10$ (17.72).

The outside volume of the coffer is 16.224 cubic cubits (4.345 × 1.867 × 2 = 16.224). The inside volume of the coffer is 8.224 cubic cubits. The solid bulk of the coffer is 8 cubic cubits (16.224 − 8.224), equal to the bulk of a cube with side lengths of 2 cubits, the same as the height of the coffer.

A sphere with a diameter of 16.224 cubits has a volume of 2,236 cubic cubits, equal to the volume of the King’s Chamber. The radius of this sphere is 8.112 cubits, equal to the area of the outside base of the coffer. After a comment I wrote in 2004 regarding these relationships, Michael Saunders commented that a sphere with a diameter of π cubits has a volume of 16.23 cubic cubits, closely approximating the outside volume of the coffer.

In 1993, Rudolph Gantenbrink surveyed the distances and angles of the diagonal shafts that extend from the king's chamber and the queen's chamber. The shafts from the queen's chamber stop underneath the point where the shafts from the king's chamber exit the pyramid. Gantenbrink's survey of the shafts produced the following angular measurements:

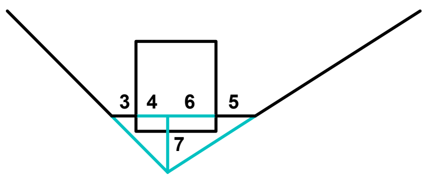
Upper southern shaft: 45˚

Upper northern shaft: 32˚ 36'

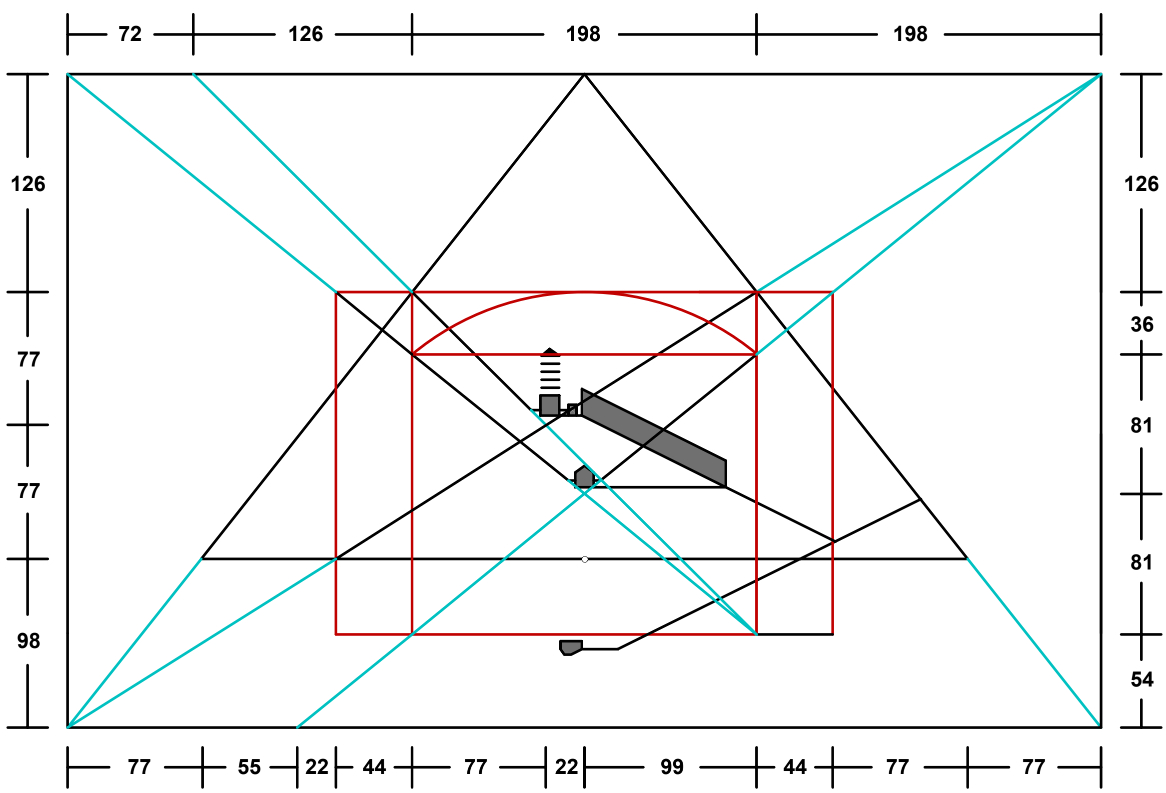
Lower southern shaft: 39˚ 36'

Lower northern shaft: 39˚ 7'

Gantenbrink concluded that the intended slope of the upper southern shaft was 1/1 (45˚) and the intended slope of the upper northern shaft was 7/11 (32˚ 28' 16"). This slope for the upper northern shaft is one half of the 14/11 slope of the exterior faces of the pyramid. The outer casing stones are missing where the upper shafts exit the pyramid, but Gantenbrink calculated that both of the king's chamber shafts exited the outer casing of the pyramid at the same height, 154 cubits above ground level. The roofs of all four shafts in the king's chamber and the queen's chamber are at joints between courses of the masonry. Petrie gave the top of the 104th course as 3175 inches above ground level and 3175/154 = 20.62 inches per cubit.



The north wall of the king's chamber is 16 cubits south of the vertical axis of the pyramid and the south wall is 26 cubits south of the vertical axis. The shaft outlets are two cubits above the floor of the chamber, or 84 cubits above ground level. The northern shaft runs horizontally for five cubits before beginning its ascent. The southern shaft runs horizontally for three cubits before beginning its ascent. Extended downward, the intersection point of the shafts is five cubits below the floor of the chamber, or 77 cubits above ground level. This height is also 77 cubits below the height the shafts exit the pyramid. The intersection point is 22 cubits south of the vertical axis of the pyramid. 22 cubits south of the vertical axis of the pyramid is 77 cubits north of the point the southern shaft exits the pyramid and 121 cubits south of the point the northern shaft exits the pyramid. The rise of 77 over the run of 121 gives the 7/11 slope of the northern shaft.



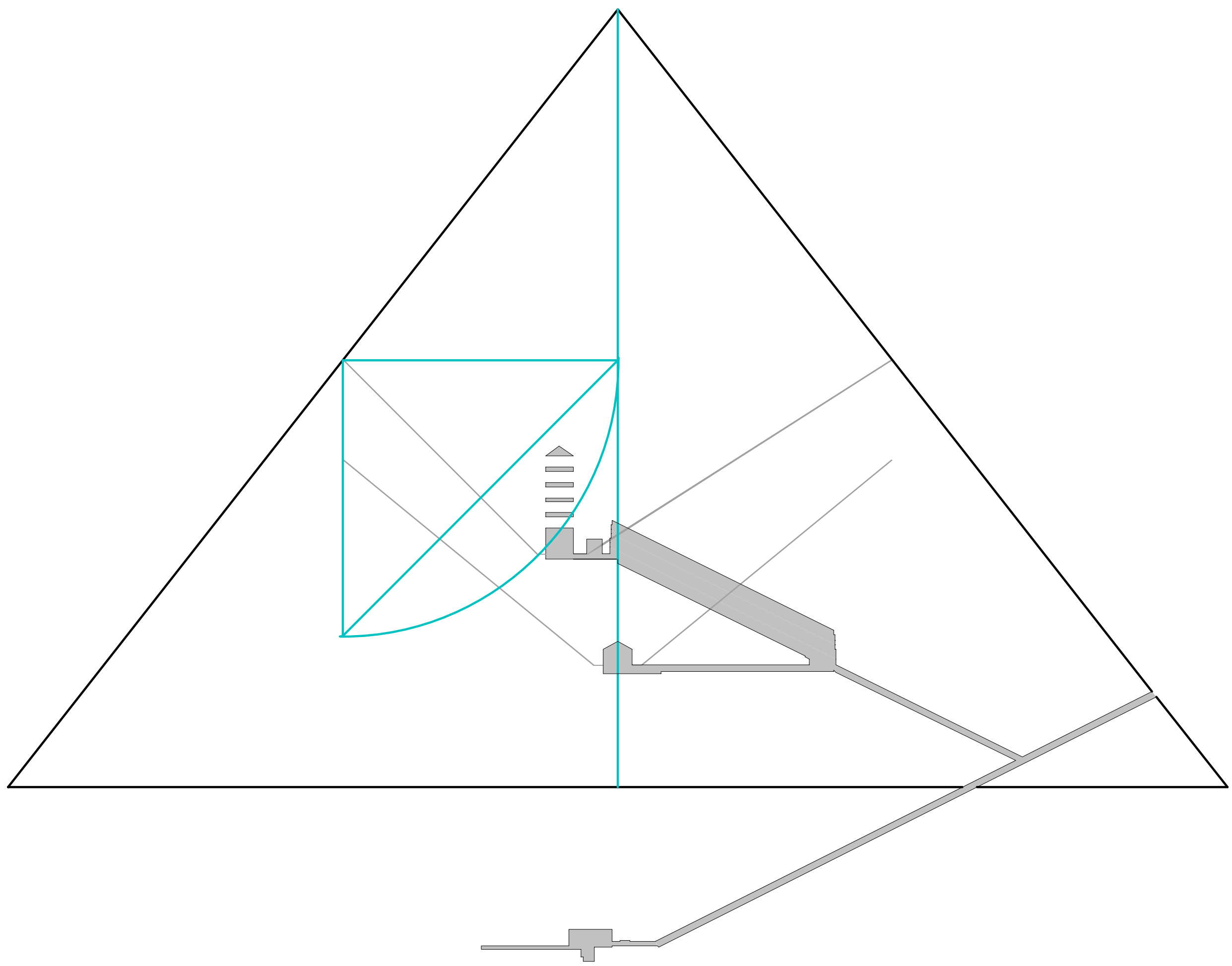
Extending the upper northern shaft beyond the face of the pyramid, it reaches the height of the apex of the pyramid 198 cubits north of its exit point (126/198 = 7/11). Extended downward, the upper northern shaft intersects ground level 77 cubits north of the southern edge of the pyramid. The upper northern shaft intersects the extended southern face of the pyramid 77 cubits south of the southern edge of the pyramid and 98 cubits below ground level, showing the slope of the pyramid (98/77 = 14/11) and the slope of the shaft (98/154 = 7/11). These points define a rectangle with a height equal to the height of the pyramid plus 98 cubits (below) and a width equal to the base length of the pyramid plus 77 cubits on each side. The 198 cubit square, centered just above the apex of the queen's chamber, has the same dimensions as the side lengths of the pyramid at the height of the exit points of the upper shafts. The upper southern shaft extended downward forms a diagonal from the upper southern corner to the lower northern corner of the square. The length of this diagonal is 280 cubits, equal to the height of the pyramid.

Unlike the upper shafts, the lower shafts do not exit the pyramid. Petrie stated that "They are exactly like the air channels in the King's Chamber in their appearance, but were covered over the mouth by a plate of stone, left not cut through in the Queen's Chamber walls, until they were discovered by Waymon Dixon." The shaft outlets in the Queen's Chamber are 44 cubits above ground level. The chamber is 10 cubits wide and the shafts are horizontal for 3.75 cubits before beginning their angles of ascent.

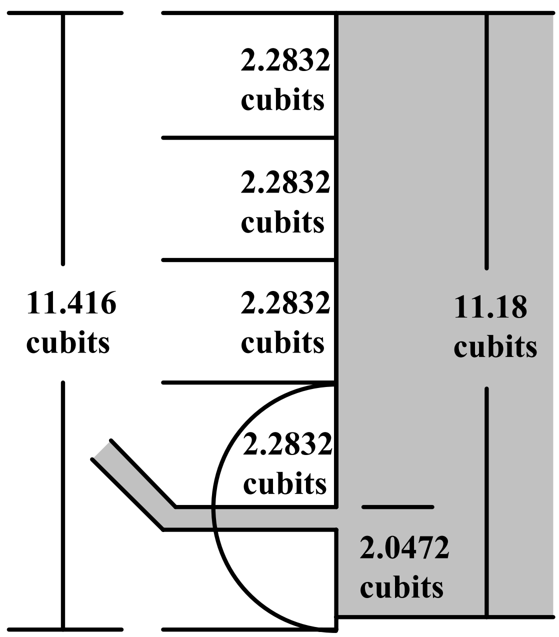
The two lower shafts (extended downward) intersect at the midline of the pyramid, 37 cubits above ground level and 81 cubits above the lower border of the square. This point is also 81 cubits below the point where the lower shafts intersect the sides of the square. The sides of the square are 99 cubits across from the intersection point of the lower shafts (81/99 = 9/11). Both of the lower shafts intersect the sides of the square 36 cubits below the upper shaft exit points and both of the lower shafts (extended upward) cross over the height of the upper shaft exit points 44 cubits across from the exit points (36/44 =9/11).

The lower northern shaft, intersects the square 36 cubits below the exit point of the upper northern shaft and 162 cubits below the apex of the pyramid. Extended upward, the lower northern shaft intersects the upper northern shaft at the height of the apex of the pyramid, 198 cubits north of the exit point of the upper northern shaft (162/198 = 9/11). The lower southern shaft intersects the square 36 cubits below the exit point of the upper southern shaft and as extended downward, intersects the upper southern shaft at the lower northern corner of the square, 198 cubits north of the exit point of the upper southern shaft (162/198 = 9/11).

The point on the midline of the pyramid at ground level is the origin point of the arc in the diagram. The height of the upper shaft exit points is 154 cubits above ground level. The points where the lower shafts cross directly under the upper shaft exit points are also 154 cubits from the midline of the pyramid at ground level. The distance of these diagonals may be calculated as the hypotenuse of the right triangle formed by the height above ground level where the lower shafts cross under the upper shaft exit points (118 cubits) and the distance across from this point to the centerline of the pyramid (99 cubits): $118^2 + 99^2 = 154^2$.



The height of the roof of the king's chamber shaft at the outer casing is 154 cubits above ground level. The height of the apex is $(4/\varphi + \varphi^2) \times 55 = 279.9593$ cubits. The height from the roof of the shaft at the outer casing to the to the apex is 125.9593 cubits (279.9593 − 154). The horizontal distance from the roof of the shaft at the outer casing to the vertical midline of the pyramid is 98.9282 cubits (125.9593 divided by $4/\pi$). This horizontal distance is arced to a point vertically below the roof of the shaft at the outer casing. The midpoint of this arc marks the end of the horizontal portion and the beginning of the ascending portion of the roof of the shaft. The length of the ascending portion of the roof is also 98.9282 cubits, and the height of the ascending portion is $98.9282/\sqrt{2} = 69.9582$ cubits.



The roof of the horizontal portion of the shaft is 154 − 69.9582 = 84.0472 cubits above ground level. The roof of the shaft marks the joint between the first and second courses of masonry in the walls of the king's chamber. The height of the floor of the king's chamber is 82 cubits above ground level. The height of the ceiling of the shaft is 84.0472 − 82 = 2.0472 cubits above the floor. The height from the floor to the ceiling of the king's chamber is 11.18 cubits ($\sqrt{5} \times 5$). The height from the roof of the shaft to the roof of the chamber is 11.18 − 2.0472 = 9.1328 cubits. The height of each of the four courses of the walls of the chamber from the roof of the shaft to the roof of the chamber is 9.1328/4 = 2.2832 cubits, giving a wall height for all five courses of 11.416 cubits (2.2832 × 5). The perimeter of the long walls of the chamber are 62.832 cubits (22.832 cubits for the height of all five courses on both sides, plus 40 cubits for the length of the floor and the roof). The perimeter divided by the floor equals π (62.832/20 = 3.1416).

In The Pyramids and Temples of Gizeh (1883), Petrie states:

The perpendicular height of the entrance passage is “the same as the very carefully wrought courses of the King's Chamber, with which the passage is clearly intended to be identical.” (p. 51) “Any theory of the height of the entrance passage can not be separated from the similar passages, or from the most accurately wrought of all such heights, the course height of the King's Chamber.” (p. 189)

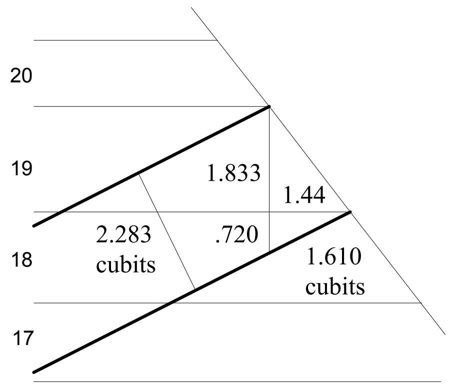
“The angle of slope of the entrance passage is 1 rise on 2 base.” (p. 221) “The old theory of 1 rise on 2 base, or an angle of 26º 33' 54"; is far within the variations of the entrance passage angle, and is very close to the observed angle of the whole passage, which is 26º 31' 23"; so close to it, that two or three inches on the length of 350 feet is the whole difference; so this theory may at least claim to be far more accurate than any other theory.” (p. 191)

“The entrance passage has a flat end, square with its axis, and out of this end a smaller horizontal passage proceeds, leaving a margin of the flat end along the top and two sides.” (p. 59) The length of the end of the passage is the same as the perpendicular height of the passage.

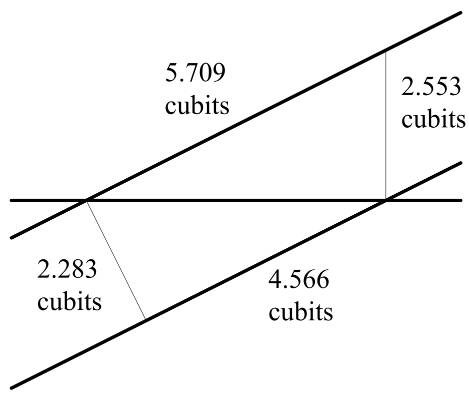
“From the entrances of the Third pyramid, the south pyramid of Dahshur, and the pyramid of Medum all of which retain their casing, there seemed scarcely a question bu that the rule was for the doorway of a pyramid to occupy the height of exactly one or two courses on the outside. That the casing courses were on the same levels as the present core courses is not to be doubted, as they are so in the other pyramids which retain their casing, and at the foot of the Great Pyramid.” (p. 51)

The entrance passage exited the finished outer casing of the great pyramid at the 19th course of masonry. (p. 52) The bottom of the 19th course is 668.2" above ground level (p. 55) and the top of the 19th course is 706" above ground level (p. 52). “The total original length of the entrance passage floor is 4143 inches and the length of the roof is 4133 inches.” (p. 188)

The height of the 19th course of masonry is 706 - 668.2 = 37.8 inches. Given a royal cubit of 20.62 inches, 37.8/20.62 = 1.833, or a vertical length of 1.833 cubits from the roof of the passage at the outer casing to the height of the floor at the outer casing. Given a slope of $4/\pi$, or 1.2732, for the outer casing, the vertical length of 1.833/1.2732 = 1.44 cubits for the horizontal distance from the point directly below the roof of the passage at the outer casing, to the floor of the passage at the outer casing. The vertical distance from the height of the floor at the outer casing to the height of the floor directly below the roof at the outer casing is .720 cubits. The length of the floor from the outer casing to a point directly below the roof at the outer casing is 1.61 cubits.



The vertical height of 1.833 + .720 = 2.553 cubits for the vertical height from the floor to the roof of the passage. Given the one over two slope of the passage, the perpendicular height of the passage equals the vertical height of 2.883 × 2/√5 = 2.2832 cubits, or 2π − 4 cubits, or the height of the courses of the walls of the king's chamber.



The lower end of the passage has the same angle as the perpendicular height of the passage. Given the one over two slope of the passage, the section of the floor that is below ground level is 4.566 cubits longer than the section of the roof that is below ground level (2.283 × 2 = 4.566).

The floor continues for 1.61 cubits to the outer casing from the point on the floor vertically beneath the roof at the outer casing. The roof continues for 5.709 cubits to ground level from the point on the roof vertically above the floor at ground level (2.553 × √5 = 5.709). The section of the roof that is above ground level is 4.099 cubits longer than the section of the floor above ground level (5.709 − 1.61 = 4.099).

In 2007, I commented that 706" × √5 = 1578.6" is the sloping length of the roof from the outer casing to ground level and 1578.6" × φ^2 = 4132.99" for the length of the roof from the outer casing to the lower end of the passage, showing ground level divides the roof in the φ proportion. In response, Michael Saunders commented that 668.2" × √5 = 1494.1", that the sloping length of 1494.1" × √π = 2648.3", and that 1494.1" + 2648.3" = 4142.4", showing ground level divides the floor in the √π proportion.

Given the perpendicular height of the passage of 2.283 cubits, the perpendicular lower end, the one over two slope of the passage and the $4/\pi$ slope of the pyramid, an equation will provide the beginning heights above ground level and the total lengths of the floor and the roof that are required for both the φ and $\sqrt{\pi}$ proportions to be present. If the above ground length of the floor is $\boldsymbol{x}$, then the below ground length of the floor is $\sqrt{\pi}\,\boldsymbol{x}$.

The above ground length of the roof is $\boldsymbol{x}$ plus 4.099 cubits. The below ground length of the roof is $\varphi\,\boldsymbol{x}$ plus $\varphi \times 4.099$ cubits. $\varphi \times 4.099 = 6.632$, giving $\varphi\,\boldsymbol{x}$ plus 6.632 cubits for the below ground length of the roof. The below ground length of the roof is 4.566 cubits shorter than the below ground length of the floor. The below ground length of the floor ($\sqrt{\pi}\,\boldsymbol{x}$) equals the below ground length of the roof plus 4.566 cubits:

$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 6.632$ cubits + 4.566 cubits

$\sqrt{\pi}\,\boldsymbol{x} = \varphi\,\boldsymbol{x} + 11.198$ cubits

$$\sqrt{\pi}\,\boldsymbol{x} = 1.7724\,\boldsymbol{x}$$

$$\varphi\,\boldsymbol{x} = 1.6180\,\boldsymbol{x}$$

$1.7724\,\boldsymbol{x} = 1.6180\,\boldsymbol{x} + 11.198$ cubits

$1.7724\,\boldsymbol{x} - 1.6180\,\boldsymbol{x} = 11.198$ cubits

$$1.7724 - 1.6180 = .1544$$

$.1544\,\boldsymbol{x} = 11.198$ cubits

$\boldsymbol{x} = 11.198$ cubits$/.1544$

$\boldsymbol{x} = 72.516$ (the above ground length of the floor of the passage).

$72.516 \times \sqrt{\pi} = 128.531$ cubits (below ground length of the floor).

$72.516 + 128.531 = 201.041$ cubits (total length of the floor).

$72.516/\sqrt{5} =$ **32.43** cubits (height of the floor at the outer casing of the pyramid).

$72.516 + 4.099 = 76.615$ (above ground length of the roof of the passage).

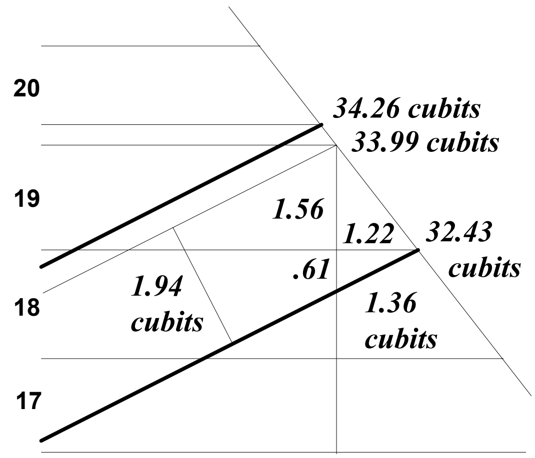
$76.615 \times \varphi = 123.966$ (below ground length of the roof).

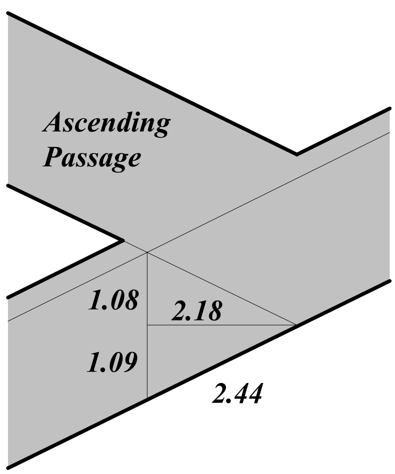
$76.615 + 123.966 = 200.482$ cubits (total length of the roof).

$76.615/\sqrt{5} =$ **34.26** cubits (height of the roof at the outer casing of the pyramid).

The height above ground level of the beginning of the roof of is fixed by the perpendicular height of the passage and the φ and √π ratios of the roof and the floor. The beginning of the diagonal of the double square is .27 cubits below the roof at the outer casing of the pyramid.

The floor of the passage at the outer casing is 32.43 cubits above ground level and the diagonal of the double square is 33.99 cubits above ground level at the casing. The vertical height from the floor at the outer casing to the diagonal at the casing is 1.56 cubits. The horizontal distance from the floor at the casing to a point vertically below the diagonal at the casing is 1.22 cubits. The length of the floor of the passage from the casing to the point vertically below the diagonal at the casing is 1.36 cubits. The height from the diagonal to the floor is 2.17 cubits. The perpendicular height from the diagonal to the floor is 1.94 cubits.





The floor of the ascending passage is projected to the floor of the entrance passage. The slope of the ascending passage is 1/2.03. The sides of the right triangle formed by the diagonal of the double square and the projection of the floor of the ascending passage have the same 1/2.03 ratio. The slope of the entrance passage is one over two. The vertical side of the right triangle formed by the floor of the entrance passage is ½ the length of the horizontal side, or 1.015 times the vertical side of the right triangle formed by the projection of the ascending passage floor. The vertical height from the diagonal of the double square to the floor of the entrance passage is 2.17 cubits. 2.17/2.015 = 1.08 and 2.17 ×1.015/2.015 = 1.09. The length of the floor of the entrance passage from the intersection point of the floors to the point vertically below the intersection of the ascending passage and the diagonal is 2.44 cubits (1.09 × √5).

The length of the diagonal of the double square from the outer casing to the intersection with the projection of the floor of the ascending passage is 55 cubits. The length along the floor of the entrance passage from the outer casing to the intersection of the floor of the entrance passage with the projection of the floor of the ascending passage is 53.92 cubits (55 + 1.36 − 2.44 = 53.92). Petrie calculated 1111 inches along the floor of the entrance passage from the outer casing to the intersection of the floor of the entrance passage with the projection of the floor of the ascending passage (53.92 cubits × 20.62 inches per cubit equals 1112 inches).

In *The Pyramids and Temples of Gizah*, Petrie gave the following base lengths, azimuths, angles, heights and relative locations of the Giza pyramids

	First pyramid:		Second pyramid:		Third pyramid:	
	Length	Azimuth	Length	Azimuth	Length	Azimuth
N	9069.4"	−3' 20"	8471.9"	−5' 31"	–––––	+16' 48"
E	9067.7	−3' 57"	8475.2	−6' 13"	4149.2"	+12' 23"
S	9069.5	−3' 41"	8476.9	−5' 40"	4157.8	+12' 57"
W	9068.6	−3' 54"	8475.5	−4' 21"	4153.9	–––––
Mean	9068.8"	−3' 43"	8474.9"	−5' 26"	4153.6"	+14' 3"
	440 cubits		411 cubits		201.5 cubits	

First pyramid angle:		Second pyramid:		Third pyramid:	
casing:	51° 46' 45"	casing:	53° 12'	granite:	50° 57'
top and base:	51° 49'	plumb line:	53° 2'	loose blocks:	51° 00'
north face:	51° 53' 20"	loose blocks:	53° 14'	limestone:	51° 58'
Conclusion:	51° 52'	Conclusion:	53° 10'	Conclusion:	51° 10' 30"

Height (calculated from base length and angle):

5776"	280 cubits	5664"	274 cubits	2580"	125 cubits

Distances from the centers of the pyramids:

Inches:	NS Distance:	EW Distance:	Total Distance and Angle West of Due South:
First to Second	13,931.6	13,165.8	19,168.4 at 43° 22' 52"
First to Third	29,102.0	22,616.0	36,857.7 at 37° 51' 6"
Second to Third	15,170.4	9,450.2	17,873.2 at 31° 55' 12"

Cubits:			
First to Second	675.5	638.5	929.5
First to Third	1,412	1,097	1,788
Second to Third	736.5	458.5	867.5

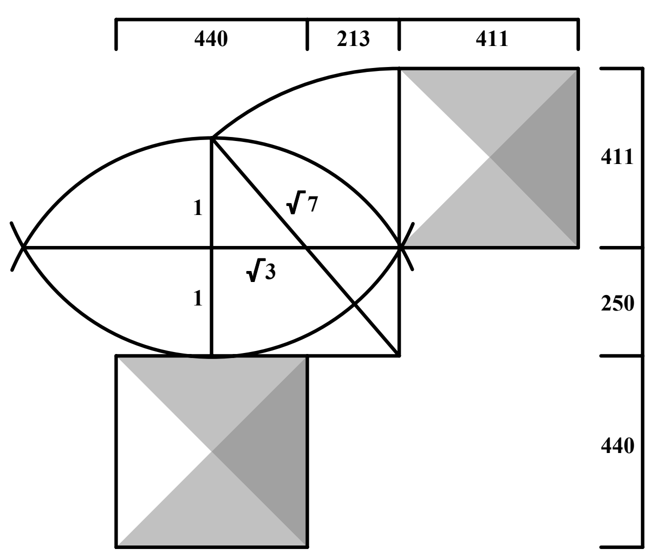
Orientations of the Giza Pyramids

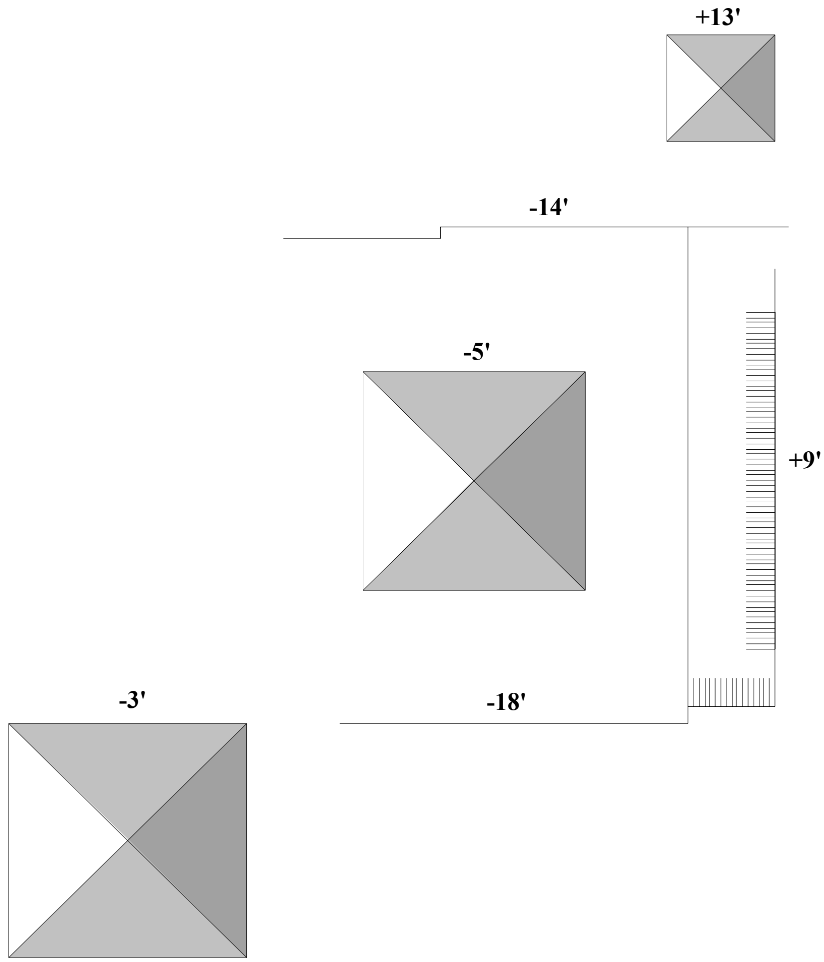
Jim Alison - September 2022

In addition to the azimuths of the sides of the pyramids, Petrie surveyed the azimuths of the entrance passages, and noted Smyth's survey of the azimuths of the entrance passages. Smyth gave an azimuth of −5' 49" for the great pyramid entrance passage and −5' 37" for the second pyramid entrance passage. Petrie gave −5' 49" for the built part and −3' 44" for the whole length of the great pyramid entrance passage, and +13' 16" for the third pyramid entrance passage.

Petrie believed that all three of the pyramids were intended to be oriented with the cardinal directions, but he measured the NS and EW distances between the centers of all three pyramids on parallels inclined −5' to true North, which he regarded as the mean azimuth of the first and second pyramids. He stated that "the third and lesser pyramids are so inferior in work, that they ought not to interfere with the determination from the accurate remains."

The NS distance from the center of the first pyramid to the center of the second pyramid is 675.5 cubits. Subtracting the half base of both pyramids gives 250 cubits from the south side of the first pyramid to the north side of the second pyramid (675.5 −220 −205.5 = 250). The EW distance is 638.5 cubits. Subtracting the half base of both pyramids gives 213 cubits from the West side of the first pyramid to the east side of the second pyramid (638.5 −220 −205.5 = 213). John Legon pointed out that the distance from the EW midline of the first pyramid to the east side of the second pyramid is equal to 250 times the square root of three ($250 \times \sqrt{3} = 433$) and the distance from the south side of the first pyramid to the south side of the second pyramid is equal to 250 times the square root of seven ($250 \times \sqrt{7} = 661$). In the diagram below, one equals 250 cubits. The 500 cubit segment of 1 + 1 = 2, or $\sqrt{4}$, produces the right triangle $\sqrt{3}^2 + \sqrt{4}^2 = \sqrt{7}^2$.



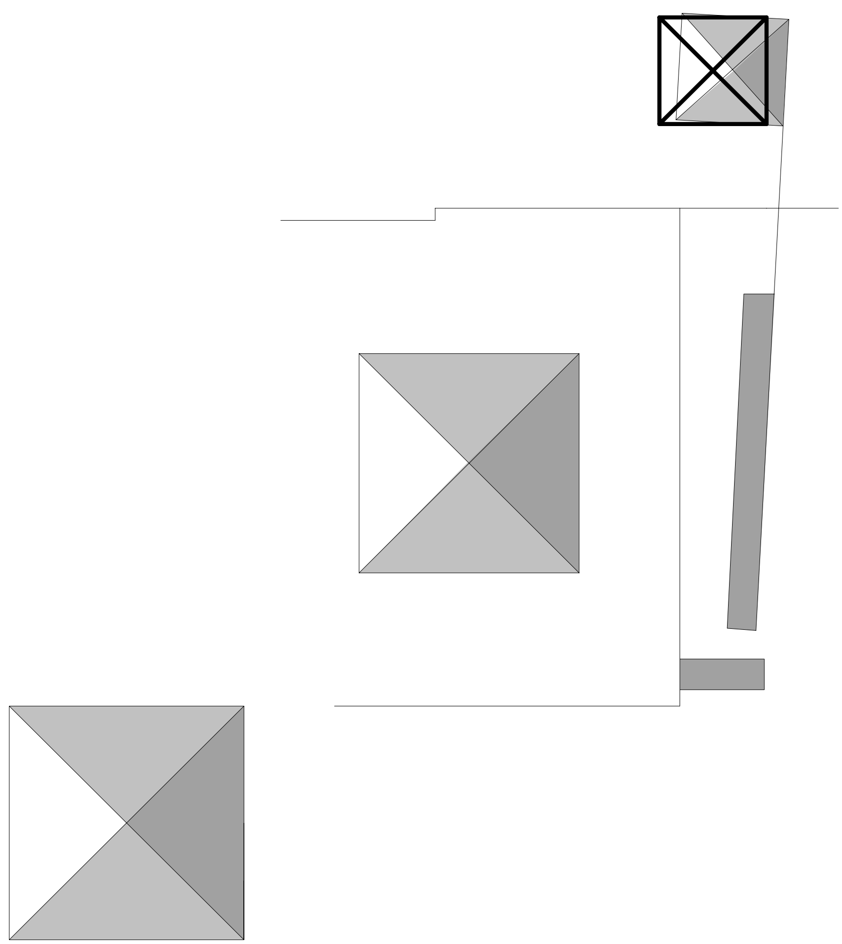


Petrie used an orientation of − 5' to survey the NS and EW distances between the centers and by extension the sides and corners of the pyramids. If the relative locations of the pyramids were planned, this would have also required an orientation for surveying the NS and EW distances between the pyramids. The almost identical deviation from cardinality of the first and second pyramids suggests that one determination of cardinality was used to orient both pyramids and to survey the NS and EW distance between the two. Given their virtually identical azimuths, even if the first two pyramids were oriented separately, and one or the other of their orientations was used to survey the NS and EW distance between the two, the results would be virtually identical.

Petrie surveyed the low walls around the north, south and west sides of the second pyramid, as well as the workers barracks. The barracks were divided into 91 long galleries and Petrie estimated they could have housed up to 4,000 workers. Petrie found the azimuth of the W. wall of the long barracks oriented clockwise +9'. He also commented that “the wall at the head of the galleries, if prolonged, would pass but 29 inches within the W. side of the third pyramid and therefore those seem to be intended for the same line.”

The 29 inch discrepancy between the west wall of the barracks and the west side of the pyramid is based on Petrie's survey orientation of − 5'. Petrie gives 6,567 inches NS from SW corner of the workers barracks to the N. Side of the third pyramid. If the orientation of the W. wall of the barracks was extended to mark the NW corner of the pyramid, the +9' azimuth of the barracks varies from Petrie's survey by 14' clockwise. 14' = .2333° and the sine of .2333° is .004072... A long side of 6567" × .004072 = 26.74" for the short side of the right triangle. The long side has Petrie's − 5' orientation from the SW corner of the barracks to the N. side of the third pyramid and the hypotenuse has the +9' orientation of the barracks from the SW corner of the barracks to the N. side of the third pyramid, less than three inches from the NW corner of the third pyramid. This suggests that the orientation used to locate and orient the third pyramid was the clockwise orientation of the barracks, rather than the counterclockwise orientation of the first two pyramids.

In the diagram below, the clockwise orientation of the workers barracks and the third pyramid, in relation to the counterclockwise orientation of the first and second pyramids, is 140', or 2°20', or 10 times the 14' deviation reported by Petrie, showing that the deviation results in an insignificant deviation from the intended N. and S. sides of the third pyramid, but a significant deviation in the E. and W. sides.



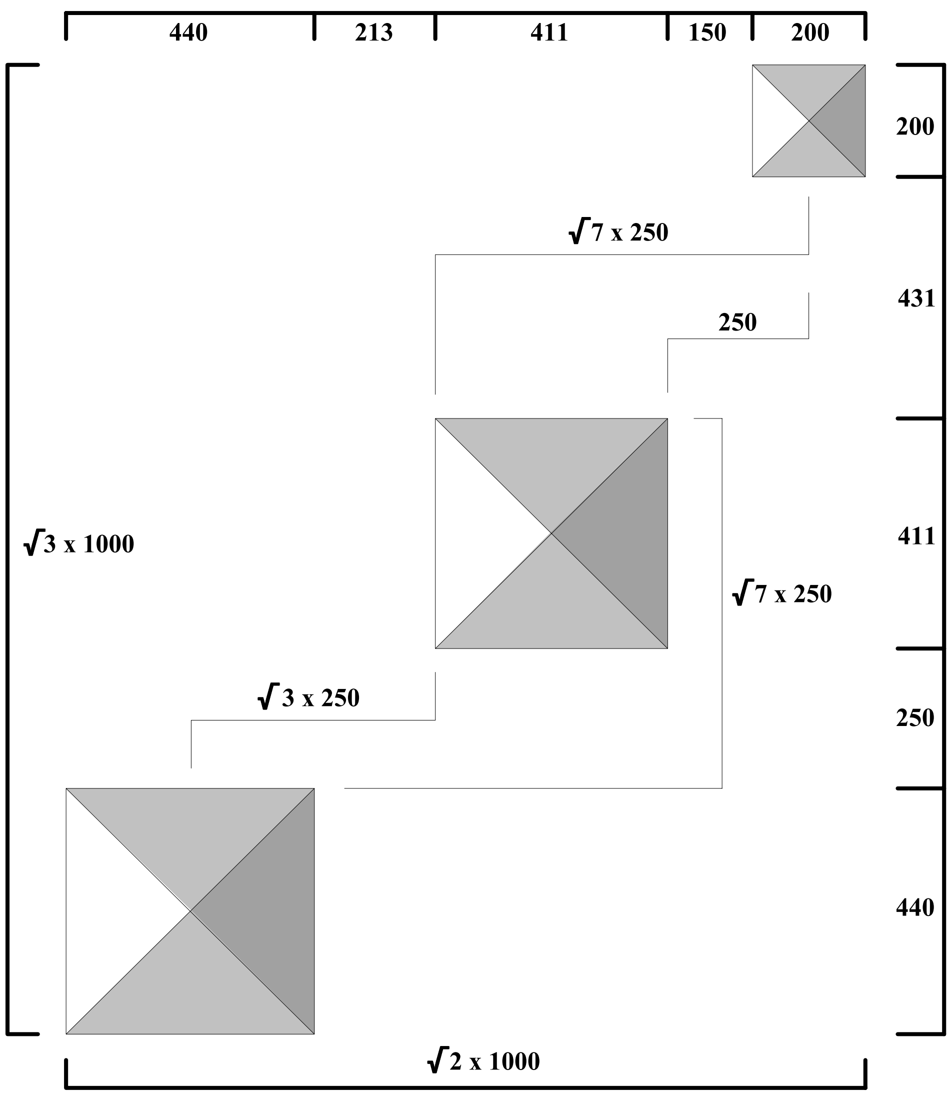
Based on the orientation of the first two pyramids, the S. side of the barracks is further west than the N. side of the barracks. The NS distance given by Petrie between the apexes of the second and third pyramids is 15,170.4 inches. If the EW distance from the second pyramid to the W. wall of the barracks was determined by a survey from the apex of the second pyramid, the 14' deviation in the orientation of the barracks gives 15,170.4 × .004072 = 62 inches west of the intended location of the apex of the third pyramid, due to the deviation in the NS orientation of the barracks. Based on Petrie's orientation for his survey of the relative positions of all three pyramids, the NS distance from the N. side of the first pyramid to the S. side of the third pyramid is 1732 cubits, and the EW distance from the E. side of the first pyramid to the W. side of the third pyramid is 1417 cubits. The W. wall of the barracks at the NS midpoint of the second pyramid is 1414 cubits from the E. side of the first pyramid. If the orientation of the barracks and the third pyramid was intended to be the same as the first and second pyramids, this suggests 1414 cubits was the intended EW distance from the E. side of the first pyramid to the W. side of the third pyramid.

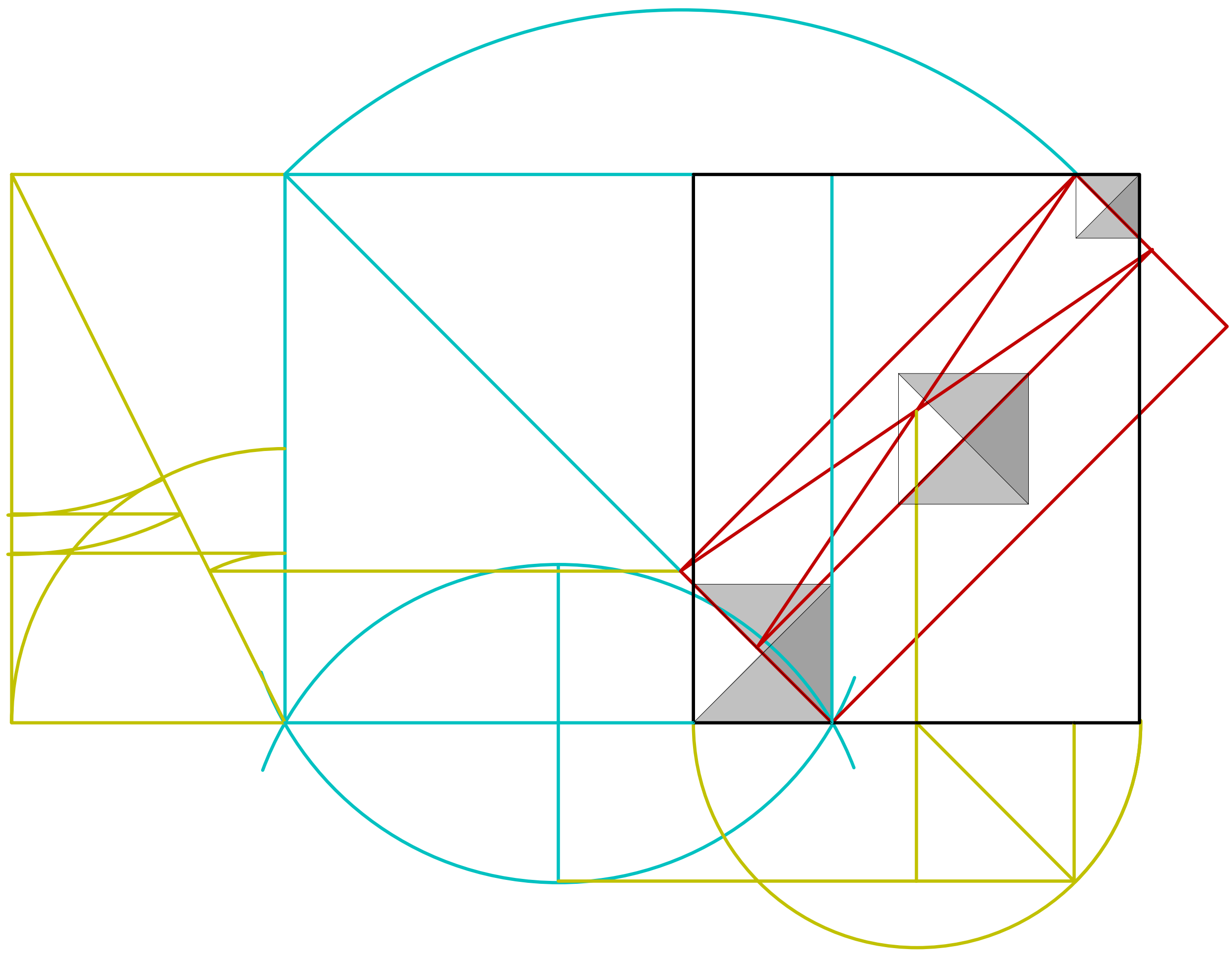
The lower part of the third pyramid was cased in granite blocks that were never dressed. The casing blocks on the first course indicate that they were intended to receive paving stones, rising above the base of the first course, with the top of the paving stones marking ground level, but the first course could not receive paving stones without first being dressed, and there are no paving stones around the third pyramid. The upper courses of the third pyramid were cased in dressed and polished limestone blocks, but no limestone casing blocks remain on the pyramid. The third pyramid was surrounded by rubble that Petrie excavated on both sides of the NE, SE, and SW corners, but he was not able to reach the NW corner.

In 1982 Gay Robins and Charles Shute surveyed the face angles of the Giza pyramids. They confirmed Petrie's findings for the first and second pyramid, but found 14/11 or 51° 52' for the face angle of the third pyramid, the same as the first pyramid and close to Petrie's measures for the fragments of the limestone casing blocks recovered from the rubble around the third pyramid. In 1997, Miroslav Verner gave Petrie's findings for the base lengths of the first and second pyramids, but reported base lengths of 104.6 meters, or 4118 inches for the third pyramid, citing Maragioglio and Rinaldi. Petrie commented that "it seems most probable that the third pyramid was designed to be 200 cubits long." Given the base lengths of 4118 reported by Margioglio and Rinaldi, 4118/20.6 inches = 199.9 cubits.

Petrie gives precise lengths for three sides of the third pyramid but not direct measurements. His calculations are from distances measured by triangulation from excavated points at the base of the first course and from points on higher courses; from an estimate of the height of the missing paving stones; from estimates of the length the granite blocks would have been reduced by dressing; and from his calculation of the angle of the pyramid. Petrie reported a weighted angle of 51° 10', concluded that the designed angle was 51° 20', for a slope of 5/4, but used 51° 0' to calculate the base lengths, producing a longer base length than the steeper angle reported by Robins and Shute, and longer than the base length reported by Margioglio and Rinaldi.

Given 1732 cubits from the N. side of the first pyramid to the S. side of the third pyramid and 1414 cubits from the E. side of the first pyramid to the W. side of the third pyramid, and if the base length of the third pyramid is 200 cubits, then the EW distance between the EW midline of the third pyramid and the W. side of the second pyramid is the same as the NS distance between the S. side of the first pyramid and the N. side of the second pyramid, and the EW distance between the EW midline of the third pyramid and the E. side of the second pyramid is the same as the NS distance between the S. side of the first pyramid and the S. side of the second pyramid.





The short axis of the vesica piscis is 1000 cubits. The long axis is 1732 cubits, or $\sqrt{3} \times 1000$. The right end of the long axis marks the NW corner of the first pyramid. The long axis is squared. The diagonal of the square is $\sqrt{3}^2 + \sqrt{3}^2 = \sqrt{6}^2$, or $\sqrt{6} \times 1000$, or 2449 cubits. The diagonal is divided into the proportion of φ^2 to one, or 1772 cubits for the long side and 677 cubits for the short side: $677 \times \varphi^2 = 1772$ and $677 + 1772 = 2449$.

The long side of 1772 cubits is $\sqrt{\pi} \times 1000$. The long side is arced 90°, marking the SE corner of the third pyramid, 1732 cubits south of the N. side of the first pyramid. The diagonal rectangle is defined by the NW corner of the first pyramid and the SE corner of the third pyramid. The long sides are 1772, or $\sqrt{\pi} \times 1000$ cubits, and the short sides are 677, or $\sqrt{\pi/\varphi^2} \times 1000$ cubits.

The distance from the E. side of the first pyramid to the end of the short diagonal is equal to the distance from the W. side of the third pyramid to the midpoint of the short diagonal. The EW midpoint from the E. side of the first pyramid to the W. side of the third pyramid is marked by the midpoint of a line from the midpoint of the short diagonal that crosses over the third pyramid, to the upper end of the short diagonal that crosses over the first pyramid. The midpoint between the E. side of the first pyramid and the W. side of the third pyramid is extended vertically and one half of the short axis of the vesica piscis is extended horizontally to this vertical line and squared. The diagonal of this square is equal to $\sqrt{2}$ x 500 cubits, and this length is arced to mark the E. side of the first pyramid and the W. side of the third pyramid, with an EW distance of $\sqrt{2} \times 1000$ cubits.

The EW distance from the W. side of the first pyramid to the E. side of the third pyramid is equal to the 1772 cubit length of the long diagonal, minus the 677 cubit length of the short diagonal, divided by $\sqrt{2}$. The exact measures are $\sqrt{6} \times \varphi^2/(\varphi^2 + 1)$, or $\sqrt{6} \times .72360679 = 1.77246742$. The square of 1.77246742 is 3.14640786. This is exactly the same π coefficient produced by $6/5\ \varphi^2$.

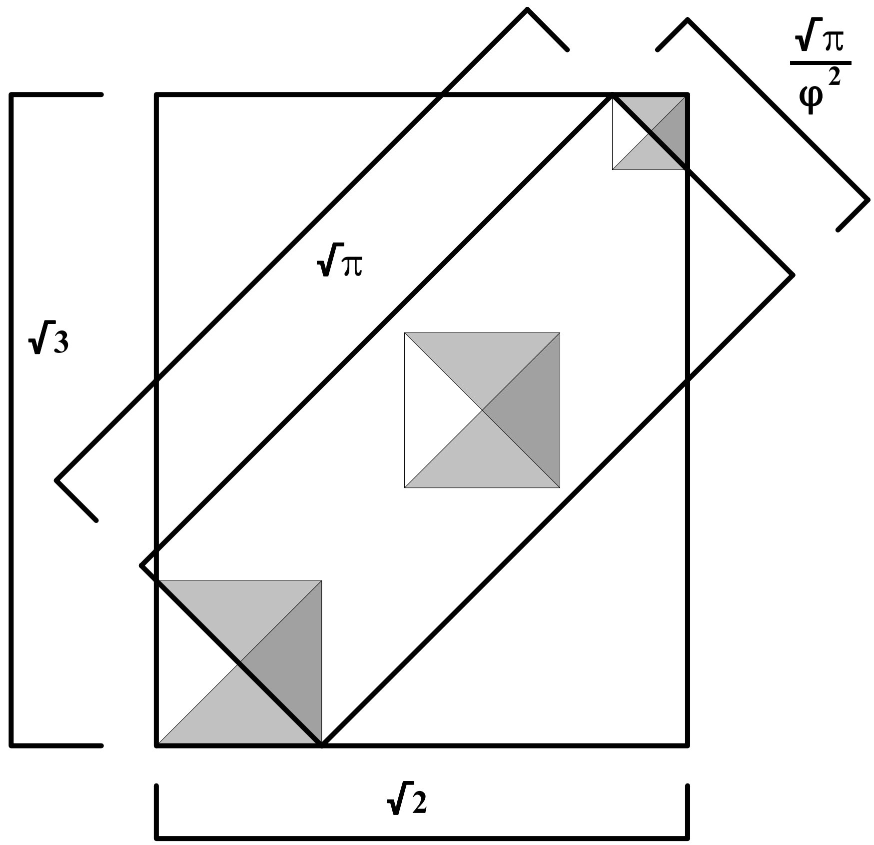
The square root of 6, or 2.449489, minus 1.772467, equals .677022, or 677.022 cubits for the short sides of the diagonal rectangle. $1772.467 - 677.022 = 1095.445$ for the diagonal distance from the W. side of the first pyramid to the E. side of the third pyramid, and 1095.445 divided by $\sqrt{2}$ gives an EW distance of 774.596 cubits from the W. Side of the first pyramid to the E. side of the third pyramid.

The distance from the E. side of the first pyramid to the end of the short diagonal is equal to the distance from the W. side of the third pyramid to the midpoint of the short diagonal. The diagonal length of the first pyramid exceeds the diagonal length of the third pyramid by 677.022/2 = 338.511, giving $338.511/\sqrt{2} = 239.363$ cubits for the base length of the first pyramid in excess of the base length of the third pyramid.

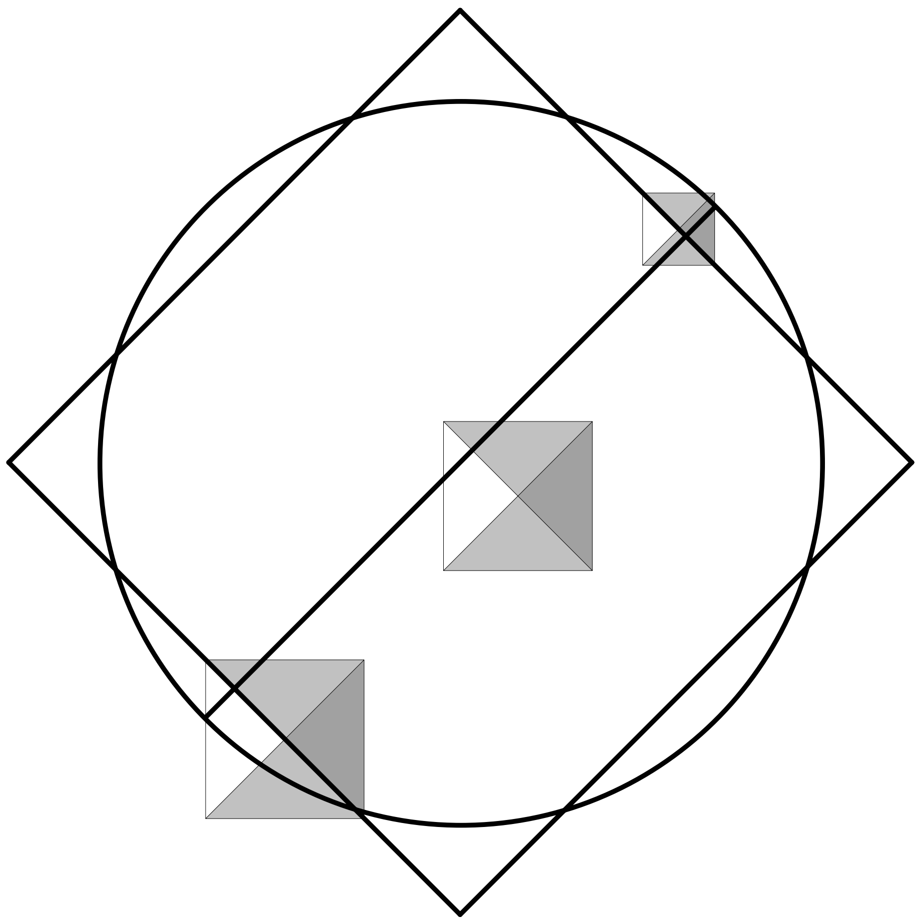
The EW distance of $\sqrt{2} \times 1000$ cubits, or 1414.213 cubits, minus the EW distance of 774.596 cubits between the W. side of the first pyramid and the E. side of the third pyramid, minus the distance of 239.363 cubits for the base length of the first pyramid in excess of the base length of the third pyramid, equals 400.254 cubits. One half of 400.254 cubits is 200.127 cubits for the base length of the third pyramid, and 200.127 plus 239.363 equals 439.49 cubits for the base length of the first pyramid.

The half base of the first pyramid is 219.745 cubits. From the EW midline of the first pyramid to the E. side of the second pyramid is 433.013 cubits ($\sqrt{3} \times 250$). The half base of the third pyramid is 100.063 cubits. From the EW midline of the third pyramid to the E. side of the second pyramid is 661.437 cubits ($\sqrt{7} \times 250$).

The 219.745 cubit half base of the first pyramid, plus 433.013 cubits from the midline of the first pyramid to the E. side of the second pyramid, plus the 100.013 cubit half base of the third pyramid, plus 661.437 cubits from the midline of the third pyramid to the E. side of the second pyramid, gives a length of 1414.258 cubits from the E. side of the first pyramid to the W. side of the third pyramid. Compared to $\sqrt{2} \times 1000$ cubits, or 1414.213 cubits, this is an error of 1414.258/1414.213 = 1.0000314, or approximately three parts in 100,000.



$\sqrt{6} \times \varphi^2/(\varphi^2 + 1) = \sqrt{\pi}$
Half perimeter - $\sqrt{3} + \sqrt{2} = 3.146$
Area - $\sqrt{3} \times \sqrt{2} = \sqrt{6}$
Half perimeter - $\sqrt{\pi} + \sqrt{\pi}/\varphi^2 = \sqrt{6}$
Area - $\sqrt{\pi} \times \sqrt{\pi}/\varphi^2 = \pi/\varphi^2 = 1.2 = 6/5$



Side lengths of square - $\sqrt{\pi} \times 1000$ cubits
Area of square - 3,141,640 square cubits
Radius of circle - 1000 cubits
Area of circle - 3,141,592 square cubits

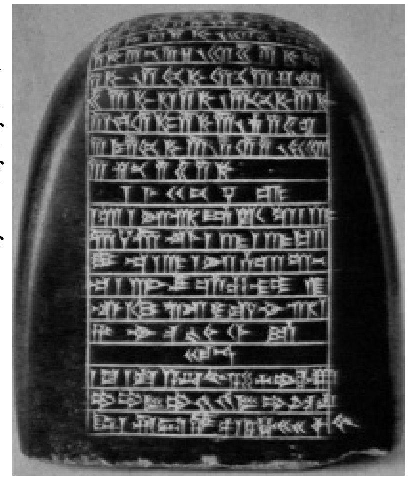
Babylonian and Roman Lengths, Volumes and Weights

Jim Alison - May 2023

The length of one meter is 100 centimeters. By design, the length of the polar circumference is four billion centimeters. One meter is 60 Babylonian shusi. The polar circumference is 2.4 billion shusi. The meter is 54 ancient Egyptian digits and the Roman digit is the same length as the Egyptian digit. The polar circumference is 2.16 billion Egyptian or Roman digits. Four Roman digits is equal to three Roman inches, or 1.62 billion Roman inches in the polar circumference.

The length of the Babylonian foot is 20 shusi, or one third of one meter. The Babylonian cubit is 30 shusi, or one half of one meter, and the double cubit is 60 shusi, or one meter. The weight of the Babylonian mina is 500 grams. The weight of the Babylonian talent is 60 mina and the weight of the Babylonian shekel is 1/60th of one mina. The cuneiform inscription on British Museum exhibit number 91148 gives a weight of two mina during the reign of Shulgi, (c. 2000 B.C.). This weight is 1000 grams, or 500 grams per mina.

In *The Treasury of Persepolis* (1939), Erich Schmidt states "Another Darius inscription appears on stone weights, all found in rooms other than the treasure halls. The beautifully finished grayish green diorite weight PT3 283 was about 1 meter above the floor of Room 3 in the southern part of the Treasury. The Persian section of the trilingual legend reads: '120 karsha. I Darius, the great king, the king of kings, the king of lands, the king of this earth, the son of Hystapes, an Achaemenid.' The Persian symbol for 'hundred' has hitherto been unknown. The Babylonian version gives '20 minae', corresponding to 120 karsha. The weight of the stone is 9.950 kilograms. Allowing for chips missing at the lower edge, a mina is almost exactly 500 grams, and a karsha is about 83.33 grams."



One of the smaller weights found by Schmidt in Persepolis, with inscriptions in both Persian and Babylonian, is exhibit number 91117 in the British Museum. The Persian inscription gives two karsha and the cuneiform inscription gives 1/3 mina. This weight is 166.29 grams, or 83.145 grams for the karsa, or 498.87 grams for the mina, without making allowance for missing chips. Schmidt stated "The core of the square tower and of at the foundation of the curtain walls at either side consists of solidly packed rubble incased in powerful walls of sun-dried bricks averaging 33×33×12 cm." Schmidt also stated "All walls of the Treasury complex are built of sun-dried bricks (32-34 cm. square and 10-13 cm. high)." The area of these bricks is one square Babylonian foot. The unit of measure is 18 digits, or 20 sushi, or one Babylonian foot, or one third of one meter.

In *Ancient Weights and Measures* (1926), Petrie reported a "fine hermatite weight of duck form, with the head and eyes carved, from Sparta, 20 darics of 128.6 grains." The grain weight given by Petrie is contained 15.4324 times in one gram. 128.6 grains, divided by 15.4324, equals 8.333 grams for one daric, or one tenth of a karsha of 83.33 grams, or 1/60 of a Babylonian mina of 500 grams.

Exhibit number 128489 in the British Museum is a two shekel duck weight from the Late Babylonian period (circa 700 B.C.) This weight is 16.7 grams, or 8.35 grams per shekel. In *The Oxford Handbook of Greek and Roman Coinage* (2016), William Metcalf states: "The weight of the daric, in comparison to the Croesus stater, increased from 8.06 g to 8.36 g, thereby restoring the ancient Mesopotamian weight standard." In *Bronze Age Weighing Systems of the Eastern Mediterranean* (2006), Alberit, Ascalone and Peyronel state "The earliest secure dating for Mesopotamian weights is Tepe Gawra VII: a variety of shapes already fit the standard unit of 8.3 g."

In *Mesopotamian Measures*, Livio Stecchini states: "Carl Lehmann-Haupt, as a young man, published the best essay of his career... His most important discovery was the reading of texts that clearly indicate that the qa (volume of a double mina of water) is a cube with an edge of six fingers. Hence, there can be no doubt that the cubic cubit contains 250 minai of water weight, or 125 qa."

In his essay, presented to The International Congress of Orientalists in1889, and published in 1891, Lehmann-Haupt states: “5000 years ago the Babylonians had a system in place very similar to our metric system. As the tenth of the meter forms the edge of a cube with a volume of one liter and a water weight of one kilogram, the tenth of the Babylonian double cubit forms the edge of a cube with a volume of one qa and a water weight of two mina.” In *A Remarkable Collection of Babylonian Mathematical Texts* (2008), Joran Friberg states “There was, indeed, an Old Babylonian sìla (qa) equal to a cube with sides of 6 fingers (c. 10 cm), which was almost exactly equal to one liter.”

In *Babylonian, Assyrian and Persian Measures* (1944), Angelo Segre states that in *Textes Mathematiques Babyloniens* (1938), “F. Thureau-Dangin correctly gives the fundamental relation one qa equals the cube of six fingers (shusi). This equivalence is based on 4669 (1-9) in the Yale Babylonian Collection and is verified in many cases.” Segre also states “The Babylonian cubit was certainly very near to mm. 500.”

British Museum exhibit number 91433 is a Babylonian duck weight of 15.06 kilograms with a cuneiform inscription reading 30 mina - true.

Segre gives this weight, of 15.06/30 = 502 grams for the mina, or 1004 grams for the double mina, giving 1004 milliliters for the water weight of the double mina that is contained in the qa, with the cube root of 1004 being 10.013 cm for the length of six shusi, or 50.066 cm for the length of the Babylonian cubit.

Six shusi cubed = 216 cubic shusi, in the base-six Babylonian system, is the equivalent of 10 cm cubed = 1000 cubic cm in the metric system. The 2.4 billion shusi in the polar circumference, or 600 million shusi for the quarter circumference from the equator to the pole, is the equivalent of one billion cm in the quarter circumference. The meter is a decimal expression of 100 cm, compared to the equivalent double cubit of 60 shusi.

The Roman foot was divided into 12 uncia (inches), and the Roman libra was divided into 12 uncia (ounces). In *A discourse of the Romane foot and denarius from whence, as from two principles, the measures and weights used by the ancients may be deduced* (1647), John Greaves cited Roman laws and inscriptions giving the volume of the amphora quadrantal as one cubic foot; the volume of the congius as one-eighth of the amphora; and the water weight of the congius as ten libra, or 120 ounces.

Given 12 inches for the edge of the cubic amphora, the cubic congius, with one-eighth the volume of the amphora, has an edge of 6 inches, or 216 cubic inches. The water weight of 120 ounces, contained in the congius of 216 cubic inches, gives a water weight for the cubic inch of 120/216 or 5/9 of one ounce. Given 60 shekels in the Babylonian mina, and two mina of water weight in the cube of six shusi, or 216 cubic shusi, the ratio between the water weight of the cubic shusi and the weight of one shekel is also 120/216, or 5/9.

Greaves determined the length of the Roman foot from inscribed measuring rods, monuments, buildings, roads with inscribed waypoints, and given distances between known locations. Because of variations in the weights of coins and inscribed Roman weights, and because purity and temperature affect the density of water, Greaves rejected the idea of determining the Roman length standard based on the remains of Roman weights. Determining the weight standard based on the length standard is also a better method because the Roman weight standard is based on the length standard of the digit being contained 2.16 billion times in the polar circumference. Greaves concluded that the length of 60 Roman feet, or 720 Roman inches, was equal to 700 English inches. Twelve inches × 35/36 = 11.666 inches for the Roman foot, 11.666/16 = .7292 inches for the digit, and .7292 × 54 = 39.375 inches for the meter.

The Roman digit is 100/54 = 1.852 centimeters. The Roman foot is 12 inches, or 16 digits, or 1.852 × 16 = 29.629 cm. The cubic Roman foot of 29.629^3 = 26012.29 cc contains 80 libra of water weight, or 26012.29/80 = 325.15 cc or 325.15 grams of water weight for the libra. The Roman ounce is 1/12 of the libra, or 325.15/12 = 27.1 grams.

When the metric system was introduced, a copper cylinder that weighed exactly one kilogram and a rod that measured exactly one meter were sent to the United States, but the ship was captured by pirates. The meter rod was lost, but the kilogram weight eventually arrived. The length of the meter may be determined as 10 times the edge of a cube with a water weight of one kilogram, but the weight of the kilogram is based on 4 billion centimeters in the polar circumference, just as the weight of the Egyptian deben and the Roman libra are based on 2.16 billion digits in the polar circumference, and the weight of the Babylonian mina is based on 2.4 billion shusi in the polar circumference.

In *The Origin of Metallic Currency and Weight Standards* (1892), William Ridgeway gives examples of inscriptions from ancient cultures indicating the weight of one carob seed was regarded as equal to the weight of three barley grains or four wheat grains. Barley weighs 4/5 of the same volume of wheat, and wheat weighs 4/5 of the same volume of water. Wheat is denser than barley, but individual wheat grains are smaller and lighter than barley grains. The weight of one wheat grain is 3/4 of one barley grain. Based on measurements of the weight of carob seeds, barley grains and wheat grains, Ridgeway gives .048 grams for one wheat grain, .064 grams for one barley grain, and .192 grams for one carob seed, or carat.

The standard for the troy (barley) grain is 15.4324 grains per gram, or .0647 grams per grain. Three grains of barley equals one carat, or 5.144 carats per gram, or .1944 grams per carat. The carat has been metricized to equal exactly 5 carats per gram, or .2 grams per carat. As a result, the ratio between the modern carat and the troy grain is .2/.0647 = 3.08 carats per gram, compared to the ancient standard of three to one. Barley weighs 4/5 of the same volume of wheat, and wheat weighs 4/5 of the same volume of water. Barley weighs 4/5 × 4/5 = 16/25, or .64 of the same volume of water. One Barley grain weighs.064 grams. One milliliter of barley weighs .64 grams and is equal to the weight of 10 barley grains.

In the Roman weight system, one Roman ounce equals 144 carob seeds, or 144 carats. The Roman congius contains 10 libra of water and the volume of the congius is 1/8 of one cubic Roman foot, giving 80 libra of water in the cubic foot. Eighty libra of water is 64 libra of wheat in the cubic foot. Ten libra of water is 8 libra of wheat in the congius. A cube with an edge of three Roman inches has a volume equal to 1/8 of the congius, containing one libra of wheat, or 12 Roman ounces. A cube with an edge of three inches is 27 cubic inches. One cubic Roman inch of wheat weighs 12/27 of one Roman ounce.

A cube with an edge of one half inch is 1/8 of one cubic inch, or 1/8 × 12/27 = 1/18 of one Roman ounce of wheat. A cube with an edge of one quarter of one inch is 1/64 of one cubic inch, or 1/64 × 12/27 = 1/144 of one ounce of wheat, or one carat, or three barley grains, or four wheat grains. 1/8 of one cubic inch of wheat weighs 8 carats, or 24 barley grains, or 32 wheat grains. One cubic Roman inch of wheat weighs 64 carats, or 192 barley grains, or 256 wheat grains. The Roman ounce is 256 × 27/12 = 576 wheat grains, or 576/4 = 144 carats, or 144 × 3 = 432 barley grains.

In the Roman linear system, the mile, or mille passus, was 5000 Roman feet, or 1000 paces, of five feet each. The pace was further divided into a step, of 2.5 feet. The amphora, has a volume of one cubic foot and contains 64 libra of wheat. A cube with an edge of two and a half feet has a volume of 2.5^3 = 15.625 cubic feet, and 15.625 × 64 = 1000 libra of wheat, or 1000 × 5/4 = 1250 libra of water, or 1000 × 4/5 = 800 libra of barley. A cube with an edge of five feet contains 8000 libra of wheat, or 10,000 libra of water, or 6400 libra of barley.

In *Historical Metrology* (1953), Berriman stated: "One of the most important documents relating to English weights and measures is the *Tractatus de Ponderibus et Mensuris (c. 1303)*; the context and other relevant evidence require that this definition of the gallon be interpreted as meaning that 8 lb. of wheat measure a gallon for wine. The Latin version mentions corn, but not wine. The opening paragraph (as translated in the *Statutes at Large*) reads:

By consent of the whole realm, the King's measure was made so that an English penny, which is called Sterling, round without clipping, shall weight thirty-two grains of wheat dry in the midst of the ear; twenty pence make an ounce and twelve ounces make a pound and eight pounds [of wheat] make a gallon of wine and eight gallons of wine make a bushel of London."

Henry VII ordered this same standard in 1497. In *The Origin of Metallic Currency and Weight Standards* (1892), William Ridgeway stated: "It was ordained by 12 Henry VII. ch. v. that the bushel is to contain eight gallons of wheat, and every gallon eight pounds of wheat, and every pound twelve ounces of Troy weight, and every ounce twenty sterlings, and every sterling to be of the weight of thirty-two grains of wheat that grew in the midst of the ear of wheat according to the old laws of this land."

The weight of thirty two grains of wheat, times twenty sterlings, equals 640 wheat grains in the Troy ounce of Henry VII. Given three grains of barley equals four grains of wheat, 640 wheat grains times 3/4 equals 480 barley grains, which is the standard for the number of troy (barley) grains in the troy ounce.

The English gallon defined in 1303 and 1497 contained eight pounds of wheat, or10 pounds of water, and the bushel contained 64 pounds of wheat, or 80 pounds of water. The Roman amphora contained 80 libra of water, or 64 libra of wheat, and the congius contained 10 libra of water, or eight libra of wheat. The English avoirdupois gallon contains 10 pounds of water, or 8 pounds of wheat, and the pint contains 20 ounces of water, or16 ounces, or one avoirdupois pound, of wheat.

In the Greek system of weights, one talent weighs 60 mina, one mina weighs 100 drachma, one drachma weighs six obols, and one obol weighs 12 barley grains. In *Archaeologies of the Greek Past*, Rachel Griffin states "There were several coins minted which were variations of the obol. Silver obols and triobols (three obol pieces) were among the most common coins in Thessaly, while central Thrace minted large numbers of tetrobols, triobols and diobols." The tetartemorion is a small Greek coin that weighs one-fourth of one obol. The obol weighs 12 barley grains and the tetartemorion weighs three barley grains, or one carat.

In *Weights of Greek Coins* (1855), W. M. Leake states "A hoard of more than 200 silver coins [from the time of Alexander the first], all weighing about 35 troy grains, were found in Macedonia in 1827." These were triobols of 35 troy grains, times 2 = one drachma = 70 troy grains, times 100 = one mina = 7000 troy grains, the same as the avoirdupois pound, although in the Greek system this was regarded as 7200 barley grains, lighter than the troy grain in the ratio of 35/36.

The volume of the Greek chous was equal to the volume of the Roman congius. The congius contained 10 libra of water, or 120 Roman ounces, or 120 × 144 = 17280 carats. The volume of the chous was equal to 72 kyathos, and the volume of the kyathos was equal to 10 kochlarion. The water weight of 17280 carats for the chous, divided by 720, equals 24 carats, or 72 barley grains, for the water weight of one kochlarion, equal to the weight of six obols, or one drachma, or the weight of one sextula, the named weight for 1/6 of one Roman ounce.

The common Jewish cubit is five handsbreadths, or 25 digits, the same length as the Greek cubit. The Jewish cubit of the alter is six handsbreadths, or 30 digits. In *An essay towards the recovery of the Jewish Measures and Weights* (1686), Richard Cumberland gives 21.87 English inches for the cubit of the alter.

In *Historical Metrology* (1953), A. E. Berriman also gives 21.87 English inches for the Talmudic cubit. The common cubit is 5/6 of 21.87 inches, or 18.23 inches, the same length as the Greek cubit of 25 digits, and 18.23 inches × 2/3 = 12.15 English inches for the Jewish foot, the same length as the Greek foot of 16.666 digits.

In the Jewish system of volumes, the liquid measure of one bath has the same volume as the dry measure of one epha. The log is a liquid and dry measure. The bath contains 72 logs of liquid volume and the epha contains 72 logs of dry volume. Like the bath and the epha, the liquid volume and the dry volume of the log is the same.

1 KINGS 7: 23-26 states that for his temple, Solomon "made the sea of cast bronze, ten cubits from one brim to the other; it was completely round. Its height was five cubits, and a line of thirty cubits measured its circumference...It was a handbreadth thick and its brim was shaped like the brim of a cup, like a lily blossom. It contained 2000 baths."

The diameter of 10 sacred cubits, of 30 digits each, is equal to 12 common Jewish or Greek cubits, or 18 Jewish or Greek feet, or three Greek fathoms. The radius and the height of five sacred cubits is equal to six common Jewish or Greek cubits. Since 5/6 = φ^2/π, the proportion of the common Jewish cubit or the Greek cubit in relation to the sacred Jewish cubit is also φ^2/π.

The biblical description of Solomon's sea is interpreted as 10 cubits for the outer diameter and 30 cubits for the inner circumference, or $30/2\pi = 4.77$ cubits for the inner radius and 4.77 cubits for the inner height of the cylinder. Given 30 digits for the cubit, the inner circumference is 30 cubits times 30 digits equals 900 digits, divided by 2π equals 143.24 digits for the inner radius and the inner height. The cube of 143.24 digits is 2,938,921 cubic digits, times π equals 9,232,892 cubic digits for the volume of the sea, divided by 2000 equals 4616 cubic digits for one bath, with a cube root of 16.65 digits, or one Jewish or Greek foot, for the edge of a cube with a volume of one bath. 4616 cubic digits, divided by 72, equals 64.1 cubic digits for the volume of one log, with a cube root of 4.002 digits for the edge of a cube with a volume of one log.

The Roman cubit is 24 digits and the Roman foot is 16 digits. A cube with a volume of one Roman amphora has an edge of one Roman foot, or 16 digits. The Roman foot is 24/25 of the length of the Greek or Jewish foot. A cube with a volume of one Roman congius has an edge of 8 digits and contains 10 libra of water, or 8 libra of wheat. A cube with a volume of 1/8 of one congius has an edge of four digits and contains 15 ounces of water, or 12 ounces, or one libra, of wheat.

The volume of one libra of wheat, times 64, is the volume of one Roman amphora, containing 64 libra of wheat, or 80 libra of water. The volume of one libra of wheat, times 72, is the volume of one Jewish bath, containing 72 libra of wheat, or 90 libra of water. These divisions in the Roman and Jewish measurement systems reflect the virtual equality of 25/24 cubed = 9/8, or 72/64.

The volume of the Greek chous and the volume of the Roman congius are eight times the volume of one Jewish log. One log is six times the volume of one Jewish egg. The volume of the chous or the congius is 48 times the volume of one egg. The congius is 48 times the volume of the Roman acetabulum. The chous is 48 times the volume of the Greek oxybaphon. Like the log and the egg, the acetabulum and the oxybaphon are named volumes for liquid and dry measure and are the same volume as the egg.

The volume of the congius weighs 120 Roman ounces of water, and 120/48 = 2.5 Roman ounces, or 360 carats, or the weight of 15 drachma, or 90 obols, for the volume of one acetabulum, or one oxybathon, or one egg. The volume of the congius weighs 96 Roman ounces of wheat and 96/48 = two ounces, or 288 carats, or 12 drachma, or 72 obols for the volume of one acetabulum, or one oxybathon or one egg. The European weight classification for large chicken eggs is 63 to 73 grams. The weight of 2.5 Roman ounces is 67.75 grams.

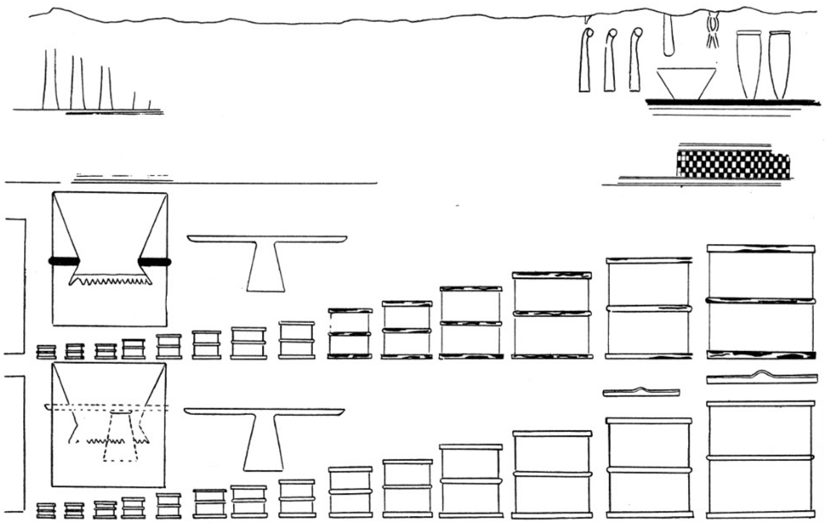
The volume of one egg weighs 2.5 ounces, or 360 carats, of water. The water weight of one log is 360 × 6 = 2,160 carats. A cube with edges of four digits, or three Roman inches, contains the volume of one log, or 27 cubic inches. The water weight of one cubic inch is 2160/27 = 80 carats, or 240 barley grains, or 480 barley grains for two cubic inches. The Roman amphora is one cubic foot, or a cube with edges of 12 Roman inches, or 1728 cubic inches. The water weight of the amphora is 80 libra, or 960 Roman ounces. Given the weight of 480 barley grains in one troy ounce, and 480 barley grains in two cubic Roman inches, 1728/2 = 864 troy ounces, or 72 troy pounds of water in the amphora, if the weight of the troy grain was equal to one third of one Roman carat.

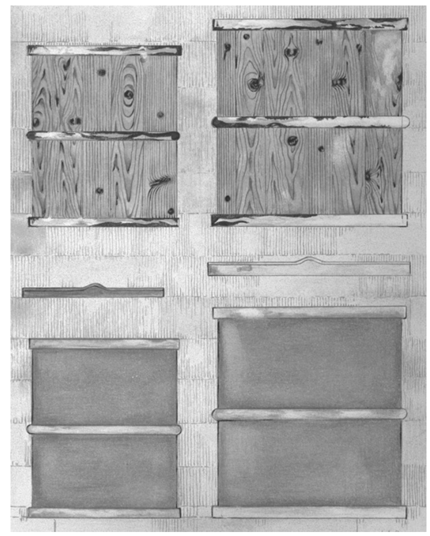
If the weight of the Roman barley grain was 35/36 times the weight of the troy grain, as suggested by the hoard of Alexandrian triobols, and possibly suggested by the length of the Roman foot being 35/36 the length of the English foot, then the amphora would have been 70 troy pounds of water. Based on the Roman foot of 11.666... inches, or 35/36 of the English foot, and based on the modern standard of 15.4324 troy grains per gram, the weight of the Roman amphora is 69.7 troy pounds of pure water at maximum density.

The Egyptian henu contains 5 deben of water. The cube of the royal cubit contains 300 henu, or 1500 deben of water. Based on 20.625 English inches, or 52.387 centimeters, for the length of the royal cubit, the cubic cubit contains 143775 milliliters, or 143775 grams of water, divided by 1500 equals 95.85 grams per deben, or 9.585 grams per qedet. A weight for the barley grain of 150 per qedet is 9.585/150 = .0639 grams, or 15.649 barley grains per gram, or 1500 barley grains per deben, giving 1500 × 1500 = 2,250,000 barley grains, or 750,000 carats, or 3,000,000 wheat grains for one cubic cubit of water, giving 500,000 carats, or 1,500,000 barley grains, or 2,000,000 wheat grains for one Egyptian khar of water.

In *Wisdom of the Egyptians* (1940), Petrie stated: "For capacity measures, the most distinctive are the plain cylinders, figured as a series in the tomb of Hesy, iiird dynasty. These were of copper for liquids, and of hooped staves for grain. The actual measures begin with the Amratian age, a basalt vase being marked 'one-half,' and intended to contain half of ten debens of water. Possibly the marks were put on later in historic times, as there is no other evidence of liquid measure or of the deben weight before the dynastic age."

Petrie also stated: "It seems not improbable that the adoption of 2/3 of the cubit cubed as the khar unit of capacity, arose from the form of the measure. It was accepted that a diameter of 9 gives an area = 8^2. A measure of diameter 9 palms, and a half cubit deep, holds .65 of a cubic cubit, the khar being .66. The form of such a vessel has the proportions of the old English bushel standard, which is a convenient form." A circumference of four royal cubits has a diameter of nine palms. A cylinder with a circumference of four cubits, and a height of $\pi/6$ cubits, or $\varphi^2/5$ cubits, or .5236 cubits, has a volume of exactly one khar. A cylinder with a circumference of four cubits and a height of $\pi/4$ cubits has a volume of exactly one cubic royal cubit.





In *Excavations at Saqqara* (1913), J.E Quibell published a drawing of the full series of cylinders in the tomb of Hesy wall painting and a close up drawing of the two larger cylinders in both rows. Another form for 2/3 of one cubic cubit is a cylinder with a diameter of one royal cubit, or 28.28 digits, and a height of one short cubit, or 24 digits. One cubic cubit is 28.28 digits cubed, or 22627 cubic digits. A radius of 14.14 digits squared, times π, times a height of 24 digits equals 15079 cubic digits for the khar, and 15079/22627 = .666 cubic cubits. A cylinder with a diameter of one royal cubit and a height of 9 palms, or 36 digits, has a volume of one cubic royal cubit. One henu is equal to 1/300 of one cubic cubit or 22627/300 = 75.4 cubic digits. A cylinder with a diameter of four digits and a height of six digits has a volume of $2^2 \times \pi \times 6 = 75.4$ cubic digits, or one henu, or five deben of water. A cylinder with a diameter of 8 digits and a height of 12 digits has a volume of 8 henu. A cylinder with a diameter of 8 digits and a height of 15 digits has a volume of 10 henu, or one hekat. A cylinder with a diameter of two digits, and a height of three digits, has a volume of 1/8 of one henu, or four ro, or four qdt of barley, or five qdt of wheat. A cylinder with a diameter of one digit and a height of three digits, has a volume of 1/32 of one henu, or one ro, weighing one qdt, or 9.585 grams, of barley, or $9.585 \times 25/16 = 14.976$ grams of water.

In *The Old Egyptian Medical Papyri* (1952), Chauncey Leake stated "old Egyptian drug measurement seems usually to have been made by capacity estimation. In the Hearst Medical Papyrus, for example, although one of the prayers refers to weighing by deben, none of the prescriptions seem to call for drug amounts by weight. All apparently specify volume amounts, usually fractions or multiples of the ro (mouthful)."

The hieroglyphic symbol for the ro is a mouth, which is also the hieroglyphic symbol for the letter r, and Leake believed this was not a coincidence. He states that several ancient Egyptian spoons on exhibit at the Metropolitan Museum contain a volume of 14 to 15 milliliters, which he compares to one ro, and one-half ounce, and one tablespoon, and one mouthful. He states that this is the dosage for several prescriptions in the papyri and he translates prescription number 212 from the Hearst medical papyrus: "As for this measuring utensil, this prescription shall be measured with it. It is the measuring utensil with which Horus measured his eye. It is tested; there is found life, well being and health. This prescription is measured with this measuring utensil in order to remove therewith every sickness which is in this body."

The handle of this spoon is in the shape of an ankh, the hieroglypic symbol for life. The description of this spoon given by the Met is from the Middle Kingdom, of travertine (Egyptian alabaster), with a length of four and one-eighth inches, a width of one and one-eighth inches, and a depth of one quarter inch.

The volume of a cube with an edge length equal to the circumference of a sphere is virtually 60 times the volume of the sphere. A cube with a volume of 60 has an edge length of 3.91...
A sphere with a circumference of 3.91... has a radius of .623..., cubed $\times \pi \times 4/3 = 1.013...$

In *An Ancient Relation between Units of Length and Volume Based on a Sphere* (2012), Zapassky, Gadot, Finkelstein and Benenson measured the circumference of several hundred intact, fully restored and partially restored Egyptian containers and found a modal circumference of one royal cubit. The authors also measured several Iron Age Phoenician spherical containers, giving 29.2 digits for the outer circumference of the pictured jug, and stating, "In this case, too, the distribution of the jugs' external maximal circumference has a clear mode at 25-30 digits. Taking into consideration a wall width of 0.5-0.7 cm, they provide a modal volume of 0.5 hekat. It is possible that the Phoenician globular jugs were used in trade of valuable liquids. The inherent relationship between the royal cubit and the hekat could have made a quick estimate of their capacity possible."

Problem 10 in the Moscow mathematical papyrus asks for the surface area of a hemispherical bowl with a diameter of four and one half, or a radius of 2.25. Problem 10 does not specify digits or palms for the unit of measure. A diameter of 4.5 digits has a circumference of one-half of one royal cubit. The modern formula for the surface area of a hemisphere is $\pi r^2 \times 2$, giving a surface area of 31.8 square digits for a radius of 2.25. In problem 10, the area of the circle is given as the square of 8/9 of the diameter, or $4.5 \times 8/9 = 4$, and $4^2 \times 2 = 32$ square units for the surface area.

The modern formula for the volume of a hemisphere is $\pi r^3 \times 2/3$. A hemisphere with a radius of 2.25 digits has a volume of 23.8 cubic digits. The cubic cubit contains $(20 \times \sqrt{2})^3 = 22627$ cubic digits. The volume of one ro is equal to 1/9600 of one cubic cubit. The volume of 10 ro is 22627/960 = 23.6 cubic digits. A hemisphere containing 23.6 cubic digits has a radius of 2.24 digits. One ro contains one qdt of barley. A hemisphere with a volume of 10 ro contains 10 qdt, or one deben of barley. A hemisphere with a diameter of 4.5 palms, or 18 digits, has a circumference of two royal cubits. Regarding the volume of this hemisphere as 4^3 or 64 times the volume of 10 ro gives a volume of 640 ro, or 20 henu, or a double hekat, or 1/10 of one khar, or 64 deben of barley, or 80 deben of wheat, or 100 deben of water.

A circumference of one royal cubit has a diameter of 9 digits. Using the ancient Egyptian method of calculating the volume of a cylinder as 8/9 of the diameter squared, times the height, a cylinder with a circumference of one royal cubit, or a diameter of 9 digits, and a height of 8 digits, has a volume of $8^2 \times 8 = 512$ cubic digits, or a cube with an edge of 8 digits, or six Roman inches, or 216 cubic Roman inches, or one congius, or 10 libra of water, or 8 libra of wheat. A cylinder with a circumference of two royal cubits, or a diameter of 18 digits, and a height of 16 digits, has a volume of $16^2 \times 16 = 6096$ cubic digits, or one cubic Roman foot, or one amphora. One cubic Greek foot is virtually 9/8 of one amphora. A cylinder with a circumference of two royal cubits, and a height of 18 digits, gives a volume of one cubic Greek foot, or one Jewish bath. Calculating volume as 8/9 of the diameter squared, times the height, a cylinder with a circumference of one half of one royal cubit, or a diameter of 4.5 digits, and a height of one palm, or four digits, has the same volume as a cube with an edge length of four digits, or three Roman inches. This cylinder has 1/8 of the volume of one congius, or the volume of one Jewish log, containing 15 Roman ounces, or the weight of 2160 carob seeds, or 2160 carats, of water.

In *The Structure of Metrology*, John Neal stated: "Old Norse cosmologists reckoned the diameter of the earth's orbit to be 216 sun diameters and this too is exactly right." In *The Sand Reckoner*, Archimedes says "Aristarchus discovered that the sun appeared to be about 1/720 of the circle of the zodiac," or .5° of the 360° circle. Archimedes then goes through a geometric proof to show that the diameter of the sun is less than a 1/164 part, and greater than a 1/200 part, of a right angle. The right angle is 1/4 of the circle, so 1/164 is 1/656 of the 360° circle, or .548°, and 1/200 is 1/800 of the circle, or .45°. Given 720 sun diameters in the circle of the zodiac, $720/\pi = 229$ times the diameter of the sun for the diameter of the earth's orbit around the sun. The diameter of the earth's orbit as 216 sun diameters gives $216 \times \pi = 678.5$ sun diameters in the circle of the zodiac, or 1/678.5 of the circle, or 360/678.5 = .53° for the sun's diameter in relation to the 360° circle. Neal's comment suggests that the Norse cosmologists made or had access to an accurate measurement the diameter of the sun in relation to the circle of the zodiac.

In REPUBLIC 8:546, Plato describes a perfect number as follows: "Now that which is of divine birth has a period contained in a perfect number, but the period of human birth is comprehended in a number in which first increments by involution and evolution obtaining three intervals and four terms of like and unlike, waxing and waning numbers, make all the terms commensurable and agreeable to one another. The base of these, with a third added, when combined with five, and raised to the third power, furnishes two harmonies; the first a square 100 times as great; and the other a figure having one side equal to the former, but oblong, consisting of 100 numbers squared upon the rational diameters of a square, the side of which is five, each of them being less by one or less by two perfect squares of irrational diameters; and 100 cubes of three. Now this number represents a geometrical figure which has control over the good and evil of births. For when your guardians are ignorant of the law of births, and unite bride and bridegroom out of season, the children will not be goodly or fortunate."

In *The Babylonian Origin of Plato's Nupital Number* (1908), George Barton stated: "The passage in which Plato introduces this mystic number is said to be the most difficult passage in his writings. Recent interpreters seem to agree that Plato refers to the world, the formation of which is controlled by a large number, and that Plato claims that human births are controlled by a smaller number which bears a certain relation to this larger number. Dupois understands that the perfect number is six, being , according to Euclid and the Greek mathematicians, a number which is equal to the sum of all its divisors. Thus $6 = 1 + 2 + 3$. Apparently, however, the meaning here is, not that six is the actual number, but that it lies at the basis of that number. There seems to be general agreement that the number which controls human births, which is obtained by 'squaring and cubing,' by which 'three intervals and four terms are produced,' is 216 ($= 6^3 = 3^3 + 4^3 + 5^3$). This is the view of Dupois (1881), Hultsch (1882), Jowett (1891), Campbell (1894), and Adam (1902)."

The sum of $1 + 2 + 3 = 6$. The product of $1 \times 2 \times 3 = 6$. A 3-4-5 triangle has an area of six. The product of $1^3 \times 2^3 \times 3^3 = 216$. The sum of $3^3 + 4^3 + 5^3 = 216$. The first of Plato's two harmonic numbers is 216 times the square of 100, or $216 \times 100^2 = 2{,}160{,}000$. The second number begins with the rational (whole number) diameter (diagonal) of a square with sides of five. The whole number diagonal is seven and Plato says this is less by one, or $7 + 1 = 8$, then times 100 equals 800 for one side of the oblong. The second side of the oblong is $3^3 \times 100$, or 2700. The product of 800×2700, or $2^3 \times 3^3 \times 100^2$, is 2,160,000, the same as $(3^3 + 4^3 + 5^3) \times 100^2$. The Greek or Roman or Egyptian digit is contained 54 times in one meter. Four meters contains 216 digits. The digit is contained 2.16 billion times in the meridian circumference. The Greek and Roman stades, of 10,000 digits, are contained 216,000 times in the circumference. If the circumference is divided into Plato's number, of 2,160,000 units, these units contain 1000 digits, or 10 Greek fathoms, or 40 Greek cubits, or 50 Egyptian remen, or 60 Greek feet.

The number of minutes in the 360° meridian circumference of the earth is $360 \times 60 = 21{,}600$ minutes. One minute of latitude is 1/21,600 of the circumference. The length of 10 Greek stades, or 10 Roman stades, or 5000 Egyptian remen, equals the length of one minute of latitude. Barton stated: "Rawlinson and Oppert, in the early days of Assyrian decipherment, discovered the sexagesimal system and the notation of the saros. Rawlinson, in 1855, suggested that this was carried beyond the saros to 216,000." The ancient world regarded the time for each one degree shift in the position of the fixed stars due to precession as 72 years. Each of the 12 zodiacal constellations occupy 1/12, or 30° of the full circle. A shift of 30°, times 72 years per degree, gives 2160 years for the astrological age of each of the constellations in the zodiac. Barton stated "Plato (may have) received from the astronomers the idea that 72 years marked a definite step of the advance of the equinoxes, and independently built up from it the full period and made this of service to his ethical-biological reflections. This may have been suggested through primeval oriental myths."

Berossus was a Babylonian who wrote a chronology of Babylonia, in Greek, during Alexandrian times. In *Fingerprints of the Gods* (1995), Graham Hancock referenced the statement by Berossus that the ancient Babylonian chronology gave a period of 2,160,000 years from creation to universal catastrophe. This statement by Berossus is also found in *Archaeological Discussions*, University of Chicago (1909). This is Plato's number, given as a duration of 1000 astrological ages, from creation to catastrophe.

The length of the Babylonian rod, or gar, or nindan, is 12 Babylonian cubits. This was the unit of measure in the Smith cuneiform tablet for the base lengths of the tower of Babylon, given as 15 gar, or 15 × 12 = 180 cubits, compared to Koldewey's survey giving 90 meters for the base lengths, or .5 m per cubit. All of the other dimensions of the base lengths of the upper steps of the tower and the height of the tower in the Smith tablet are also given in gar (nindan).

In *Late Babylonian Surface Mensuration* (1984) Marvin Powell says "A cylinder with a circumference of one Babylonian cubit and a height of 0;1 nindan (six shusi) contains two qa, and each of these qa is the equivalent of a cube with an edge of 0;1 nindan, each cube being equal to 3,36 (216) cubic shusi and identical with 0;0,0,1 (the 216000th part of a) cubic nindan." The side lengths of a cube containing one qa are six shusi, or 1/5 of one cubit. The length of the nindan (12 cubits) is 60 times the side length of the qa. The number of qa in the cubic nindan is 60 × 60 × 60 = 216000.

In *Numbers and Measures in the Earliest Written Records* (1984), Joran Friberg cites cuneiform inscriptions (c. 1900 - 1500 BC) giving the breadth of six grains per shusi. Friberg also gives one-half meter for the length of the Babylonian cubit and one liter for the volume of the qa. In his *Discourse on the Romane foot*, Greaves cites Almanon, the Caliph of Babylon in the 8th century AD, and the Geographia Nubiensis from Rome in 1619 AD, also giving the breadth of six grains per digit. Six grains × 30 = one cubit × 12 = one nindan = 2160 breadths of grain in one nindan. The measure of one dana is 1800 nindan, or 1800 × 12 = 21,600 cubits, or 10.8 km.

Powell cites cuneiform inscriptions stating that each grain in a row is planted two sushi apart, and that one row, of 12 cubits, or one nindan, is one shekel of grain. One grain every two sushi is 15 grains per cubit, or 180 grains per nindan, or 180 grains per shekel. Powell regards the shekel in these inscriptions as a volumetric measure of 1/60 of one qa, and he regards the grain as barley. Powell states that "F. Thureau-Dangin made the experiment of counting modern barleycorns and came to the conclusion that 10800 barley corns (the number of se in a sila) actually fill a space of about .6 liter." Powell states "The fact that 10,800 barley corns do not add up to anything nearly approximating a liter must have puzzled everyone who has thought about Sumero-Babylonian metrology and its agricultural implications, because, if the original sila was almost a liter in size, there is no logical reason why it should be defined as 3,0,0 barley corns, rather than 5,0,0 or 6,0,0. The definition is probably taken over from the weight system."

Ridgeway also states that cuneiform inscriptions give 180 grains per shekel but he believed the weight of 180 wheat grains was being given as the weight of one shekel. Given 500 grams for the weight of the mina, 500/60 = 8.333 grams per shekel, and 8.333/180 = .0463 grams per wheat grain: 180 × 60 = the weight of 10,800 wheat grains in one mina, and 10,800 × 2 equals the weight of 21,600 wheat grains in one qa of water, or 21,600 × .0463 grams = 1000 grams, or one kilogram.

In 1917, England commissioned E.T. Richmond to survey and evaluate the Dome of the Rock to determine what works of preservation and renovation might be needed. He examined the building during the autumn and winter of 1918. In *The Dome of the Rock in Jerusalem* (1924), Richmond stated the inner diameter of the cylinder that supports the dome is 20.6 meters, the height of the cylinder, from floor level to the bottom of the dome, is also 20.6 meters, and the length of each of the sides of the exterior octagon is also 20.6 meters. Richmond stated the height from floor level to the top of the dome is equal to the inner diameter of the cylinder times the square root of three, or 20.6 x 1.732 = 35.679 meters, and the height of the outside walls of the octagon are 1/3 of this height, or 35.679/3 = 11.89 meters.

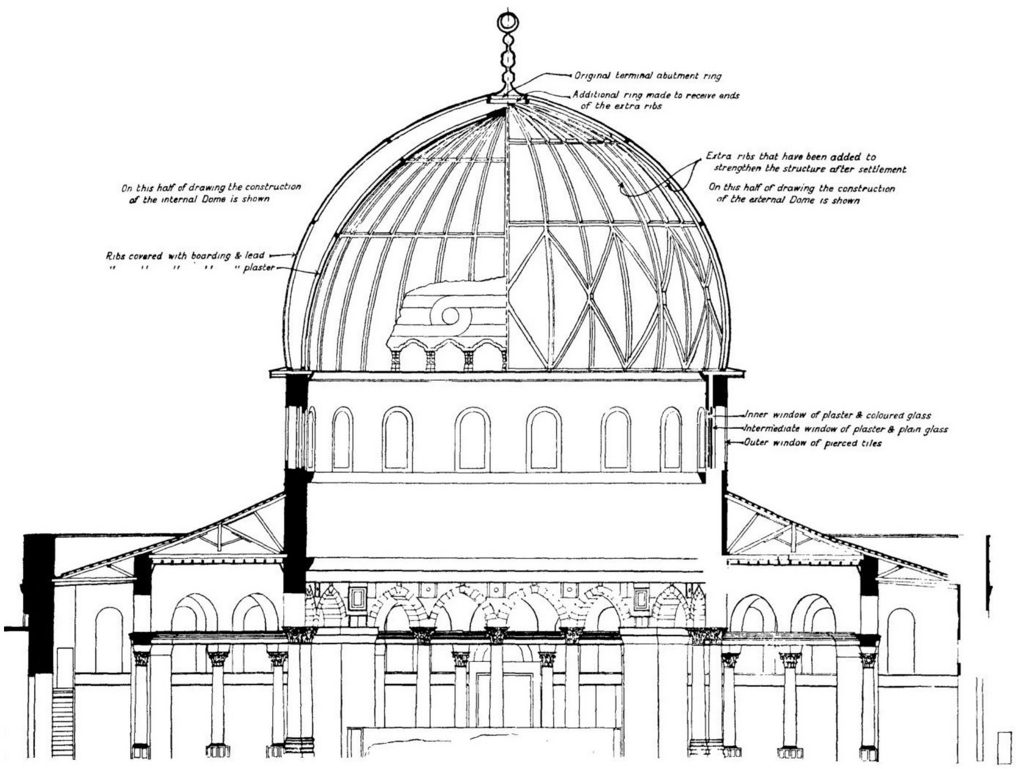
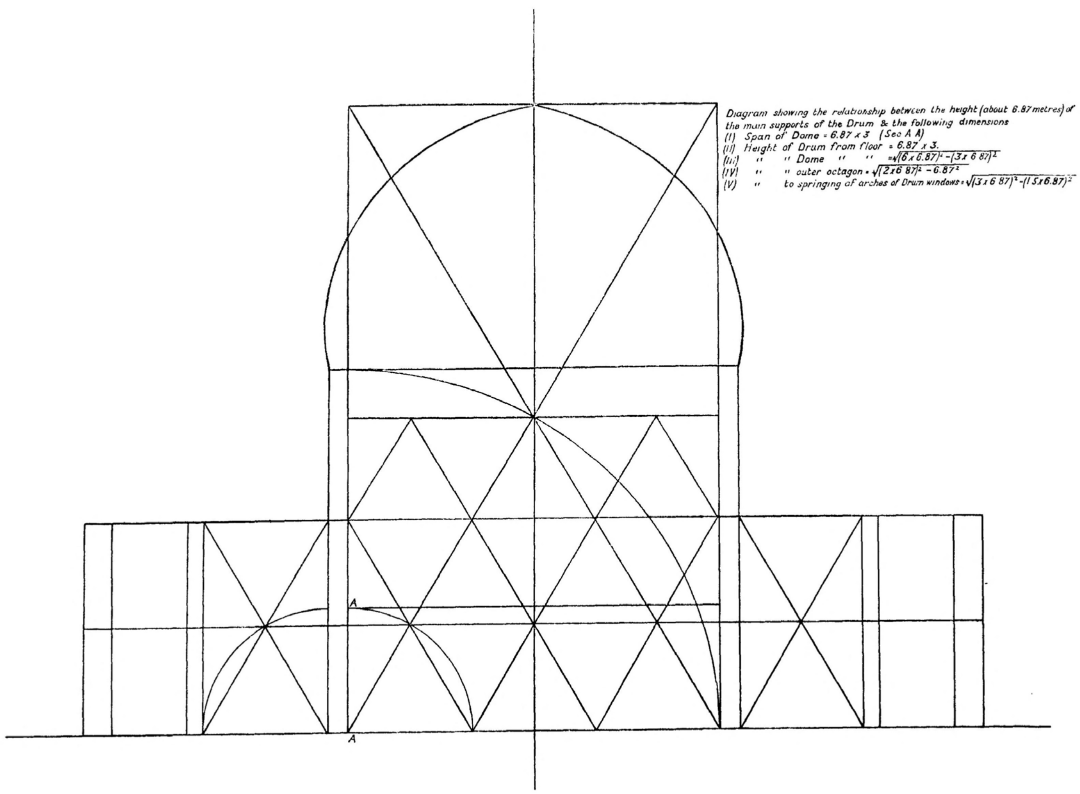


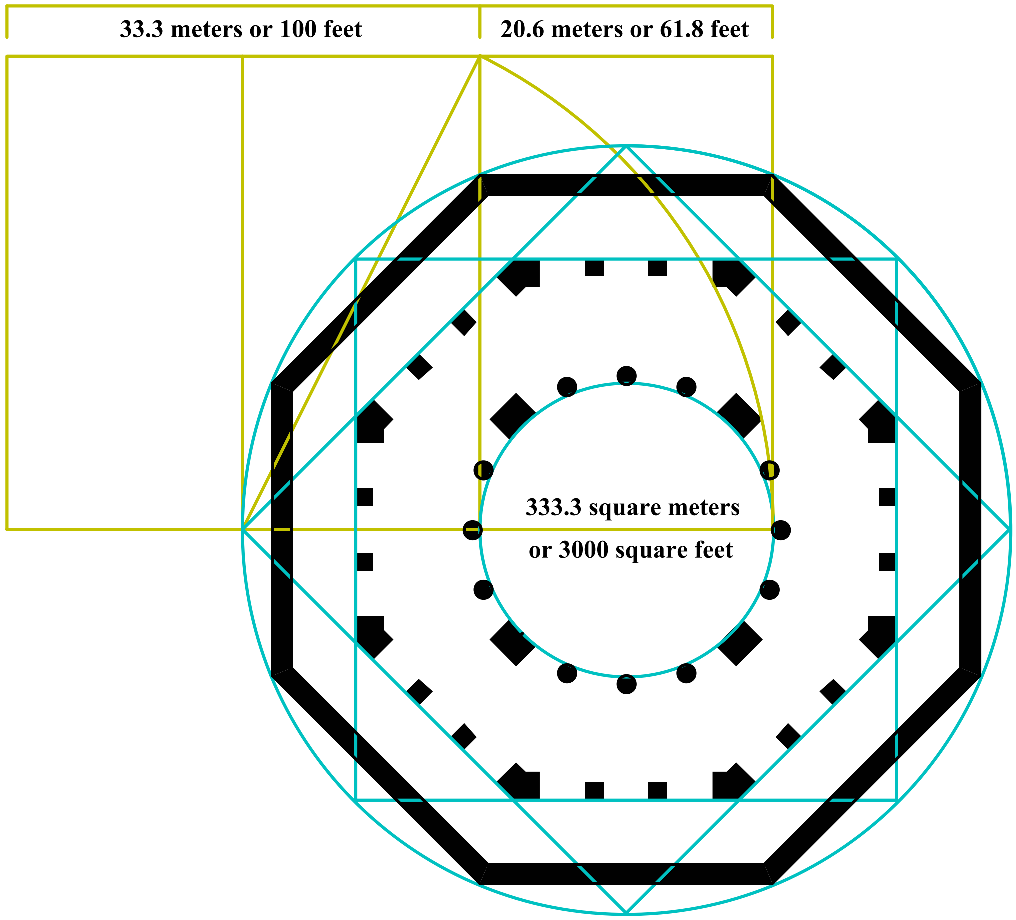
Fig.2. SECTION ON EAST & WEST AXIS

1 0 1 2 3 4 5 6 7 8 9 10
METRES

In *The Dome of the Rock: Origin of its Octagonal Plan* (2007), A. Islam and Z. F Al-hamad gave 20.6 meters for the inner diameter of the cylinder, 20.44 meters for the height from floor level to the beginning of the dome, and 36 meters for the height of the top of the dome. In *The Origin of* *the Plan of the Dome of the Rock* (1924), K. A. C. Creswell reported an average length of 20.82 meters for the sides of the exterior octagon and 15.82 meters for the sides of the interior octagon. Creswell proposed that the interior octagon is defined by two squares that are circumscribed by the same circle that circumscribed the exterior octagon. Based on this theory, a length of 20.82 meters for the sides of external octagon would give a calculated length of 15.92 meters for the sides of the interior octagon, and a length of 15.82 meters for the sides of the interior octagon would give a calculated length of 20.67 meters for the sides of the exterior octagon. Creswell believed his theory was validated by the proximity of his measured and calculated lengths for the sides of the octagons. Creswell also pointed out that the exterior octagon has 56 pillars, which is the same as the number of post holes in the Aubrey circle at Stonehenge, which also suggests an octagon, particularly given the positions of the station stones at Stonehenge, marking two opposing octagonal sections.

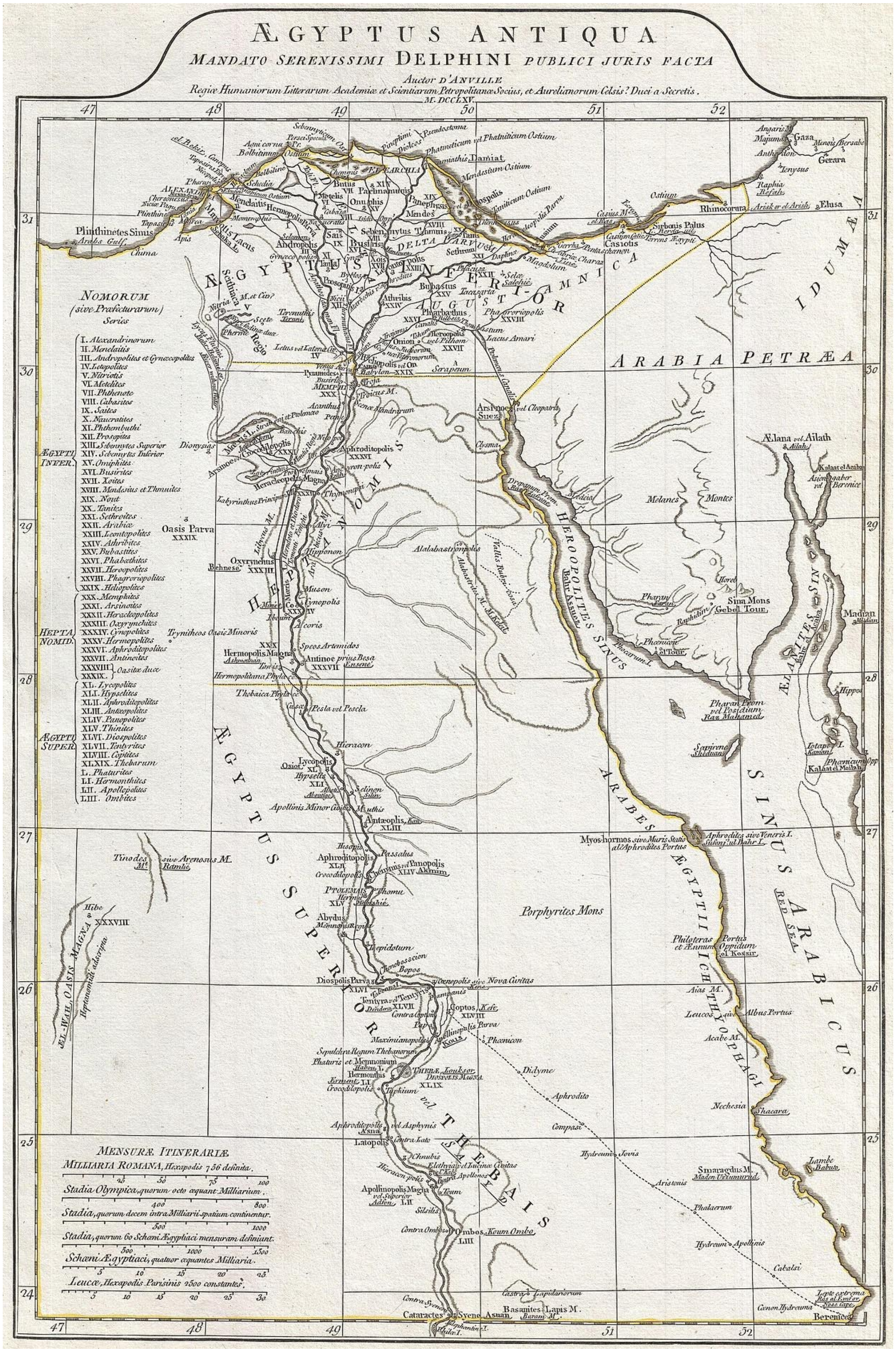


The length of one side of an octagon in relation to the diameter of a circumscribing circle is $\sqrt{(4 + 2\sqrt{2})} = 2.613...$ or 1/2.613 of the diameter of the circle for the length of each side of the octagon. For the length of the sides of the exterior octagon to be equal to the length of the inner diameter of the cylinder that supports the dome, the diameter of the circle circumscribing the exterior octagon has to be 2.613 times the length of the inner diameter of the cylinder. This ratio is close to $\varphi^2 = 2.618$. If the diameter of the circle that circumscribes the exterior octagon is 2.618 times the diameter of the inner diameter of the cylinder that supports the dome, then 20.6 meters times 2.618/2.613 = 20.63 meters for the sides of the exterior octagon.



One meter contains 60 shusi, or 54 digits, or 30 Indus Valley inches. The length of the Babylonian foot of 20 shusi, the old Northern foot of 18 digits and the Indus Valley foot of 10 Indus Valley inches, are all one-third of one meter. The length of 20.6 meters × 3 = 61.8 feet for the inner diameter of the cylinder, or 100/φ feet, and $100/\varphi \times \varphi^2 = 100\varphi$ feet. If the distance from the edge of the inner diameter the cylinder to the circle circumscribing the outer octagon is 50 of these feet, then the diameter of the circumscribing circle would be 61.8 + 50 + 50 = 161.8 feet, or 100φ feet, and the length of the walls of the outer octagon would be 161.8 /2.613 = 61.9 feet, or within one inch of the inner diameter of the cylinder.

The inner diameter of the cylinder is100/φ feet. The radius is 50/φ feet and the area is $(50/\varphi)^2 \times \pi$, or $2500/(\varphi^2) \times \pi$. Since $\varphi^2 \times 6/5 = \pi$, φ^2 cancels out, giving 2500 × 6/5 = 3000 square Babylonian feet. A more accurate statement of 100/φ is 61.8034 for the diameter, or 30.9018 for the radius, and $30.9018^2 \times \pi = 3000$ square Babylonian feet. Since a square meter contains nine square Babylonian feet, the area is 333.333 square meters. The length of 20.6 meters for the diameter gives a radius of 10.3 meters, and $10.3^2 \times \pi = 333.3$ square meters. A 12 meter height for the exterior walls is 36 Babylonian feet, and a 36 meter height for the top of the dome is 108 Babylonian feet, or 108 × 20 = 2160 Babylonian shusi, or the number of years in each astrological age.


ÆGYPTUS ANTIQUA
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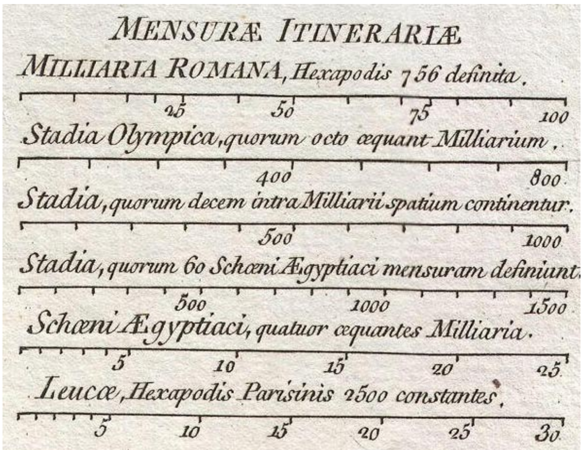

This 1765 map of Egypt was drawn by Jean-Baptiste Bourguignon d'Anville. All of the scales in the lower left corner of the map appear to be the same length. The first scale is 100 Roman miles, with each mile being 5000 Roman feet. The next scale is the Olympic stadia, which is 400 Greek cubits, or 500 remen, or 600 Greek feet, or 625 Roman feet, or 1/8 of one Roman mile. The next scale is 1/10 of one Roman mile, or 500 Roman feet. The next scale is 1/60 of one Egyptian schoeni. This is taken from Herodotus, who gave 60 furlongs in the Egyptian schoenus, called stadia by d'Anville.

Herodotus gave 1500 furlongs from Heliopolis to the sea. The next scale is Egyptian schoeni. Herodotus also gave 25 Egyptian schoenus from Heliopolis to the sea, and this is the same measure d'Anville equates with 100 Roman miles. This is taken from Pliny, who said the Egyptian schoenus contained 32 Roman stadia, or four Roman miles. It appears that d'Anville's scale of 100 Roman miles or 25 Egyptian schoenus is equal to the distance on his map due north from Heliopolis to the northern limit of Egypt.

This map was drawn before Petrie determined the length of the royal cubit, of 20 digits times the square root of 2, and before he determined the length of the Egyptian schoenus, of 12,000 royal Egyptian cubits. This gives 200 royal cubits for the 1/60 of the Egyptian schoenus. The Egyptian stadia of Eratosthenes was 1/40 of one Egyptian schoenus, or 300 royal cubits. The Egyptian *itr* was 15,000 royal cubits.

The statement of Herodotus that the distance from Heliopolis to the northern limit of Egypt is 25 Egyptian schoenus is correct, based on the Egyptian schoenus of 12,000 royal cubits, and based on the distance of 1.4142° of latitude from Heliopolis to the northern limit of Egypt. If 25 Egyptian schoenus, from Heliopolis to the northern limit of Egypt, is regarded as the baseline for d'Anville's map, the scale is:

Northern limit to Heliopolis (1.414°)		Northern limit to Elephantine (7.5°)
20	Egyptian *itr* of 15,000 royal cubits	106
25	Egyptian schoenus of 12,000 royal cubits	132.5
1000	Egyptian stadia of 300 royal cubits	5300
1500	Egyptian furlongs of 200 royal cubits	7950
848.5	Olympic stadia of 10,000 digits	4500
94.3	5000 Babylonian/Indus V./Northern feet	500
106	Roman miles of 5000 Roman feet	562.5